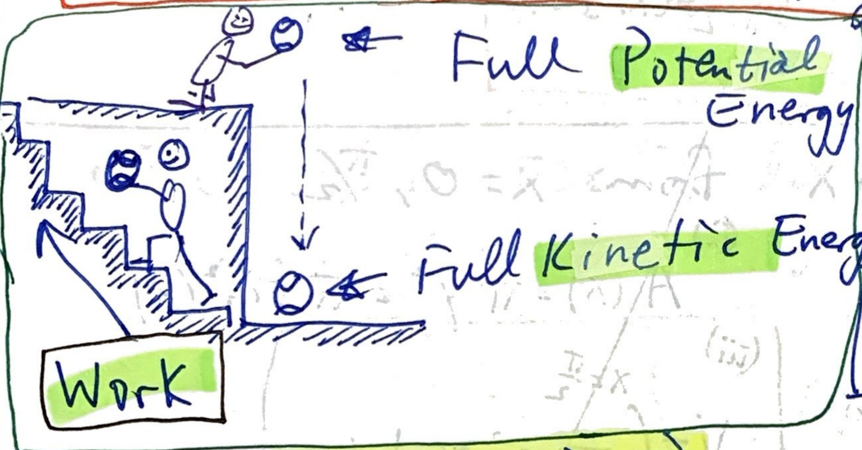
5.4 Work (Application #3)

- In mechanics there is a quatity known as **Work** – Work is an energy form like Kinetic energy, potential energy, thermal energy atomic energy.

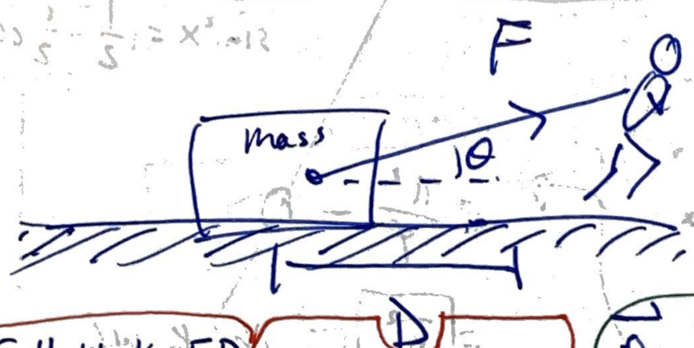
- **Conservation of Energy** says that we can transform energy from one type to another. But we cannot destroy energy.



At any position we have:

$$\boxed{K_0 + P_0 = K_f + P_f + W_{perform}}$$

⊗ Work is $\vec{F} \cdot \vec{D}$ where $\vec{D}$ = displacement
$\vec{F}$ = Force applied.



vectors

$$\boxed{W = \vec{F} \cdot \vec{D}}$$

From Trig

$$\vec{A} \cdot \vec{B} = \|\vec{A}\| \, \|\vec{B}\| \cos\theta$$

non-vectors

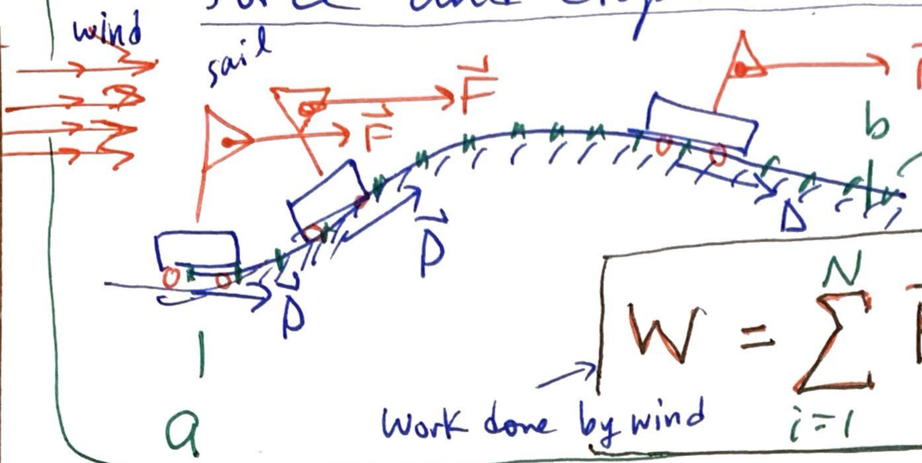
- Full Work = FD

$F \parallel D$

- No Work = 0

$F \perp D = 0$

- Over the course of transversal the force and displacement angles may change:



