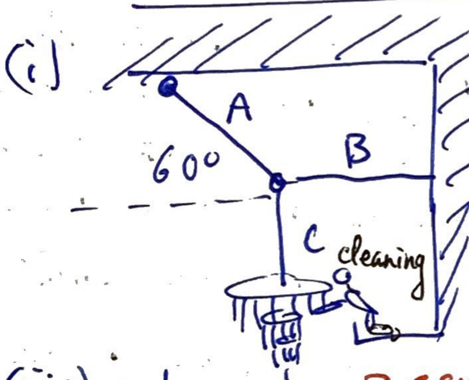
9 B Statics

- When there is **no motion** then we call a system of forces in its **static form**

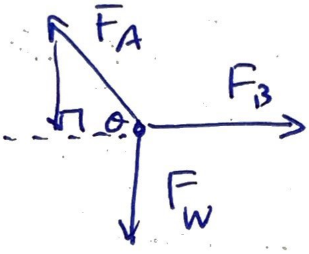
- Two **requirements** of static form
  - $\sum F = 0$ { No linear acc'ln }
  - $\sum \tau = 0$ { No angular acc'ln

Ex Consider a chandellier. What are the force in the cables: see diagram below

(i)



(ii)



(iii) $\sum \vec{F} = \vec{0}$ ($m\vec{a} = 0$)

$$\begin{cases} x: \sum F_x = 0 \\ y: \sum F_y = 0 \end{cases}$$

(iii) cont. — 2 eqns 2 unks.

$$x: \quad F_B - F_A \cos\theta = 0$$
$$y: \quad -F_W + F_A \sin\theta = 0$$

$F_W = 200\,kg$, $F_B = ?$, F_A

(iv) Do the math

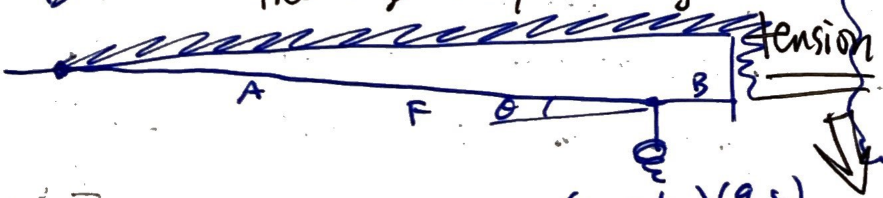
$$F_A = \frac{F_W}{\sin\theta} = \frac{(200\,kg)(9.8\,m/s^2)}{\sin 60°}$$

$$\boxed{F_A = 2263.2\,N}$$

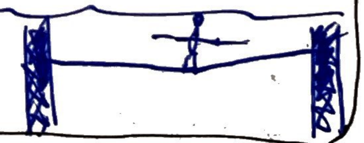
$$F_B = F_A \cos\theta = (2263.2\,N)\cos 60°$$

$$\boxed{F_B = 1131.6\,N}$$

BTW: The longer A, the higher the tension

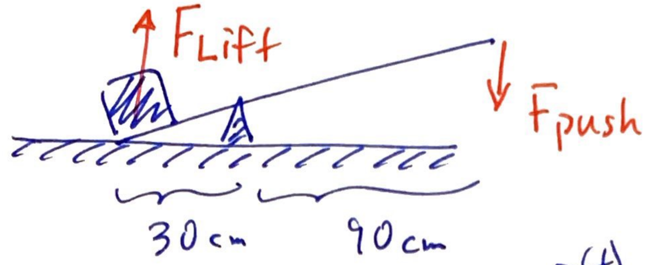


let $\theta = 2°$: $F_A = \frac{(200\,kg)(9.8)}{\sin 2°} = 56{,}161\,N$



ex Fulcrum Problem (Levers)

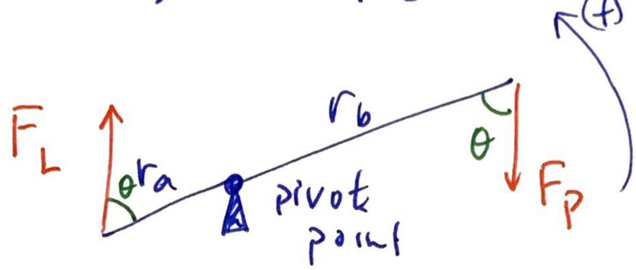
(i)



$W = 800\,N$

Q: What F_{push} do we need to lift the package?

