Math 211 Fa25 CoC R. Erickson 40/40 TEST 1 Name KEY

Show ALL work for FULL credit. All problems are 5 pts each unless otherwise noted. Show all work for FULL credit.

Go for the most points... Don't get bogged down on any one problem.

1. Sec1.4 Tangents & Rates of Change (10 pts) Consider the function $W(x) = \ln(1+x^4)$. Let the point P given at x=1. (a) Calculate the slope go the secant between P and Q, m_{PQ}, for the points Q given below: (accurate to at least 5 decimal places).

$y_1 = \ln(1 + 1^4) = \ln(2) =$ 0.693147

x_2	$y_2 = \ln(1+x^4)$	$m_{PQ} = (y_2 - y_1) / (x_2 - x_1) =$
1.1	$\ln(1+(1.1)^4) = 0.901826629$	$\frac{(0.901826629 - \ln 2)}{1.1 - 1.0} = 2.0867944$
1.01	$\ln(1+(1.01)^4) = 0.713245847$	$\frac{(0.713245847 - \ln 2)}{1.01 - 1.0} = 2.00986668$
1.001	$\ln(1+(1.001)^4) = 0.695148179$	$\frac{(0.695148179 - \ln 2)}{1.001 - 1.0} = \boxed{2.000998667}$
0.999	$\ln(1+0.999^4) = 0.691148182$	$\frac{(0.691148182 - \ln 2)}{0.999 - 1.0} = \boxed{1.998998667}$
0.99	$\ln(1+0.99^4) = 0.673248514$	$\frac{(0.673218514 - \ln 2)}{0.99 - 1.0} = 1.989866681$
0.9	$\ln(1+0.9^4) = 0.50446$	$\frac{(0.50446 - \ln 2)}{0.9 - 1.0} = 1.886817$

8

Calculator keystrokes:

0.999 $\boxed{x^2}$ $\boxed{x^2}$ $\boxed{+}$ 1 $\boxed{=}$ record #

$\boxed{-}$ 2 $\boxed{\ln}$ $\boxed{=}$ $\boxed{\div}$ 0.001 $\boxed{=}$ record # { ignore sign }

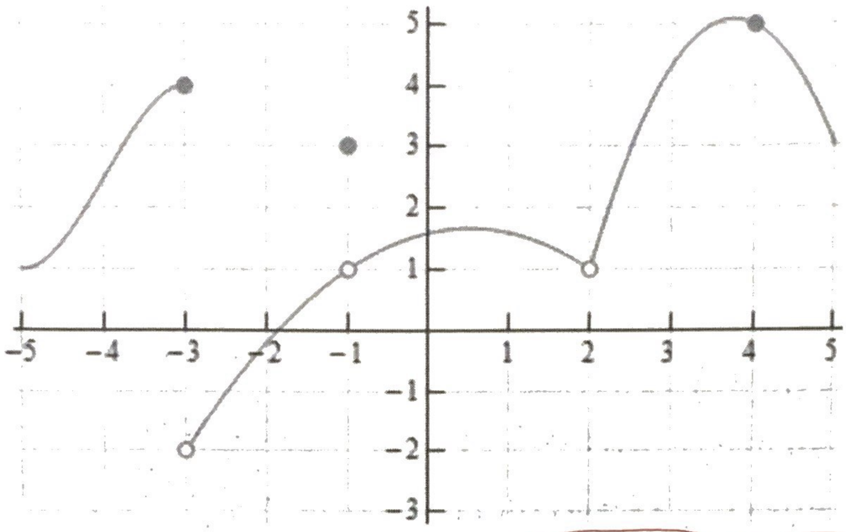
(b) Use information from (a) to estimate the slope of the tangent line to the curve W(x) at x=1

$$\boxed{\lim_{x \to 1.0} \ln(1+x^4) = 2.0}$$

2

10

2. **Sec 1.5 Limits (10 pts)** Below is the graph of f(x). For each of the given points determine the value of f(a) and $\lim_{x\to a} f(x)$. If any of the quantities do not exist (DNE) clearly explain why.