To approximate work we chop path up into pieces and sum

$$\boxed{W = \sum_{i=1}^{N} \vec{F} \cdot \Delta \vec{D}}$$

Work done by wind

- To get most acc'y we take the limit and the results is a Riemann sum:

$$\boxed{W = \int \vec{F}(x) \cdot d\vec{x}}$$

- For this section let $d\vec{x}$ always be in the Same direction (vertical or horizontal)

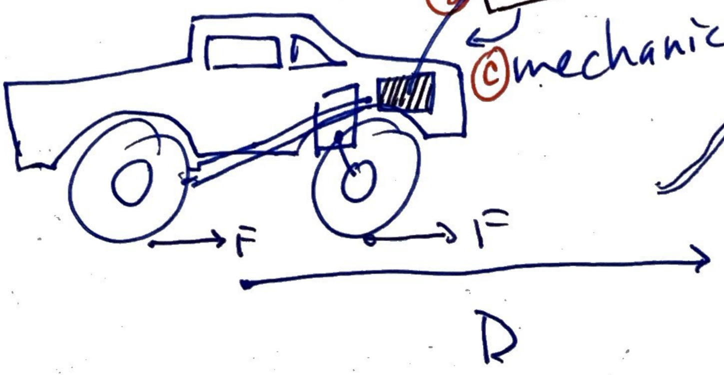
So
$$\boxed{W = \int_{x=a}^{b} F(x)\,dx}$$

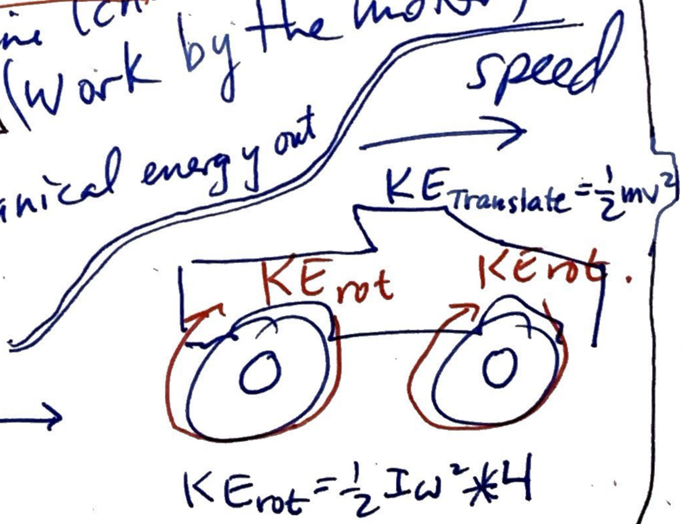
An EX Transfer of Energy:

ⓐ gasoline (chemical) energy in

ⓑ motor (work by the motor)

