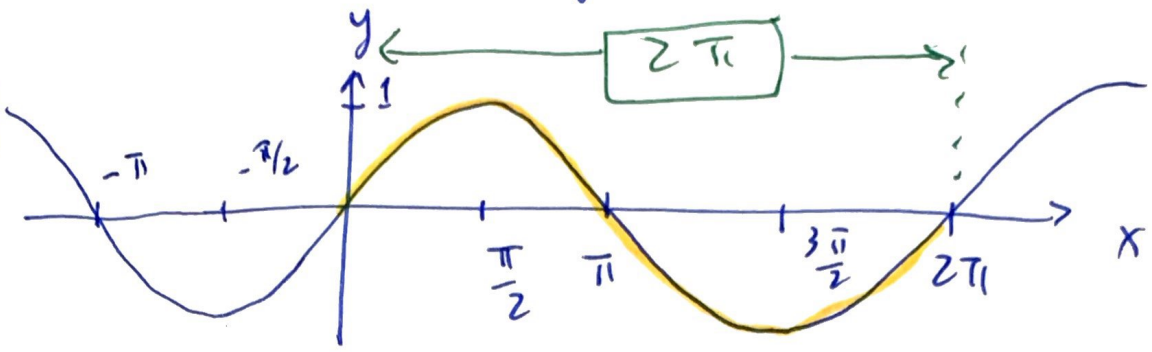


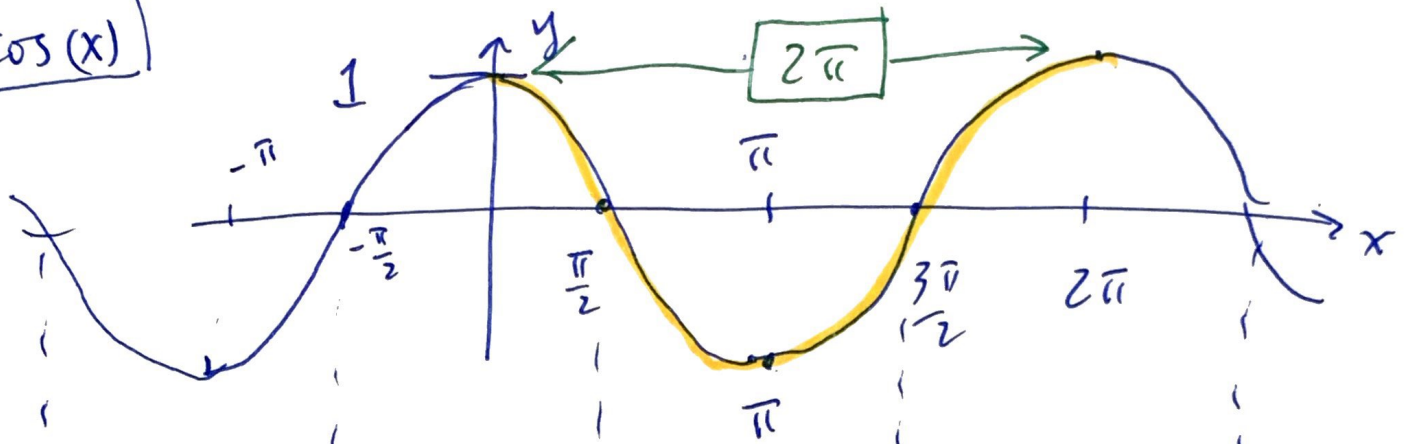
2.4 Derivatives of Trig Functions

⊗ Review

$f = \sin(x)$



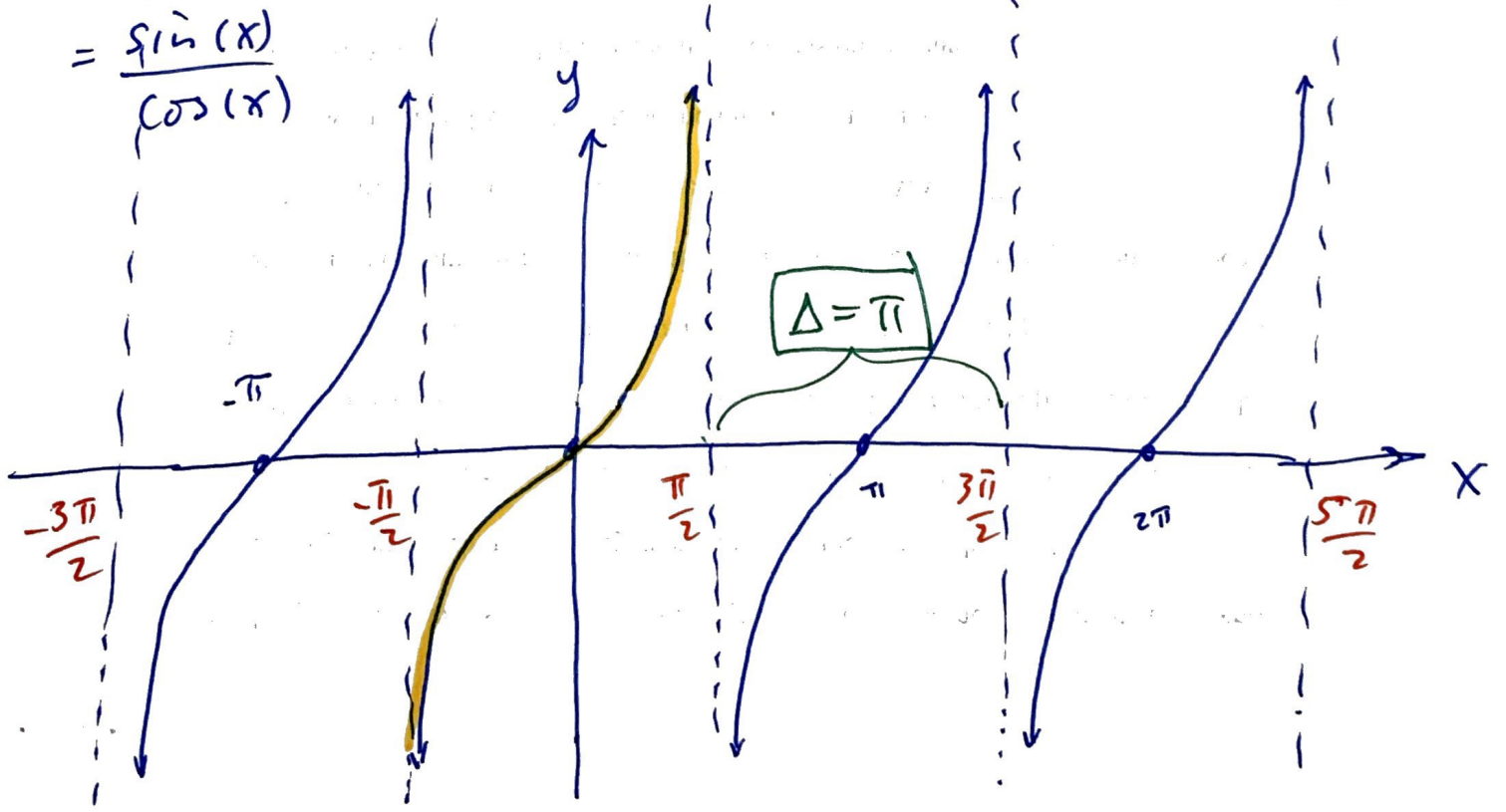
$f = \cos(x)$



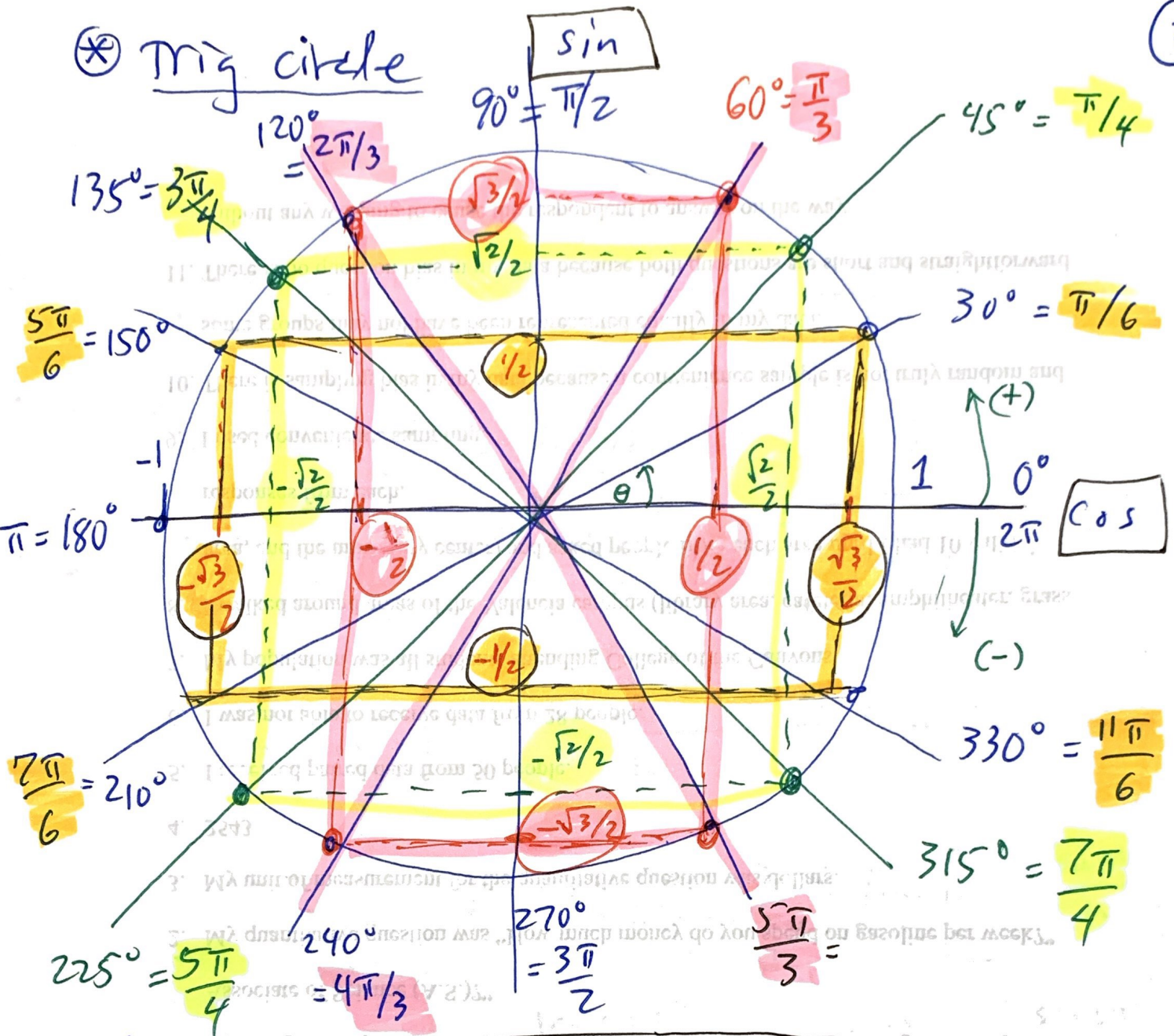
$f(x) = \tan(x)$

Shift: $\cos(x) = \sin(x + \frac{\pi}{2})$

$= \frac{\sin(x)}{\cos(x)}$



* Trig circle



* examples

$$\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$$

$$\cos\left(\frac{7\pi}{6}\right) = -\frac{\sqrt{3}}{2}$$

$$\sin\left(\frac{5\pi}{6}\right) = \frac{1}{2}$$

$$\cos\left(\frac{5\pi}{3}\right) = \frac{1}{2}$$

$$\sin\left(\frac{7\pi}{6}\right) = -\frac{1}{2}$$