(ii)



Data

$r_a = 0.30\,m$

$r_b = 0.90\,m$

$F_\ell = 800\,N$

$F_p = ?$

(iii) $\sum \vec{F} = \vec{0}$

$$\boxed{\sum \vec{\tau} = \vec{0}}$$ ← Start with this since rotation only.

(v) $\tau_A + \tau_B = 0$ $\qquad \boxed{\tau = r F_\perp \sin\theta}$

$$\ominus\left[r_a F_L \sin\theta\right] + \left[\ominus r_b F_p \sin\theta\right] = 0$$

clockwise "motion"

$$\Rightarrow -r_a F_\ell \sin\theta - r_b F_p \sin\theta = 0$$

$$\Rightarrow F_p r_b = -r_a F_\ell$$

$$\Rightarrow \boxed{F_p = -\left(\frac{r_a}{r_b}\right) F_{Lift}}$$

apply to our case

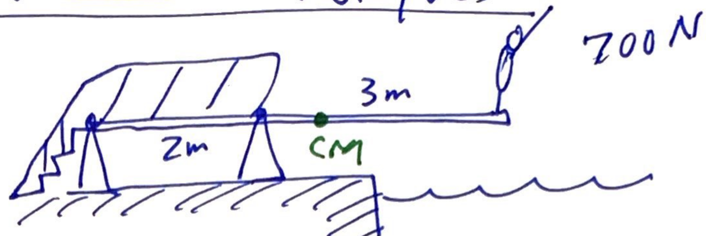
$$F_{push} = -\left(\frac{30\,cm}{90\,cm}\right)(800\,N)$$

$\boxed{F_p = -270\,N \text{ push down to lift } 800\,N}$

Ex: use both Forces and Torques

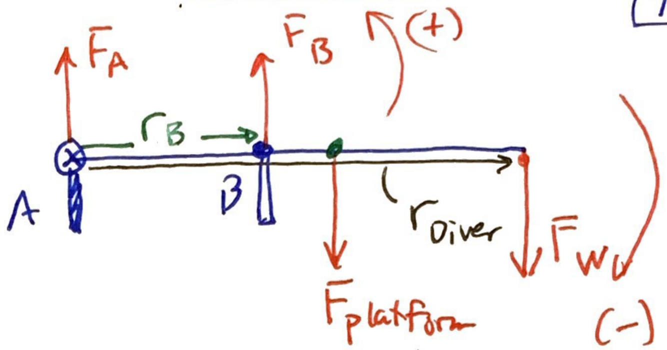
- Diving Platform

(i)



Q: Find the Forces on the two pylons supporting the board.

(ii) Freebody



DATA

$r_B = 2m$

$r_D = 2+3 = 5m$

$F_W = 700N$

$F_A = ?$

$F_B = ?$

(iii) Eqns.

- $y: \sum F_y = 0 \rightarrow F_A + F_B - F_W - F_P = 0$
- $\tau: \sum \tau = 0 \rightarrow r_A F_A + r_B F_B - r_D F_W - r_P F_P = 0$ ($r_A = 0$; $r_B F_B$: c-cw; $r_D F_W$, $r_P F_P$: cw.)

(iv) Solve using torque first:

$$r_B F_B - r_D F_W - r_P F_P = 0$$

$$F_B = \frac{r_D F_W + r_P F_P}{r_B}$$

$$F_B = \frac{(5m)(700N) + \left(\frac{5m}{2}\right)(100N)}{2m}$$

$$\boxed{F_B = 1875N \text{ upward}}$$

- For F_A use the $\sum F = 0$ eqn:

