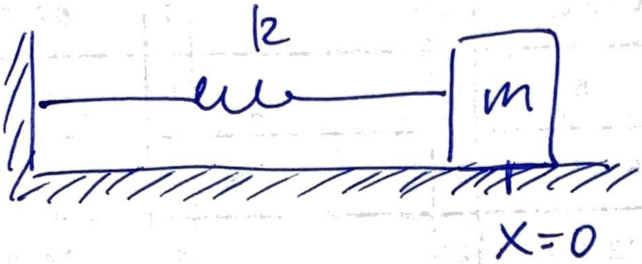
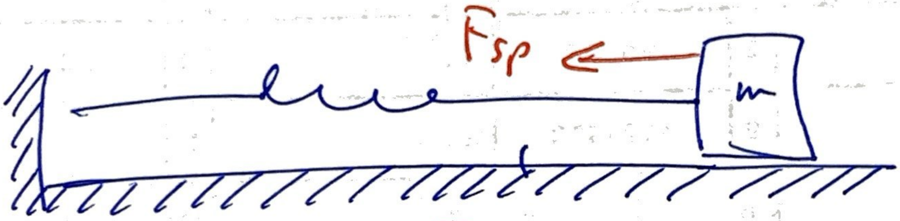
Chapter 9 Elasticity [A] and Statics [B]

← $\vec{a} = \vec{0}$

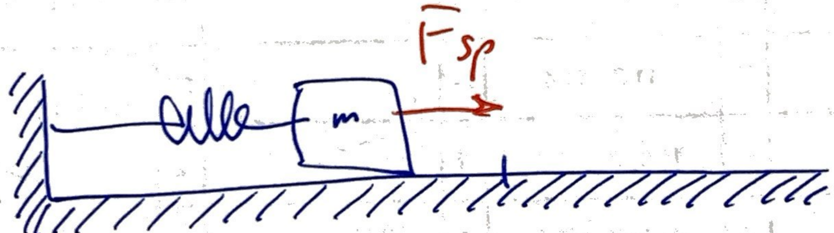
[A] Elasticity and Fracture

- **Hooke's Law** Any spring or spring-like material will resist your motions. I.E. if you pull a spring out it pulls back. If you push a spring together it pushes back.
  - Restorative Force



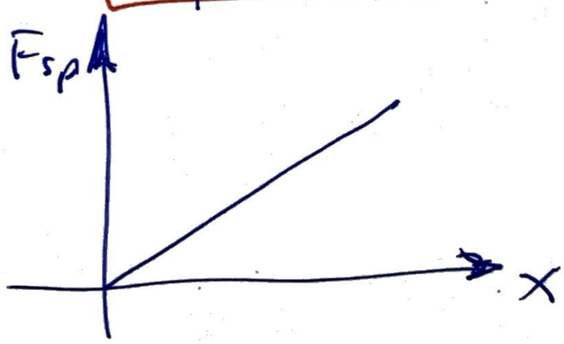


Tensile forces



Compressive forces.

$$F_{sp} = -kx$$ Hookes Law. spring

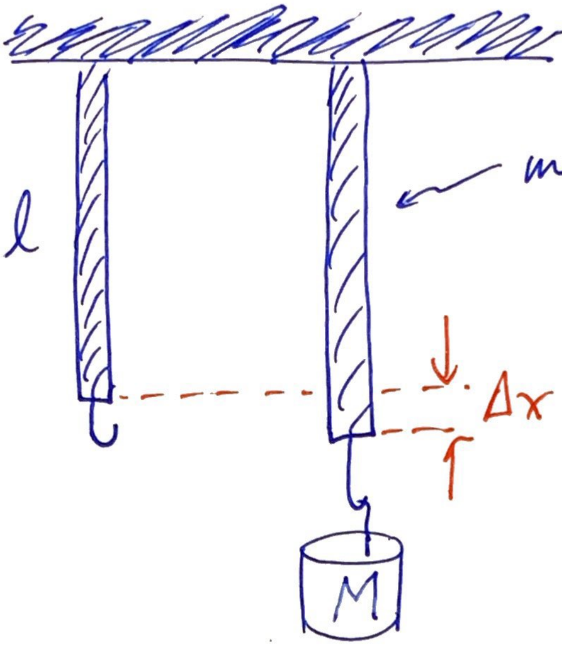


Linear Response

- This principle applies to more than the spring:

"Hanging masses"

