

- * Except for your name DO NOT WRITE on these test sheets.
- * Please do all your work on blank paper and staple those sheets to the back of these sheet before submitting.
- * Show ALL work for FULL credit.
- * All problems, or parts therein, are 5 points *each* unless otherwise noted. Please BOX all answers.

Skill Evaluation

1. (a) Find the limit: $\lim_{x \rightarrow \infty} \frac{\sqrt{x} - 3x + 4}{3\sqrt{x} + 3x - 3}$ (b) Find the limit: $\lim_{x \rightarrow 1^+} \frac{\sqrt{x} - 1}{x - 1}$
2. (a) Differentiate: $y = \int_1^x \sqrt{\cot(\sin t)} dt$ (b) Differentiate: $y = \sin(2x + 1)$
3. (a) Find $g''(x)$: $g(x) = (x^3 + 1)^2$ (b) Differentiate: $f(x) = \frac{\sqrt{2t + 1}}{(t + 1)^3}$
4. (a) Find the derivative: $y = \cos\left(\frac{\sqrt{x} + 1}{\sin x}\right)$ (b) Find the derivative: $y = \ln(e^{4x} + 3)$
5. (a) Evaluate: $\int_0^1 x^2(1 + 2x^3)^5 dx$ (b) Evaluate: $\int x \sin(x^2) dx$
6. (a) Evaluate: $\int \cos(x) \sin^6(x) dx$ (b) Find the differential: $f(x) = \frac{1}{x^2}$
7. (a) Use the definition of the derivative to find the derivative of the function $f(x) = \frac{1}{x}$
(b) Approximate the area under $f(x) = \frac{1}{x}$ from $x = 1$ to $x = 2$, by using 5 LHS rectangles.

More fun on the back >>>>

Applications

8. Consider $f(x) = \frac{1}{3}x^3 - x^2 + x + 1$

- (a) find the intervals where the function f is increasing and where it is decreasing, find the location and value of the relative extrema of f
- (b) find the intervals where the graph of f is concave upward and where it is concave downward, and label the inflection points, if any, and *then sketch the curve* using this information.

9. Find the *slope* of the tangent line at the point $(1, 2)$ to the curve $x^2 + 2xy - y^2 + x = 2$

10. (10 points) 900 square centimeters of material is to be used to make an open-topped box which has a square base. What should be the dimensions of the box in order that its volume will be as large as possible?

11. Sales of Mobile Processors (in billions of dollars) is given by $S(t) = 6.8(t + 1.03)^{0.49}$ ($0 \leq t \leq 4$) with $t = 0$ corresponding to 2003.

- (a) Show using calculus that S is increasing on the interval $(0,4)$ and interpret your results.
- (b) Show using calculus that the graph of S is concave down on the interval $(0,4)$ and interpret the results?

12. (10 points) Find the area enclosed by the curves $y = x^2 - 1$, $y = -1$ between $x = 1$ and $x = 2$.

13. (10 points) Using the method of cylindrical shells find the volume of the solid obtained by rotating the region bounded by the curves $y = 1 - x^2$ and $y = 1 - x$ about the x -axis

14. (10 points) Find the work needed to drain water out of the top of a spherical tank of radius R if only the bottom ' h ' feet is full. Set up but DO NOT EVALUATE. {Work = Force * Distance, but Force = weight here.}

THE END