ⓒ mechanical energy out



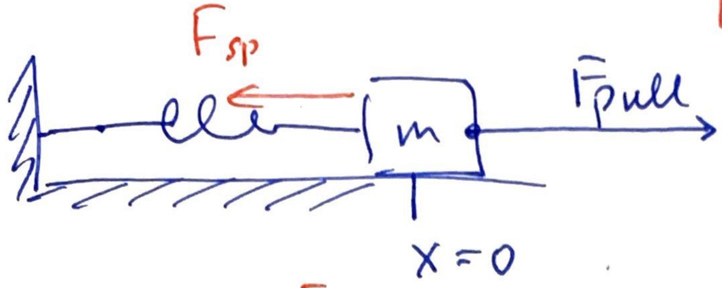


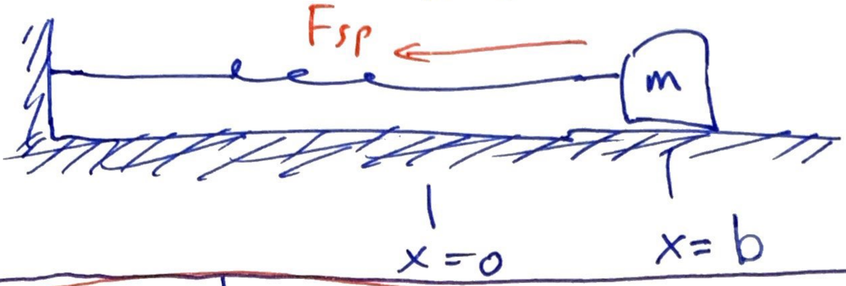
Ex Spring - mass

Hooke's Law

$$\boxed{F_{sp} = -kx}$$

The more a spring is stretched the harder it is to pull





Find the work done by the spring

$$\boxed{W = \int_{x=0}^{b} F(x)\,dx}$$

let $b = 5\text{ cm} = \underline{\underline{0.05\text{ m}}}$

let $k = 250\text{ N/m}$

$$= \int_0^{0.05} kx\,dx$$

$$= \int_0^{0.05} 250x\,dx$$

$$= 250 \frac{x^2}{2}\Big|_0^{0.05}$$

$$= 250 \frac{(0.05\text{ m})^2}{2}$$

$$= 0.3125\ \text{N-m}$$

$\underline{F \cdot D}$

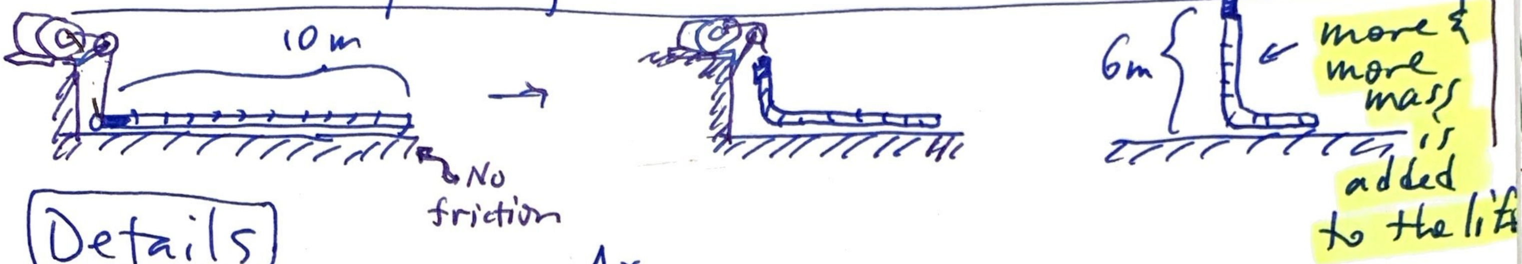
Joule

$$\boxed{0.3125\ \text{J}}$$

1 Newton of force applied over 1 meter is 1 Joule of Work

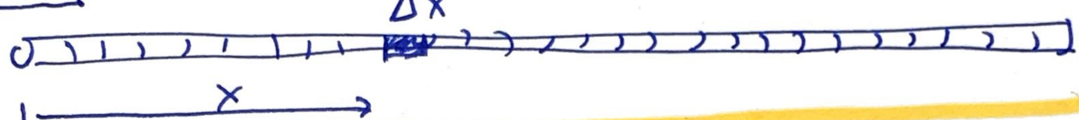
Ex Lifting a chain from deck to ship

A 10m chain lies on a deck. The end is attached to a cable (massless) which lifts the cable end to a height of 6m. The weight of the chain is 8000kg. What work does the motor perform that is pulling the cable attached to the chain?