$$\cos\left(-\frac{5\pi}{6}\right) = -\frac{\sqrt{3}}{2}$$

$$\sin\left(\frac{5\pi}{3}\right) = -\frac{\sqrt{3}}{2}$$

$$\cos\left(-\frac{\pi}{3}\right) = \frac{1}{2}$$

$$\sin\left(-\frac{11\pi}{6}\right) = \frac{1}{2}$$

$$\sin\left(\frac{7\pi}{3}\right) = \frac{\sqrt{3}}{2}$$

$$\sin\left(-\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$$

$$\cos\left(-\frac{13\pi}{6}\right) = \frac{\sqrt{3}}{2}$$

2.4 Derivatives of Trig Functions

1

I. $(\sin(x))'$

• Def $\frac{d \sin(x)}{dx} = \lim_{h \rightarrow 0} \left(\frac{\sin(x+h) - \sin(x)}{h} \right)$

• Identity use $\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b)$

• So $\frac{d \sin(x)}{dx} = \lim_{h \rightarrow 0} \left[\frac{[\sin(x)\cos(h) + \cos(x)\sin(h)] - \sin(x)}{h} \right]$

$= \lim_{h \rightarrow 0} \left[\frac{\sin(x)\cos(h) - \sin(x)}{h} + \cos(x) \frac{\sin(h)}{h} \right]$

$= \lim_{h \rightarrow 0} \left[\sin(x) \left[\frac{\cos(h) - 1}{h} \right] + \cos(x) \frac{\sin(h)}{h} \right]$

$= \sin(x) \lim_{h \rightarrow 0} \left[\frac{\cos(h) - 1}{h} \right] + \cos(x) \lim_{h \rightarrow 0} \left[\frac{\sin(h)}{h} \right]$

$= \sin(x) \cdot 0 + \cos(x) \cdot 1$

h	$\frac{\cos(h) - 1}{h}$
0.1	$[\cos(0.1) - 1] / 0.1 = -0.04$
0.01	$[\cos(0.01) - 1] / 0.01 = -0.004$
0.001	$[\cos(0.001) - 1] / 0.001 = -0.00005$
-0.001	$-[\cos(-0.001) - 1] / 0.001 = +0.00005$
-0.01	$-[\cos(-0.01) - 1] / 0.01 = 0.004$
-0.1	$-[\cos(-0.1) - 1] / 0.1 = +0.04$

$\lim_{h \rightarrow 0} \frac{\cos(h) - 1}{h} = 0$

$\frac{d \sin(x)}{dx} = \cos(x)$

EX Find $\lim_{x \rightarrow 0} \left[\frac{\sin(2x) \sin(6x)}{x^2} \right]$

• "Trick" $\lim_{x \rightarrow 0} \left[\frac{\sin(2x)}{x} \cdot \frac{\sin(6x)}{x} \right]$

Substitution: let $u = 2x$ and $v = 6x$

$$\lim_{x \rightarrow 0} \left[\frac{\sin(u)}{u/2} \cdot \frac{\sin(v)}{v/6} \right]$$

$$= 2 \cdot 6 \lim_{u \rightarrow 0} \left[\frac{\sin(u)}{u} \right] \cdot \lim_{v \rightarrow 0} \left[\frac{\sin(v)}{v} \right]$$