Tensile Forces

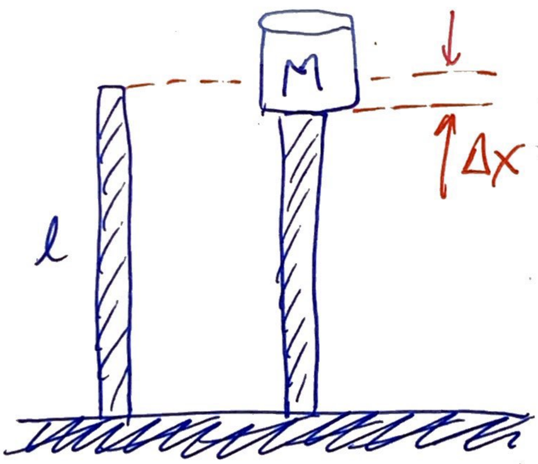


metal, brass, plastic, cement

$\Delta x \propto M$

"supporting mass"

Compressive Forces



$\Delta x \propto M$

- rods and springs have both Tensile & Compressive
- cables and ropes have only Tensile

⊛ Definitions

- Stimulus or cause

$$\text{Stress} \equiv F/A$$

(F: applied force; A: cross-sectional area)

and

- Response or effect

$$\text{Strain} \equiv \frac{\Delta l}{l}$$

(Δl: displacement length or volume; l: original)

⊛ Hookes Law in Elasticity:

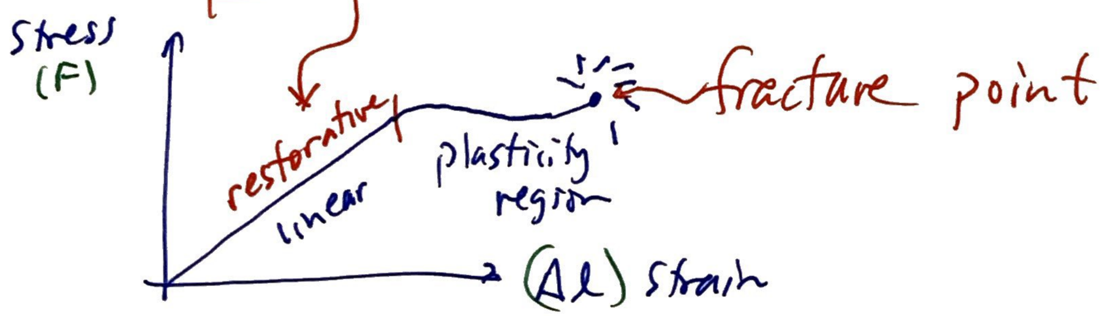
$$\text{Stress} = (\text{Modulus})\,\text{Strain}$$

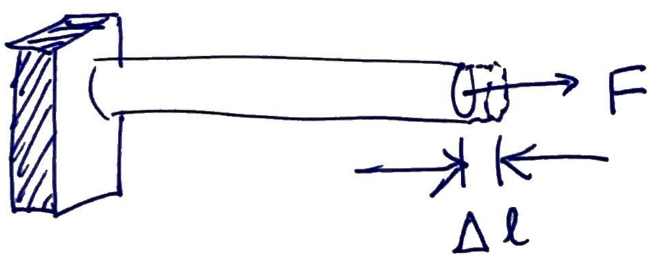
$$((F = k \, \Delta x))$$

÷ Strain

Solve for modulus

$$\text{Modulus of Elasticity} \equiv \frac{\text{Stress}}{\text{Strain}}$$





EX (a) A 10 m steel wire is stretched by 3.08 mm when a 200 N weight is hung from the wire. What is the strain?

$$\text{strain} \equiv \frac{\Delta l}{l}$$

$$= \frac{0.00308\,m}{10\,m}$$

$$\text{Strain} = 3.08 \times 10^{-4}$$

Dimensionless

(b) What is the applied stress if the wire has a crossection of 2 mm diameter?

$$\text{Stress} \equiv F/A$$

$A = \pi r^2$

$$= \frac{200\,N}{\pi \left(\frac{0.002\,m}{2}\right)^2}$$

$N/m^2 \equiv$ Pascal

$$= 6.37 \times 10^{7}\ Pa$$

Young's Modulus is E and refers to **1-Dim.** stretching or compression.