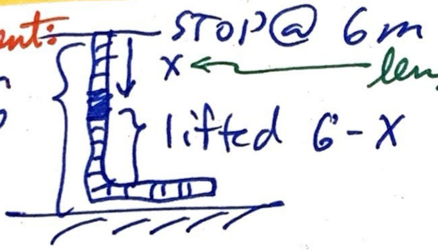
Details

- Chain:



- **Strategy**: Chop the chain up into pieces and add up how high each piece will be lifted off the deck until the 1st piece hits the 6m limit

- The displacement:



- The Force: Newton's Law

$$\boxed{F_g = mg} \quad g = 9.8\,\frac{m}{s^2}$$

- **Work (Vertical)**

vertical height

horizontal position of chain element.

$$\Delta W = \Delta F_g \cdot h$$

$$\Delta W = \Delta m \cdot g \cdot (6 - x) \qquad m = \rho x$$

$$\Delta W = \rho \Delta x \cdot g \cdot (6 - x)$$

$$\boxed{W = \int_{x=0}^{6m} \rho g (6-x)\,dx}$$

$$\to \left(\frac{8000\,kg}{10\,m}\right)\left(\frac{9.8\,m}{s^2}\right)\int_0^6 (6-x)\,dx$$

$$= 7840\,\frac{kg}{s^2}\left(6x - \frac{x^2}{2}\right)\Big|_0^6$$

$$= 7840\left[\left(36 - \frac{36}{2}\right) - (0-0)\right]$$

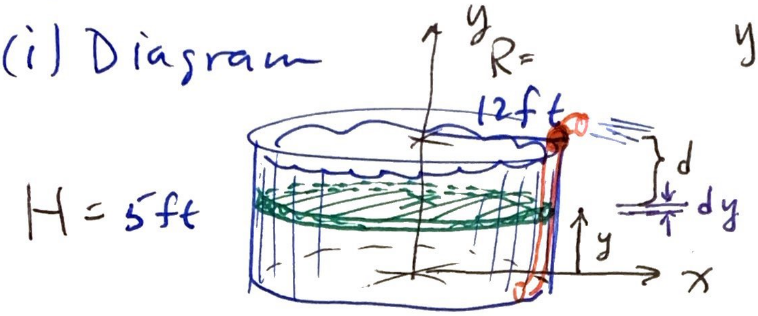
$$= 141{,}120\ \frac{kg\cdot m^2}{s^2}\quad N\cdot m$$

$$= \boxed{141\ kJoules}$$

Math 211 — **CW 14b** — Name ______

Find the work to drain water from a cylindrical swimming pool of radius 12 ft and depth of 5 ft. Use $\rho = 62.5\ \text{lbs}/\text{ft}^3$

(i) Diagram



$y + d = H \rightarrow \boxed{d = H - y}$ height

$d_1 = 5 \quad d_2 = 0$

The Basics
- $F = mg$
- $m = \rho V$
- $\Delta V = A(\Delta y)$
- $A = \pi r^2$

$$\boxed{\text{Work} = F \cdot \text{dist}}$$

Force : gravity
Dist : height

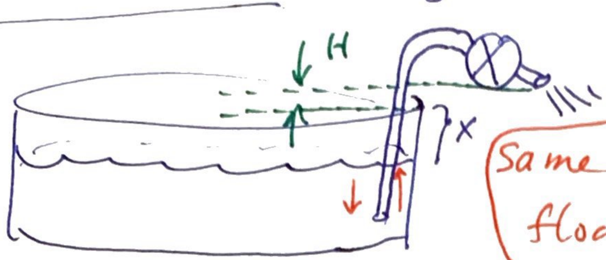
strategy: Lift a wafer shaped piece of water over the edge

$$\begin{aligned}
\Delta W &= \Delta F \cdot \text{Dist} \\
&= \Delta m g \cdot d \\
&= \rho \Delta V g d \\
&= \rho A \Delta y\, g (H - y)
\end{aligned}$$

