Math 211 Fa25 CoC R Erickson 40/40 TEST 1 Name KEY

Show ALL work for FULL credit. All problems are 5 pts each unless otherwise noted. Show all work for FULL credit.

Go for the most points... Don't get bogged down on any one problem.

1. Sec1.4 Tangents & Rates of Change (10 pts) Consider the function $W(x) = \ln(1+x^3)$. Let the point P given at x=1. (a) Calculate the slope go the secant between P and Q, m_{PQ}, for the points Q given below: (accurate to at least 5 decimal places).

$y_1 = \ln(1 + 1^4) = \ln(2) =$ 0.693147 (↙3)

x_2	$y_2 = \ln(1+x^3)$	$m_{PQ} = (y_2 - y_1) / (x_2 - x_1) =$	
1.1	$\ln(1+1.1^3) = 0.846297$	$\frac{(0.846297 - \ln(2))}{1.1 - 1.0}$	$= 1.5315018$
1.01	$\ln(1+1.01^3) = 0.708184058$	$\frac{(0.7081840 - \ln 2)}{1.01 - 1.0}$	$= 1.503687736$
1.001	$\ln(1+1.001^3) = 0.694647755$	$\frac{(0.6946476 - \ln 2)}{1.001 - 1.0}$	$= 1.500374375$
0.999	$\ln(1+0.999^3) = 0.691647556$	$\frac{(0.69164756 - \ln 2)}{0.999 - 1.0}$	$= 1.499624375$
0.99	$\ln(1+0.99^3) = 0.678185308$	$\frac{(0.6781346 - \ln 2)}{0.99 - 1.0}$	$= 1.496187268$
0.9	$\ln(1+0.9^3) = 0.5475432$	$\frac{(0.547543207 - \ln 2)}{0.9 - 1.0}$	$= 1.4560397$

8

Calculator key strokes:

0.999 [2nd] [x^3 / 1] [+] 1 [=] record #

[−] 2 [ln] [=] [÷] 0.001 [=] record #

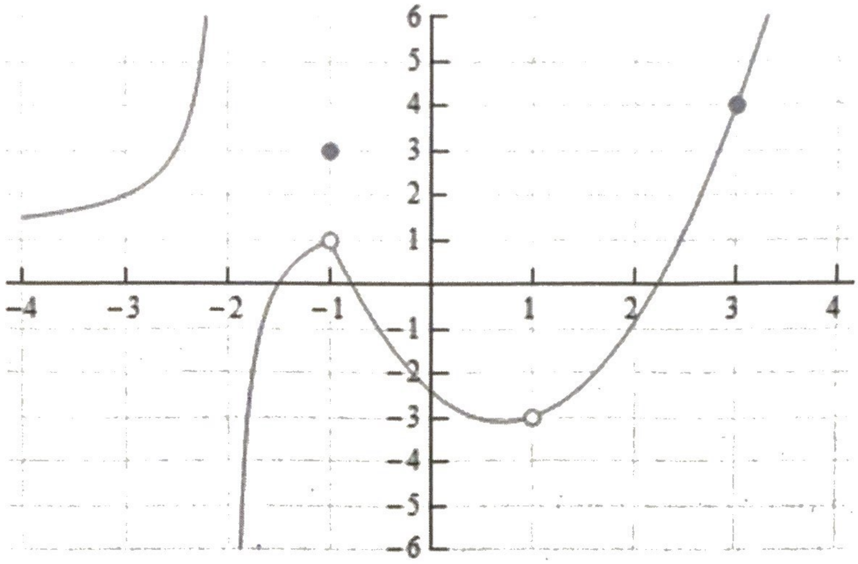
(b) Use information from (a) to estimate the slope of the tangent line to the curve W(x) at x=1

$$\boxed{\lim_{x \to 1.0} [1+x^3] = 1.5}$$

2

10

2. **Sec 1.5 Limits (10 pts)** Below is the graph of f(x). For each of the given points determine the value of f(a) and $\lim_{x\to a} f(x)$. If any of the quantities do not exist (DNE) clearly explain why.



not graded

Comments

asymptote

$\lim_{x\to -1} f(x) = f(-1)$

$\lim_{x\to 1} f(x) = f(1)$

$\lim_{x\to 4} f(x) = f(4) \Rightarrow$ continuous

3 (a) f(-2) = ∞ ; $\lim_{x\to -2} f(x) =$ DNE

2 (b) f(-1) = 3 ; $\lim_{x\to -1} f(x) =$ 1

3 (c) f(1) = DNE ; $\lim_{x\to 1} f(x) =$ -3

2 (d) f(3) = 4 ; $\lim_{x\to 3} f(x) =$ 4

10

3. Sec 1.6 Limit Properties (10 pts) Given that $\lim_{x\to -4} f(x) = 2$, $\lim_{x\to -4} g(x) = 8$ and $\lim_{x\to -4} h(x) = -6$ use the limit properties to compute each of the following limits. If it is not possible to compute the limits clearly explain why.