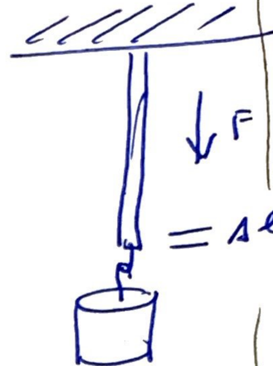
$$E = \frac{\text{stress}}{\text{strain}} = \frac{F/A}{\Delta l / l}$$

(Ex cont)

(c) Find youngs modulus from the part (a) & (b)

$$E = \frac{\text{Stress}}{\text{Strain}} = \frac{6.37 \times 10^{7}\ N/m^2}{3.08 \times 10^{-4}} = 2.07 \times 10^{11}\ N/m^2$$

Ex A brass rod of original length 8 m & diam of 1.5 mm is supporting a weight of 120 N. By how much does it stretch when the weight was attached? (5)



$E_{Brass} = 8.96 \times 10^{10}$ Pa (N/m²)

Solve for Δl:

$$E \equiv \frac{F/A}{\Delta l / l} \longrightarrow \left(\frac{\Delta l}{l}\right) E = F/A$$

$$\Longrightarrow \frac{F \cdot l}{A \cdot E} = \Delta l$$

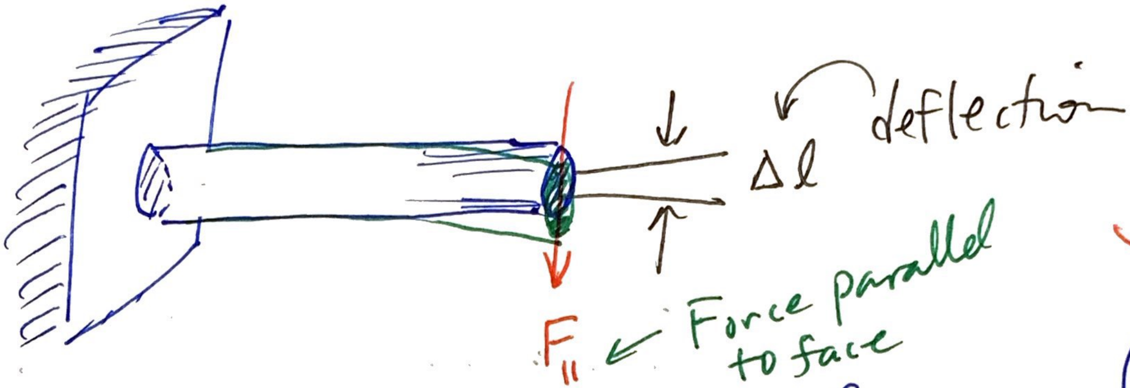
$$\Delta l = \frac{(120\ N)(8\ m)}{\pi \left(\frac{1.5\ mm}{2}\right)^2 \left[8.96 \times 10^{10}\ N/m^2\right]}$$

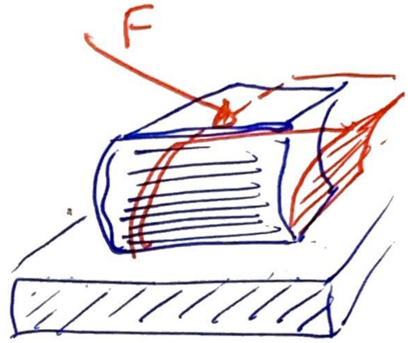
$$\frac{\frac{kg \cdot m}{s^2} \cdot m}{m^2 \cdot \frac{kg \cdot m}{s^2 \cdot m^2}}$$

0.0015 m

$$\boxed{\Delta l = 0.605\ mm}$$

⊛ 2-Dimensional Stress and Strain

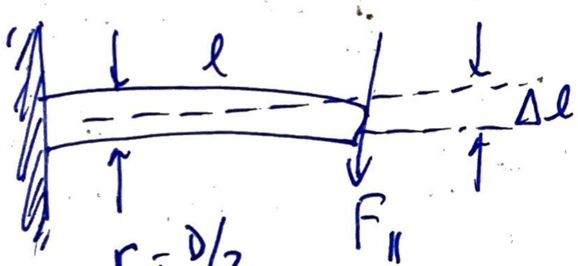




• We call this a shear force

Shear Modulus $\equiv \frac{\text{stress}}{\text{strain}} = \boxed{\frac{F_{\parallel}/A}{\Delta l/l}}$