$$\Delta W = \rho \pi R^2 \Delta y\, g (H - y)$$

$$\boxed{W = \rho \pi R^2 g \int_5^0 (H - y)\, dy}$$

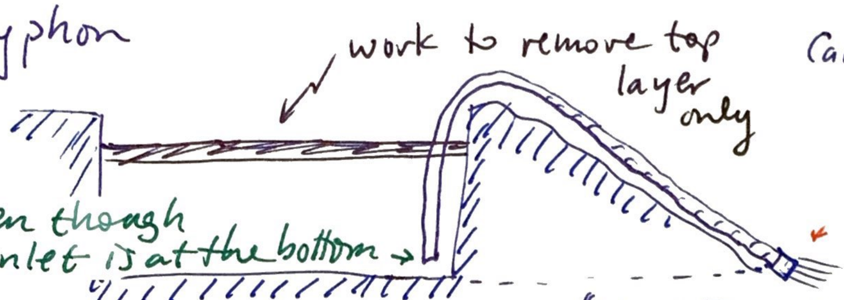
Notes on draining fluids

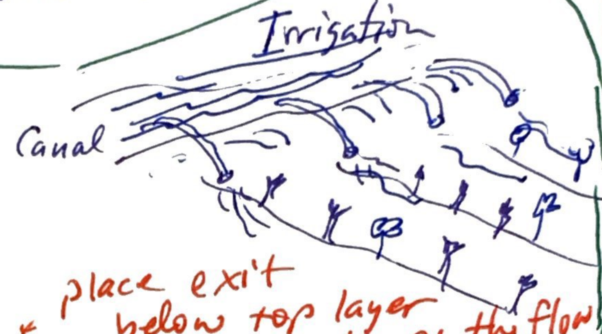


Work is only that of removing top slice. Same answer as if the tube floats on the surface.