$$= 12 \cdot 1 \cdot 1 = \boxed{12}$$

II $(\cos(x))'$

We can use similar identities and show that

$$\frac{d \cos(x)}{dx} = -\sin(x)$$

III $(\tan(x))'$

3

$$\frac{d \tan(x)}{dx}$$

$$= \frac{d \left(\frac{\sin(x)}{\cos(x)} \right)}{dx}$$

$$= \frac{(\sin(x))' \cos(x) - \sin(x) (\cos(x))'}{\cos^2 x}$$

$$= \frac{\cos(x) \cos(x) - \sin(x) (-\sin(x))}{\cos^2 x}$$

$$= \frac{c^2 x + s^2 x}{\cos^2 x}$$

$$= \frac{1}{\cos^2(x)}$$

$$= \sec^2(x)$$

$$\frac{d \tan(x)}{dx} = \sec^2(x)$$

CW2b
(Cont.)

#4 Find $\frac{d}{dx} (\sin(x) - x^2)$

$$= \boxed{\cos(x) - 2x}$$

#5 derive the formula for $\frac{d \sec(x)}{dx}$

Hint: use $\sec(x) = \frac{1}{\cos(x)}$ then use quotient rule

$$\frac{d\left(\frac{1}{\cos(x)}\right)}{dx} = \frac{1' \cdot \cos(x) - 1 \cdot (\cos(x))'}{\cos^2(x)}$$

IV

Summary of trig derivatives

4

$\frac{d}{dx} (\sin(x)) = \cos(x)$	$\frac{d}{dx} (\csc(x)) = -\csc(x)\cot(x)$
$\frac{d}{dx} (\cos(x)) = -\sin(x)$	$\frac{d}{dx} (\sec(x)) = \sec(x)\tan(x)$
$\frac{d}{dx} (\tan(x)) = \sec^2(x)$	$\frac{d}{dx} (\cot(x)) = -\csc^2(x)$

EX

diff't $f(x) = \frac{1 - \sec(x)}{\tan(x)}$

$$\begin{aligned} \left(\frac{1 - \sec(x)}{\tan(x)}\right)' &= \frac{(1 - \sec(x))' \tan(x) - (1 - \sec(x)) (\tan(x))'}{\tan^2(x)} \\ &= \frac{(0 - \sec(x) \tan(x)) \tan(x) - (1 - \sec(x)) \sec^2(x)}{\tan^2(x)} \\ &= \frac{-\sec(x) \tan^2(x) - \sec^2(x) + \sec^3(x)}{\tan^2(x)} \\ &= \frac{-\sec(x) [\tan^2(x) - \sec^2(x)] - \sec^2(x)}{\tan^2(x)} \\ &= \frac{\sec(x) - \sec^3(x)}{\tan^2(x)} \end{aligned}$$

$c^2 + s^2 = 1 \quad \div c^2$
 $1 + \frac{s^2}{c^2} = \frac{1}{c^2}$
 $+ \tan^2 = \sec^2$
 $\Rightarrow \boxed{\tan^2 - \sec^2 = -1}$

⊗ 2nd level of trig

6

$$\csc(x) \equiv \frac{1}{\sin(x)}, \quad \sec(x) = \frac{1}{\cos(x)}$$

$$\cot(x) = \frac{1}{\tan(x)} = \frac{\cos(x)}{\sin(x)}$$

since

$$\tan(x) = \frac{\sin(x)}{\cos(x)}$$

use

$$\frac{d}{dx} (\sin(x)) = \cos(x)$$

$$\frac{d}{dx} (\cos(x)) = -\sin(x)$$

$$\frac{d}{dx} (\tan(x)) = \sec^2(x)$$

$$\cos^2 x + \sin^2 x = 1$$

$$\div \cos^2$$

$$1 + \frac{\sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$$

$$1 + \tan^2 x = \sec^2 x$$

$$\div \sin^2 x$$

$$\cot^2 x + 1 = \csc^2 x$$

to get

$$\frac{d}{dx} (\csc(x)) = -\csc(x) \cot(x)$$

$$\frac{d}{dx} (\sec(x)) = \sec(x) \tan(x)$$

$$\frac{d}{dx} (\cot(x)) = -\csc^2(x)$$