2-DIM



$r = D/2$

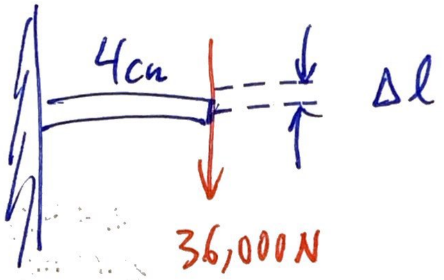
$A = \pi r^2$

↙ shear force, Not Push/pull

$$\boxed{G = \frac{F_{\parallel}/A}{\Delta l/l}}$$

↖ deflection, not stretch or Compression

EX A steel stud of 1 cm in diameter protrudes from a wall mount by 4 cm. A 36,000 N shear force is applied to the face at the floating end of the rod. Q: Find the "droop" of that face?



$$G \equiv \frac{F_{\parallel}/A}{\Delta l / l}$$

Solve for Δl : $\frac{\Delta l}{l} G = F_{\parallel}/A$

$$\Delta l = \frac{F_{\parallel} \cdot l}{A \cdot G}$$

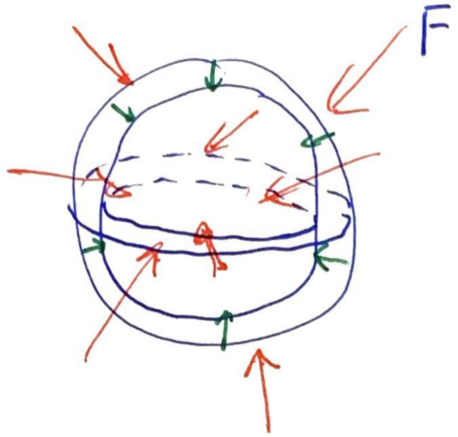
- Shear modulus for steel : $80 \times 10^9 \, N/m^2$

$$\Delta l = \frac{(36000 \, N)(0.04 \, m)}{\pi \left(\frac{.01 \, m}{2}\right)^2 \cdot 80 \times 10^9 \, N/m^2}$$

$$= 0.00023 \, m$$

or $\Delta l = 0.023 \, mm$ very small

⊛ 3-Dimension Stress and Strain



Bulk Modulus = $\frac{\text{stress}}{\text{strain}}$

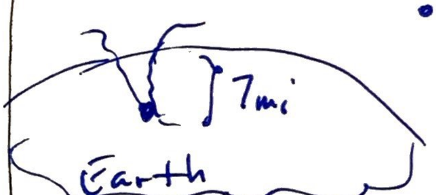
$$B = \frac{F_\perp / A}{\Delta V / V}$$

V = volume

A fluid will allow all forces to distribute ⊥ to the surface.

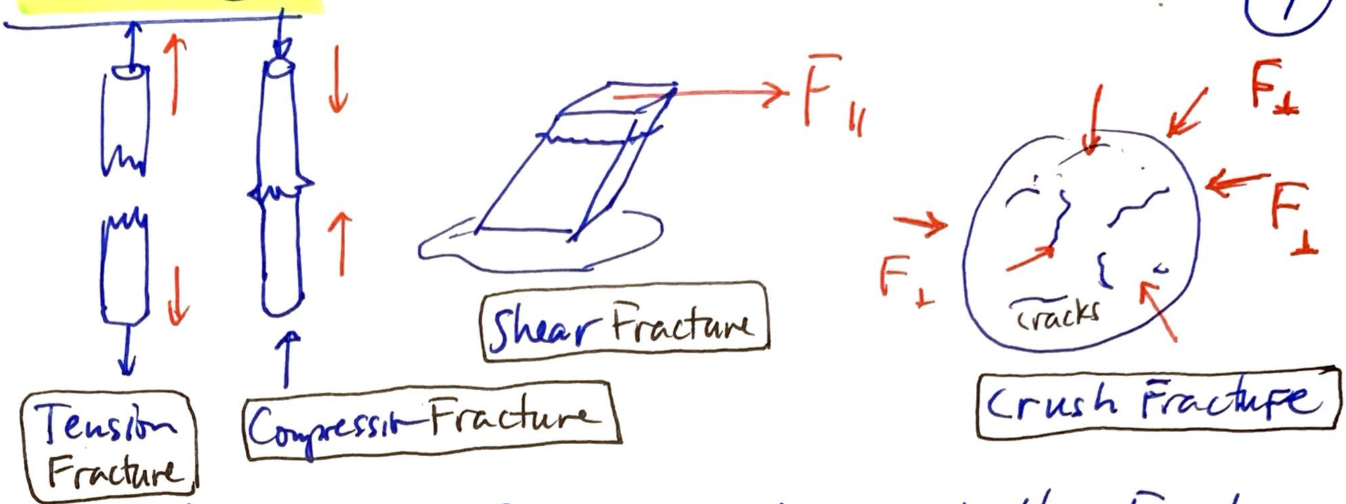
Ex A solid beachball sized steel sphere falls off the deck of a ship over the Marianas Trench (7mi deep)