• extending the inlet to the bottom

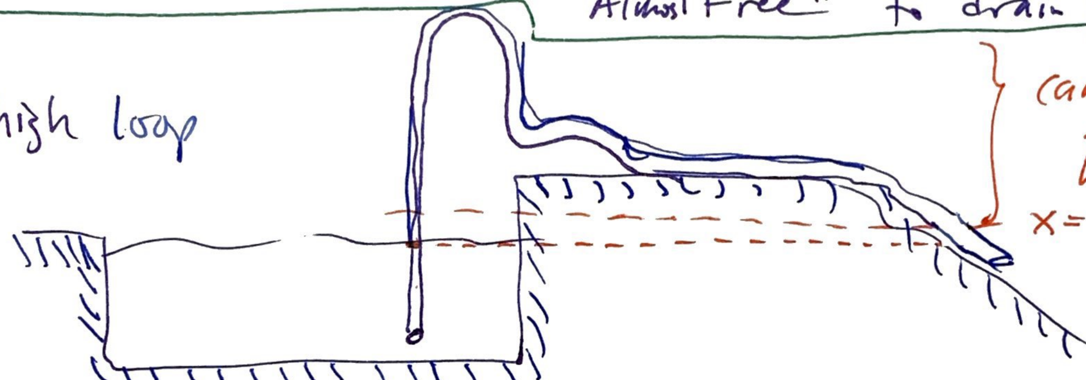
* syphon



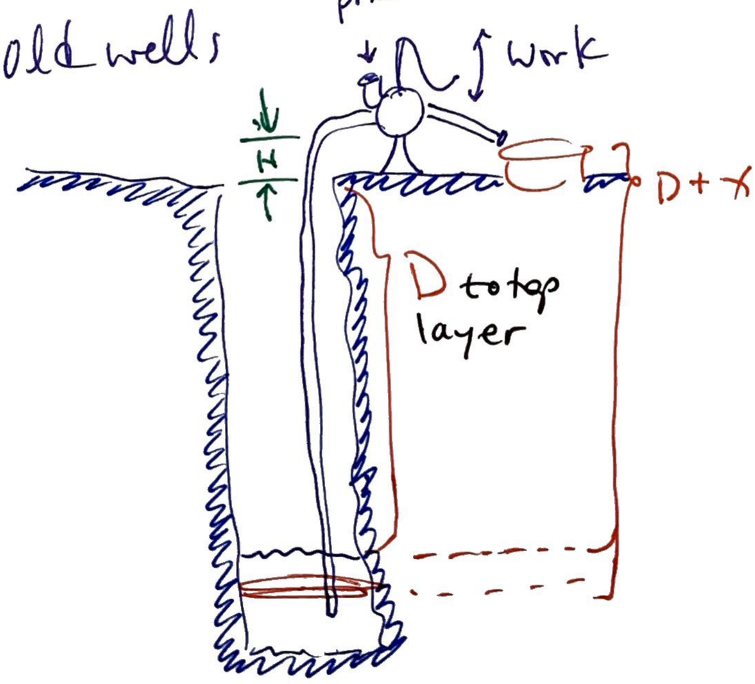


"Almost Free" to drain pool.

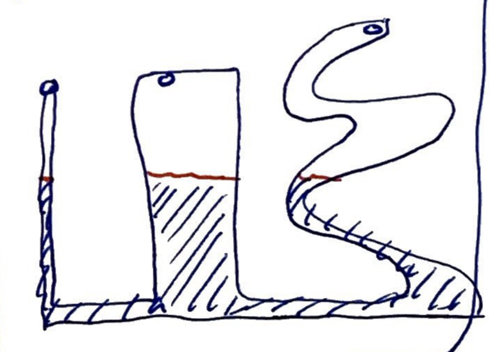
* high loop



* old wells

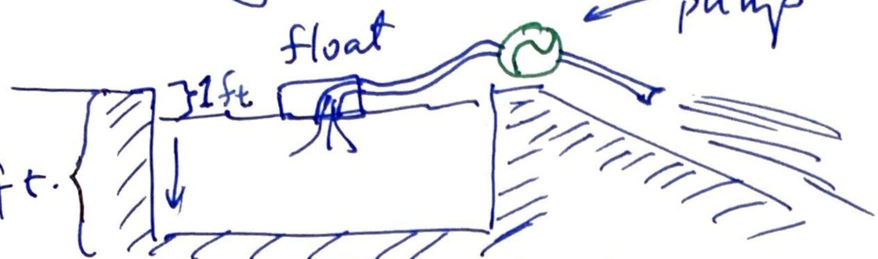


Pascal's principle



EX Pumping water from a pool.

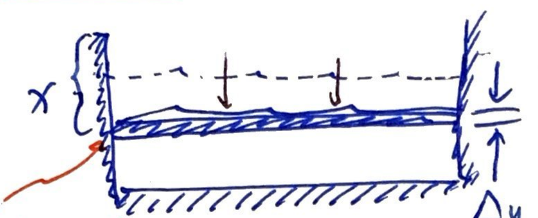
S.I. units: m, kg, s
vs
USCS: ft, lbs, s



$A_0 = \pi r^2$

$\text{Circ} = 2\pi r$

- Circular pool is 24 ft in diameter
- Density of water is 62.5 lbs/ft^3
- We lift a horizontal slice x units high



$\Delta W = \Delta \text{weight} \cdot \text{height}$

$\Delta W = \rho \Delta \text{Vol} \cdot g \cdot (5-y)$

$\Delta W = \rho \cdot A \cdot \Delta y \cdot g \cdot (5-y)$

$A = \pi r^2$; $r = \text{fixed} = (\text{Circ.}/\pi)/2 = \frac{24 \text{ ft}}{2\pi} = \frac{12}{\pi} \text{f}^2$