$$F_A + F_B - F_W - F_P = 0$$

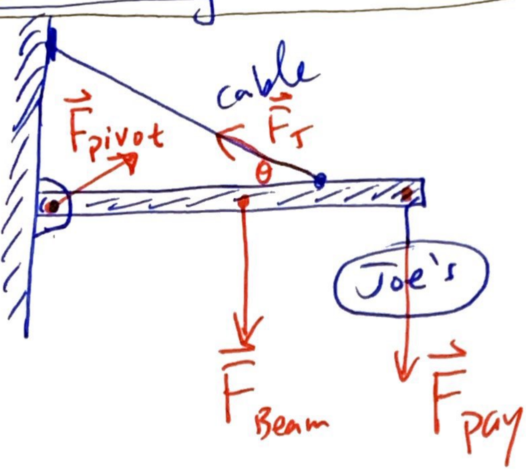
$$F_A = -F_B + F_W + F_P$$

$$F_A = -1875N + 700N + 100N$$

$$\boxed{F_A = -1075N} \text{ downwards}$$

⊛ Street-Sign Problem

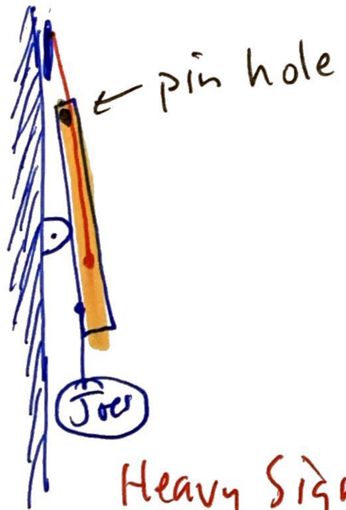
EX



We know $F_{payload}$, W_{Beam}, we want the Tension in the cable and the $\vec{F}_{pivot}$.

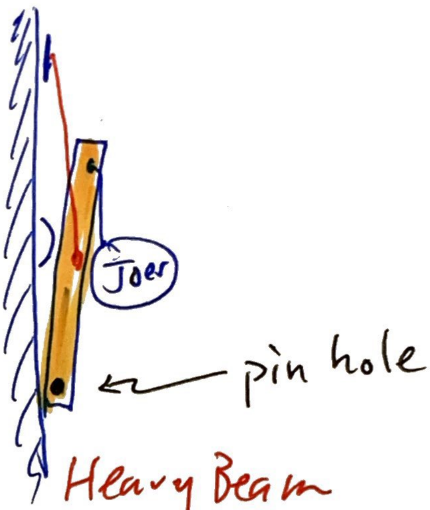
Thought: Remove the pin in the wall mount

Two possible scenarios:

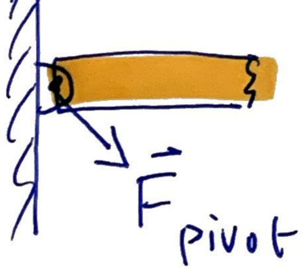


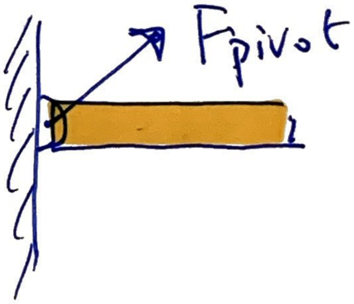
Heavy Sign or Closer Cable support

OR



Heavy Beam



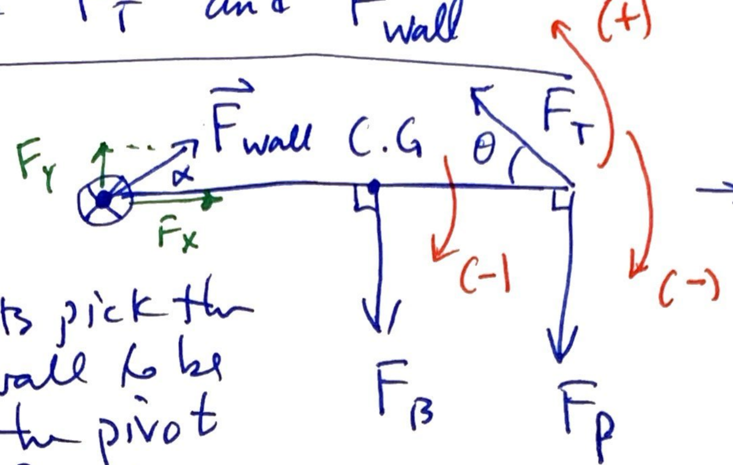


$F_y < 0$ $\qquad$ $F_y > 0$

The math will tell us which happens.