not graded

Comments

3 (a) f(-3) = 4 ; $\lim_{x\to -3} f(x)$ = DNE — LHS ≠ RHS

2 (b) f(-1) = 3 ; $\lim_{x\to -1} f(x)$ = 1 — $\lim_{x\to -1} f \neq f(-1)$

3 (c) f(2) = DNE ; $\lim_{x\to 2} f(x)$ = 1 — f(2) not defined

2 (d) f(4) = 5 ; $\lim_{x\to 4} f(x)$ = 5 — $\lim_{x\to 5} f = f(5)$ ⇒ continuous

10

3. Sec 1.6 Limit Properties (10 pts) Given that $\lim_{x\to-4} f(x) = 1$, $\lim_{x\to-4} g(x) = 10$ and $\lim_{x\to-4} h(x) = -7$ use the limit properties to compute each of the following limits. If it is not possible to compute the limits clearly explain why.

(a) $\lim_{x\to-4} [\frac{f(x)}{g(x)} - \frac{h(x)}{f(x)}]$

3

$= \left[\frac{1}{10} - \frac{-7}{1}\right] = \left[\frac{1}{10} + 7\right] = \boxed{\frac{71}{10}}$

(b) $\lim_{x\to-4} [f(x)g(x)h(x)]$

$= [(1)(10)(-7)$

2

$= \boxed{-70}$

(c) $\lim_{x\to-4} [\frac{1}{h(x)} + \frac{3 - f(x)}{g(x) + h(x)}]$

$= \left[\frac{1}{-7} + \frac{3-(1)}{10+(-7)}\right]$

2

$= \left[-\frac{1}{7} + \frac{2}{3}\right] = \left[\frac{-3+14}{21}\right] = \boxed{\frac{11}{21}}$

(d) $\lim_{x\to-4} [2h(x) - \frac{1}{h(x) + 7f(x)}]$

$= \left[2\cdot(-7) - \frac{1}{-7+7\cdot(1)}\right]$

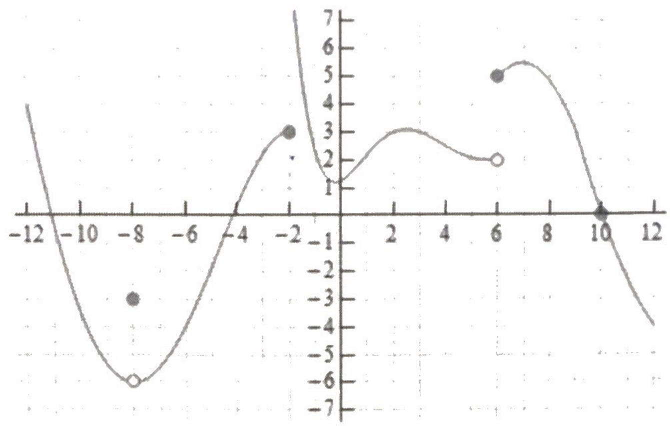
3

$= \left[-14 - \frac{1}{0}\right]$

$\boxed{\text{DNE}}$

10

4. **Sec 1.8 Continuity (10pts)** (a) For the graph of f(x) given below, determine where the function is discontinuous and state the type of discontinuity.



	x	type
i.	-8	hole
ii.	-2	infinite
iii.	6	jump
iv.	—	—

4

(b) Use the Intermediate Value Theorem to determine if the given equation has at least one solution in the indicated interval. {Note that you are NOT asked to find the solution only show that at least one must exist in the indicated interval.}

i. $25 - 8x^2 - x^3 = 0$ on [−2, 4]

LH: $25 - 8(-2)^2 - (-2)^3$

$= 25 - 32 + 8$

$= -7 + 8 = \boxed{1}$

RH: $25 - 8(4)^2 - 4^3$

$= 25 - 128 - 64$

$= \boxed{-167}$

3

yes, the root is between x = -2 & 4

ii. $w^2 - 4\ln(5w + 2) = 0$ on [0, 4]

LH: $0^2 - 4\ln[5\cdot 0 + 2]$

$= -4\ln 2$

$= \ln(2)^{-4}$

$= \ln\left(\frac{1}{16}\right)$

$(-)$

RH: $4^2 - 4\ln[5\cdot 4 + 2]$

$= 16 - 4\ln[22]$

$= 16 + \ln 22^{-4}$

$= 16 + \ln\left(\frac{1}{22^4}\right)$

$= (+)$

yes, the root is between [0, 4]

3

10