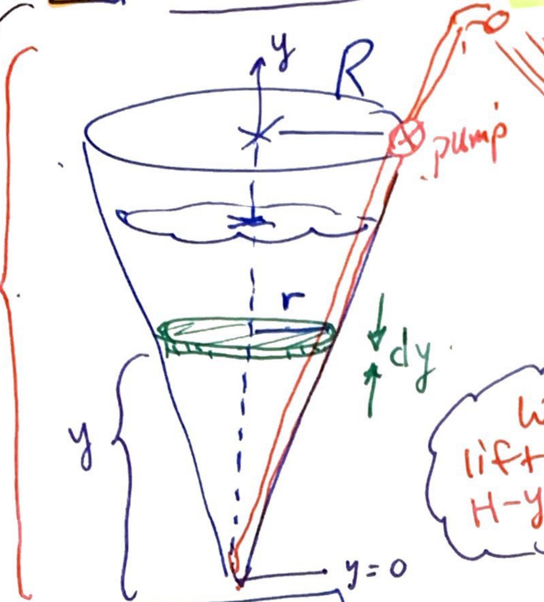
Work

- $W = \sum_{\text{slices}} \Delta W \rightarrow W = \int_{y=0\text{ft}}^{4\text{ ft}} \rho \cdot A(y) \cdot g(5-y)\,dy$

$$W = \underbrace{\overset{\rho ?}{\left(62.5 \frac{\text{lbs}}{\text{f}^3}\right)} \left(\pi 12^2 \text{f}^2\right) \overset{g ?}{\left(32.5 \frac{\text{ft}}{\text{s}^2}\right)}} \int_0^4 (5-y)\,dy$$

$$= \boxed{108{,}000\,\pi \text{ lb-ft}}$$

Ex Drain a Conical Tank of height D & Radius R



$$\Delta W = \Delta F \cdot \text{distance}$$
$$= \Delta F \cdot (H-y)$$
$$= \Delta m \cdot g \cdot (H-y)$$
$$= \rho \Delta V \cdot g \cdot (H-y)$$

Work to lift one slice H-y units

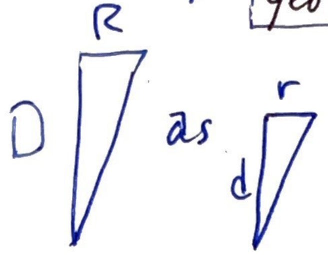
$$\Delta W = \rho\, A(x) \cdot \Delta x \cdot g \cdot (H-y)$$

geometry

Cross-section

• $A(y) = \pi r^2$

circular, but if square then $A(x) = (2r)^2$



so $\boxed{\dfrac{r}{d} = \dfrac{R}{D}}$

$d = D - y$

$$\boxed{r(y) = \frac{R}{D}(D-y)}$$

Work / slice

$$\bullet\; W = \rho g \cdot \int_{y=\text{depth }1}^{\text{depth }2} A(y)(H-y)\,dy$$

Any cross-section cone

$$= \rho g \cdot \int \pi \left[\frac{R}{D}(D-y)\right]^2 (H-y)\,dy$$

$$\boxed{W = g\rho\pi R^2 \int_{\text{depth }1}^{\text{depth }2} \left(1-\frac{y}{D}\right)^2 (H-y)\,dy}$$

R, D, H given

Formula Set-Up

EX For the swimming pool C.Work …

(a) Lets now use a fluid that is really dense at the bottom and less dense at the top. ((like tar → oil → gas, in an oil refinery "distiller"))