(a) $\lim_{x\to -4}\left[\frac{f(x)}{g(x)} - \frac{h(x)}{f(x)}\right]$

3

$= \frac{2}{8} - \frac{(-6)}{2} = \frac{1}{4} + 3 = \boxed{13/4}$

(b) $\lim_{x\to -4}[f(x)g(x)h(x)]$

$= 2\cdot 8\cdot(-6)$

2

$= \boxed{-96}$

(c) $\lim_{x\to -4}\left[\frac{1}{h(x)} + \frac{3-f(x)}{g(x)+h(x)}\right]$

$= \left[\frac{1}{-6} + \frac{3-2}{8+(-6)}\right]$

2

$= \left[-\frac{1}{6} + \frac{1}{2}\right] = \left[-\frac{1}{6} + \frac{3}{6}\right] = 2/6$

$= \boxed{1/3}$

(d) $\lim_{x\to -4}\left[2h(x) - \frac{1}{h(x)+7f(x)}\right]$

$= \left[2\cdot(-6) - \frac{1}{(-6)+7\cdot(2)}\right]$

3

$= \left[-12 - \frac{1}{8}\right]$

$= -\left[\frac{96}{8} + \frac{1}{8}\right]$

$= \boxed{-97/8}$

10

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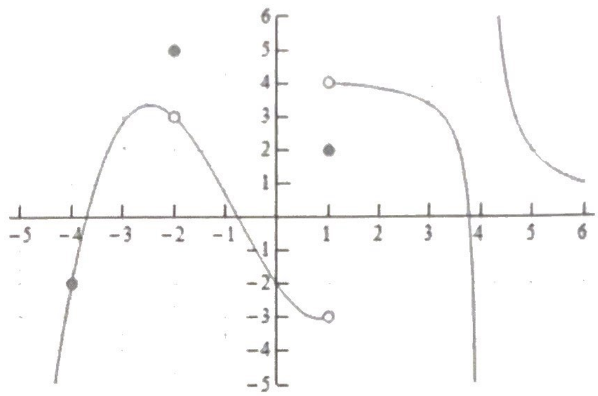
4. Sec 1.8 Continuity (10pts) (a) For the graph of f(x) given below, determine where the function is discontinuous and state the type of discontinuity.



	x	type
i.	-2	hole
ii.	1	jump
iii.	4	infinit
iv.	—	—

4

aka. zero or root

(b) Use the Intermediate Value Theorem to determine if the given equation has at least one solution in the indicated interval. {Note that you are NOT asked to find the solution only show that at least one must exist in the indicated interval.}

i.) $25 - 8x^2 - x^3 = 0$ on $[-2, 4]$

$$LH: 25 - 8(-2)^2 - (-2)^3$$
$$= 25 - 32 - (-8)$$
$$= -7 + 8$$
$$= \boxed{1}$$

$$RH: 25 - 8(4)^2 - (4)^3 = 0$$
$$25 - 8(16) - 64$$
$$= 25 - 128 - 64$$
$$= \boxed{-167}$$

3

Yes, the root is betwee $x = -2$ & 4

ii.) $\ln(2t^2 + 1) - \ln(t^2 + 4) = 0$ on $[-1, 2]$

$$LH: \ln[2\cdot(-1)^2+1] - \ln[(-1)^2+4]$$
$$= \ln[3] - \ln[5]$$
$$= \underbrace{\ln\left[\tfrac{3}{5}\right]}_{(-) \text{ since } \frac{3}{5} < 1}$$

$$RH:$$
$$\ln[2(2)^2+1] - \ln[2^2+4]$$
$$= \ln[9] - \ln[8]$$
$$= \ln(3^2) - \ln(2^3)$$
$$\ln(9/8)$$
$$(+)$$

3

yes, the root is between $[-1, 2]$

10