Q: If the sphere has a diameter of 75cm what is the change of its volume 7mi deep?



- Siri reports that the pressure at the bottom of trench is 1000 atm or 17,750 psi
- $V_0 = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left(\frac{0.75}{2}\right)^3 = 0.22\,m^3$
- Convert N/m² : $17{,}750 \frac{lbs}{in^2}\left(\frac{4.448\,N}{lb}\right)\left(\frac{39.37\,in}{m}\right)^2$
  $= 1.224 \times 10^8\, N/m^2$ ← pressure
- $\Delta V = \frac{F/A}{B_s / V_0} = \frac{(1.224\times10^8\,N/m^2)}{(140\times10^9\,N/m^2)/\frac{4}{3}\pi\left[\frac{0.75}{2}\right]^3}$
- $\Delta V = 0.000192\,m^3 \cdot \left(\frac{100\,cm}{m}\right)^3 = 192\,cm^3$

Ⓐ Fracture



- Fracture is stress so we will see in the Fracture Tables the stress values @ the point of failure

Ex A steel piano wire is 1.60 m long and has a diameter of 0.2 cm (2 mm). Q: Under what tension will the cable break?

- Stress = F/A $\Rightarrow$ F = Stress · Area
- Tensile fracture for steel $S = 2500 \times 10^6 \frac{N}{m^2}$
- Calculate Fracture forces:

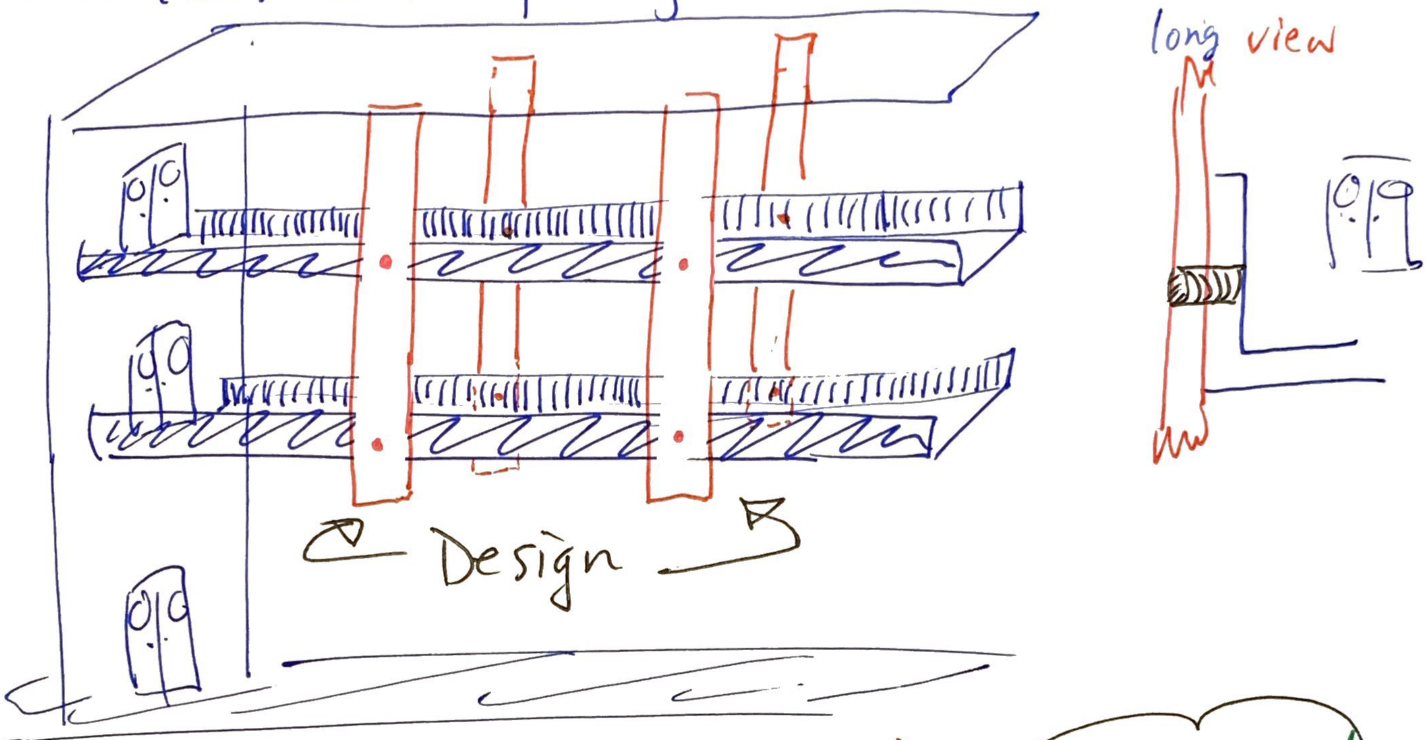
$$F = (\text{Stress of fracture})(A) \quad \leftarrow \pi r^2$$