- Here lets say $\rho = \rho_0 \cdot y^{3/2}$

then ρ stays inside the integral

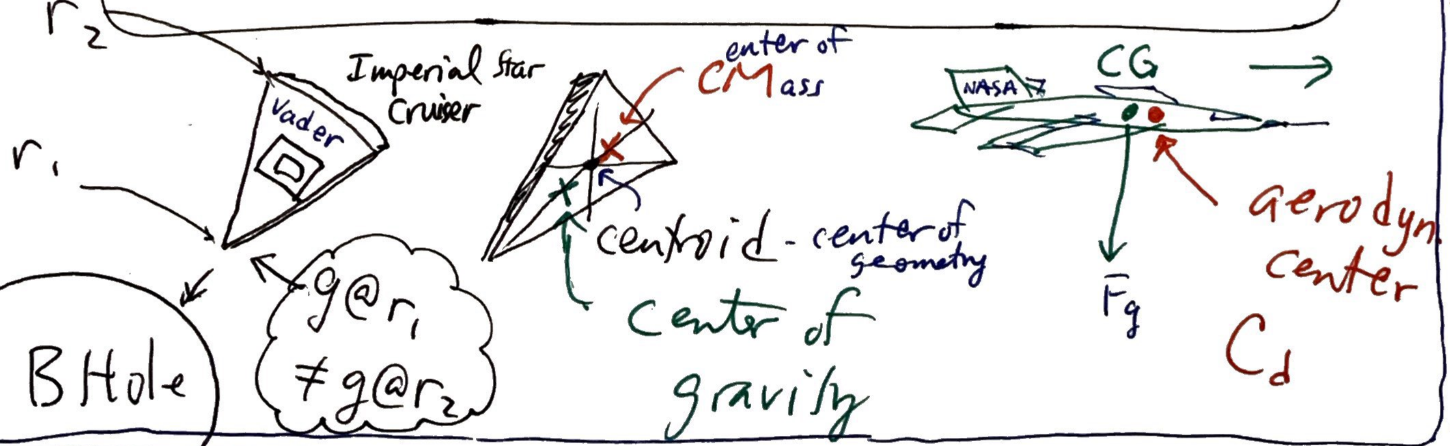
$$W = \pi R^2 g \int_5^0 [\rho_0 y^{3/2}] (H-y)\, dy$$

(b) If Radius changes with y, it stays inside too:

$$W = \pi g \int_5^0 [R(y)]^2 [\rho_0 y^{3/2}] (H-y)\, dy$$

(c) If Near a massive black hole g = changes also:

$$\boxed{W = \pi \int_5^0 g(y)\, [R(y)]^2 [\rho(y)] (H-y)\, dy}$$



Practice Problem

9

EX A bucket weighs 70 lbs when filled with water and is lifted from a 60 ft deep well.

(a) what is the work? — Constant force

physics

$W = mgh$

$F = mg$

$W = F \cdot d = mg \cdot h$

$$W = \int F(x)\, dx$$

$$W = \int_{x=0}^{x=60} (mg)\, dx$$

$$= 70 \cdot x \Big|_0^{60} = 70(60-0) = \boxed{4200 \text{ ft-lb}}$$

Just is $m \cdot g \cdot h$

(b) The bucket leaks water! It weighs only 35 lbs by the time you get it to the top. What is the work now?

Assume the loss of water is linear. Lets try out different linear arrangements...

$$F(x) = 35\text{lbs}\left(1 + \frac{x}{60}\right)$$

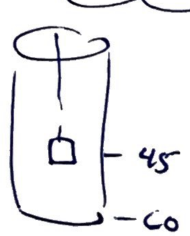
TEST:

$x = 0$ @ top: $35\left(1 + \frac{0}{60}\right) = 35$

$x = 60$ @ bottom: $35\left(1 + \frac{60}{60}\right) = 70$

So
$$W = \int_{x=0}^{60} \overbrace{35\left(1 + \frac{x}{60}\right)}^{\text{weight = Force}(x)} dx$$

equivalence:



needed only 2/3 work

$$= 35\text{ lbs} \cdot \int_0^{60} \left(1 + \frac{x}{60}\right) dx$$

$$= 35\text{lbs}\left(x + \frac{x^2}{2 \cdot 60}\right)\Big|_0^{60} = 35\left(60 + \frac{60^{\cancel{2}}}{2 \cdot \cancel{60}} - 0\right)$$

$$= 35\text{ lbs} \cdot 90\text{ft} = \overset{\text{BTW}}{70\text{ lbs} \cdot 45\text{ft}} = \boxed{3150 \text{ ft-lbs}}$$