Ex cont. Find F_T and F_{wall}

(ii)



lets pick the wall to be the pivot for the torque eqn.

DATA

$\ell = 2.2\,m$

$m_B = 25\,kg$

$m_p = 28\,kg$

$\theta = 30°$

$F_T = ?$

$\vec{F}_{wall} = ?$

(iii) $\sum \vec{\tau} = 0$, $\sum \vec{F} = 0$ $\begin{cases} \sum F_x = 0 \\ \sum F_y = 0 \end{cases}$

[τ] $r_{wall} F \sin\alpha - \left(\frac{\ell}{2}\right) F_B - (\ell) F_p + (\ell) F_T \sin 30° = 0$ ($r_{wall} \to 0$; want F_T)

$$F_T = \frac{\left(\frac{2.2}{2}\right)(25)(9.8) + (2.2)(28)(9.8)}{(2.2)\sin 30°} = \frac{396.9}{\sin 30°}$$

$\boxed{F_T = 793.8\,N}$

[F] x: $F_x - F_T \cos 30° = 0$

$(\vec{F}_w)_x$ → F_x

$\Longrightarrow$ $F_x = F_T \cos 30°$

$F_x = 793.8\,N \cos 30°$

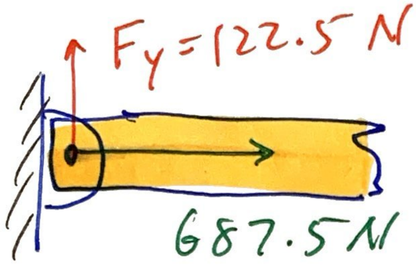
$\boxed{F_x = 687.45\,N}$

$y: \quad F_y - F_B - F_P + F_T \sin 30° = 0$

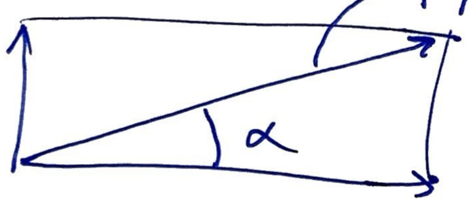
$F_y = F_B + F_P - F_T \sin 30°$