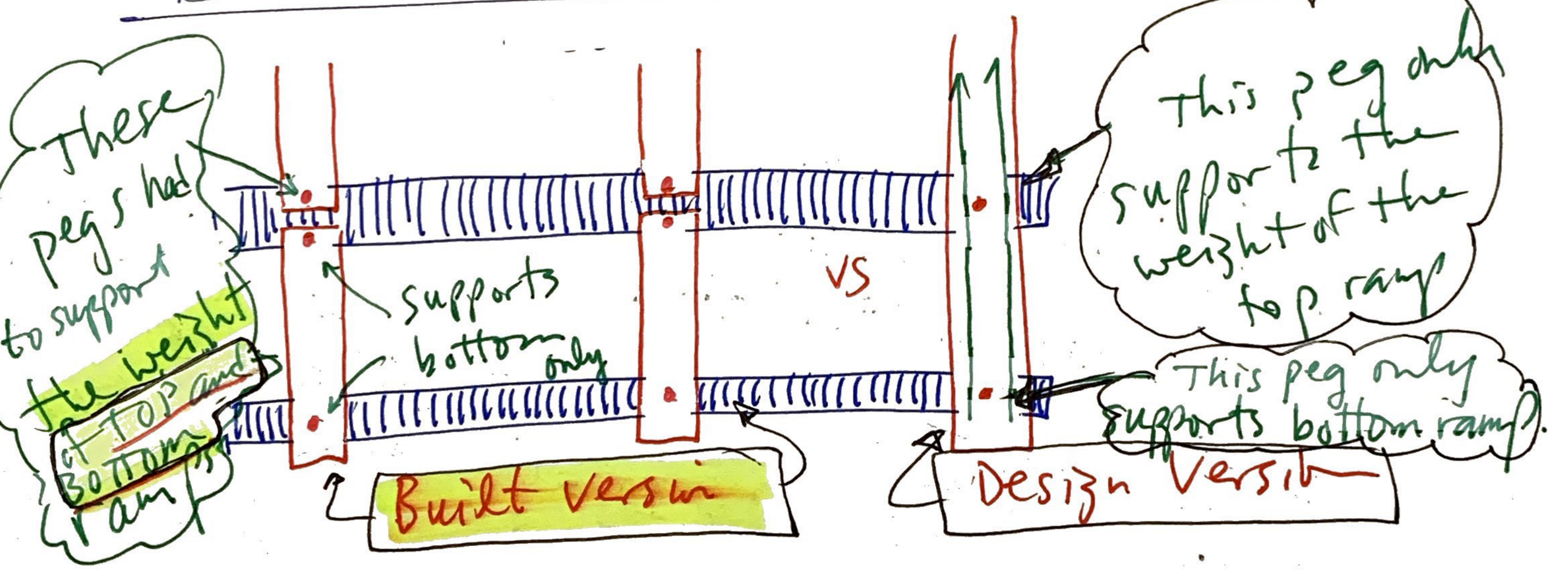
$$= (2500 \times 10^6 \, N/m^2)\left(\pi \left(\frac{0.002\,m}{2}\right)^2\right)$$

$$\boxed{F = 8000 N}$$ = 2 tons = weight of small car.

(*) Construction Fails

New Years Eve party at a hotel in St. Louis





- see "grady" and "engineering" on youtube