$F_y = (25\,kg)(9.8) + (28\,kg)(9.8) - (793.8)\sin 30°$

$\boxed{F_y = 122.5\,N}$



pin keeps beam from sliding down wall

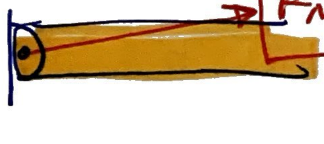


$F_{Net} = \sqrt{(687.5)^2 + (122.5)^2}$

$= \underline{698.3\,N}$

$\alpha = \tan^{-1}\left(\frac{122.5}{687.5}\right)$

$= \underline{10.10°}$

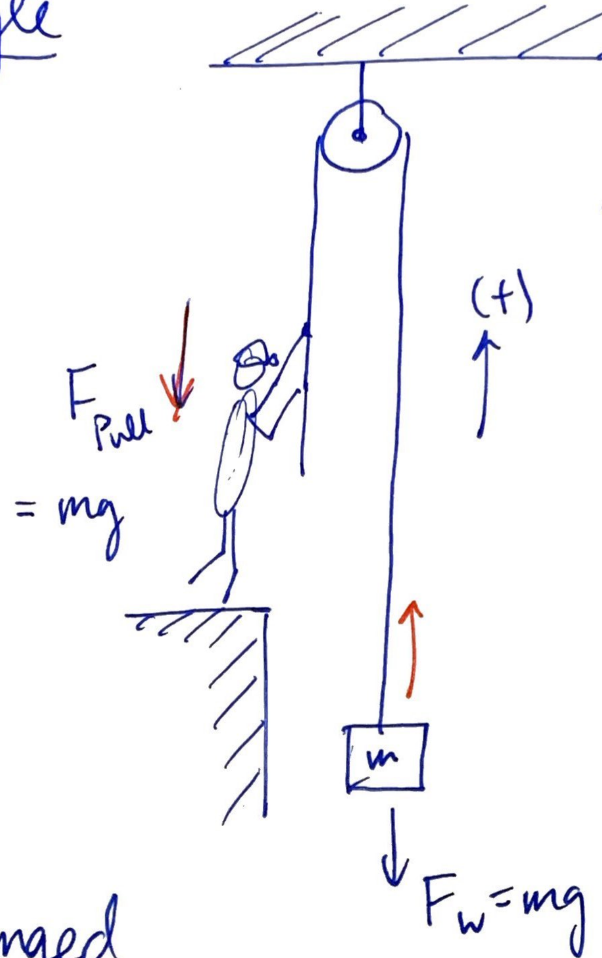


$\boxed{\vec{F}_{Net} = 700\,N,\ 10°\ \text{North of East.}}$

⊗ pulleys

• Single

No Advantage



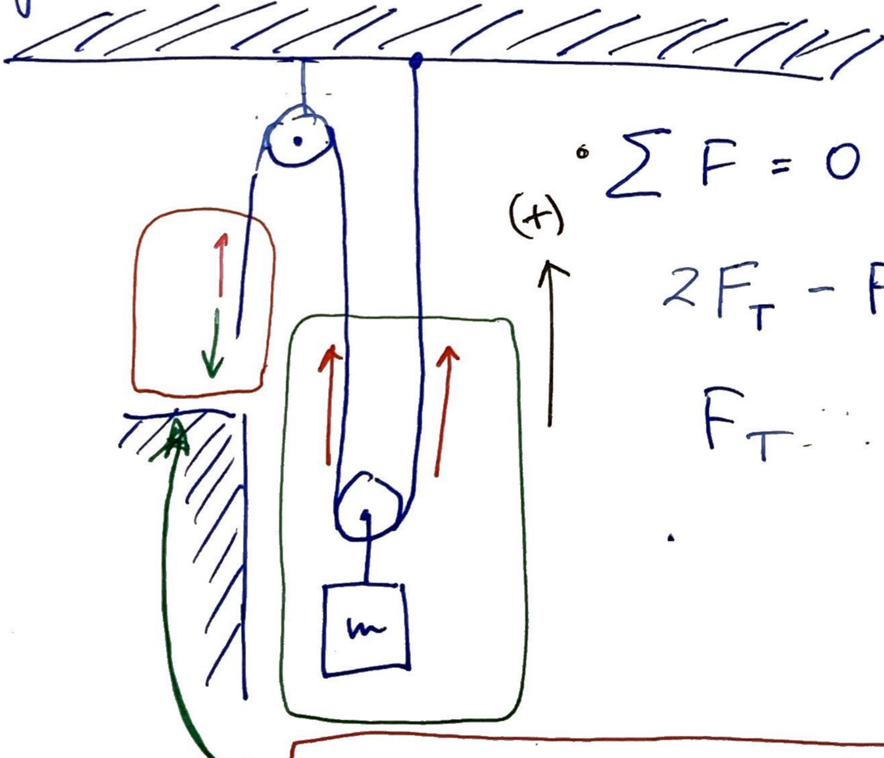
- Pull on foot down the crate moves one foot up
- I need to pull mg newtons down to get the packet to go up.
- $\sum F = 0$

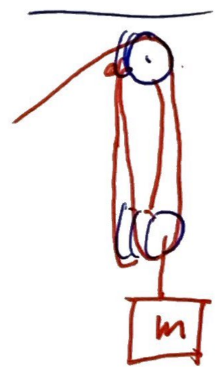
$$-F_{pull} + F_W = 0$$

$$\boxed{F_{pull} = F_W}$$

• ganged

Has an advantage





- $\sum F = 0$

$$2F_T - F_W = 0$$

$$F_T = \frac{1}{2} F_W$$

- pull down $\frac{1}{2}$ the weight
- pull through 2ft to get box to rise 1ft.

$$\boxed{F_{pull} = F_T = \frac{1}{2} F_W}$$