8b Rotational Dynamics

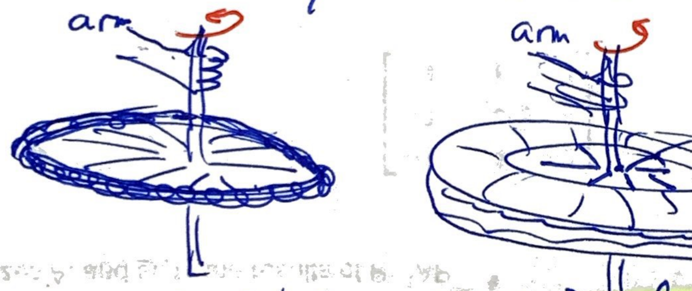
(Newton's Laws in rotation set-ups)

- **mass**: Recall that a refridgerator full of Kambucha is harder to acc'lt than an empty refridgerator. We called this "inertia" and scientists later called it mass.

$F \propto a \quad F \propto mass$

$\Rightarrow \boxed{F = ma}$ linear.

- In the rotational world we will find it easier to twist a bicycle tire's axel to get it rotating than to twist a motorcycles' tire



- We call this rotational inertia

Newton's Second Law

Linear	Rotational
$\boxed{F = ma}$ →	$\boxed{\tau = I\alpha}$

"I" is the rotational inertia also called "moment of inertia"

⊛ How do we define "I"

- Start with $\boxed{\tau = I\alpha}$ Newton's 2nd Law in Rotation
- Substitute in $\tau = rF$; $a = r\alpha$ also sub in

$$rF = I\left(\frac{a}{r}\right)$$

$$I = \frac{F}{a} r^2$$

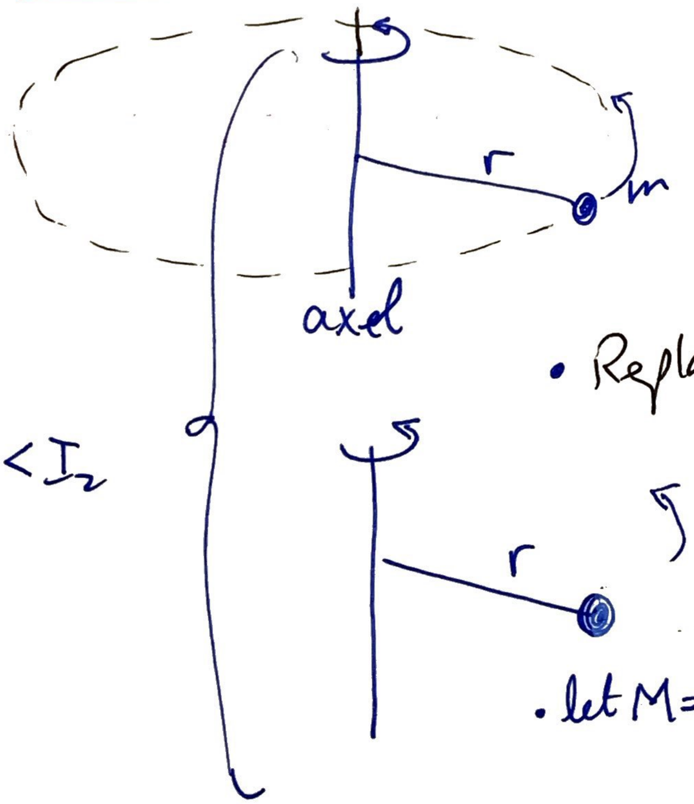
- replace F with ma

$$I = \frac{m\cancel{a}}{\cancel{a}} r^2$$

$$\boxed{I = mr^2}$$

Formula for calculating the moment of inertia

- $\boxed{\text{Dimensions: } [I] = kg \cdot m^2}$



$I_1 = mr^2$

((less rota. inertia))

- Replace with heavier mass

$I_1 < I_2$

$I_2 = Mr^2$

((more rotational inertia))

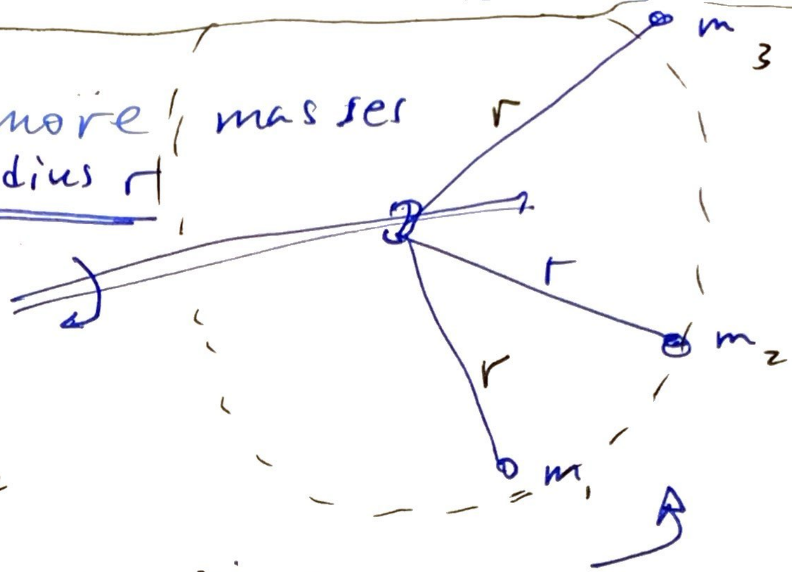
- let $M = 2m$

$I_2 = 2mr^2$

⇒ "Double the mass, double the inertia"

EX

- Add more masses at radius r



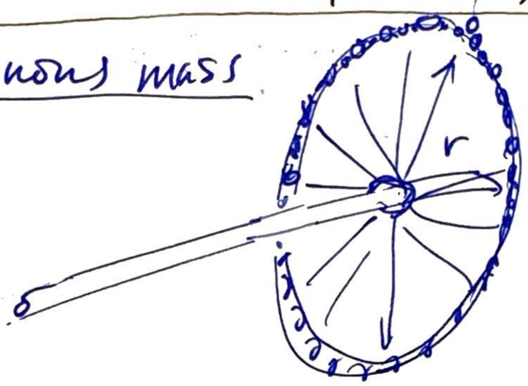
(Not Balanced)

$I = \sum m_i r_i^2$

$= m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2$

$m_1 = m_2 = m_3$ and $r_1 = r_2 = r_3 \Rightarrow \boxed{I = 3mr^2}$

- Continuous mass



$\boxed{I = M_{TOT}\, r^2}$

(balanced)

- NOW Double the radius...

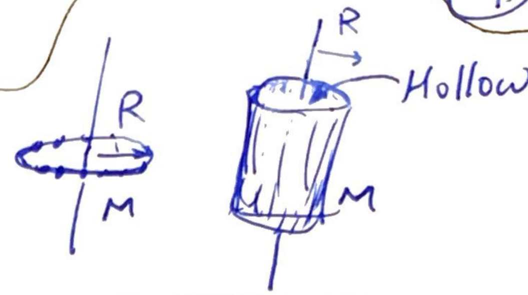
We need ~~twice~~ four times the twist, torque, to acc'lt this longer spoked mass.

$I = m(2r)^2$

$$\underline{\underline{I = 4mr^2}}$$

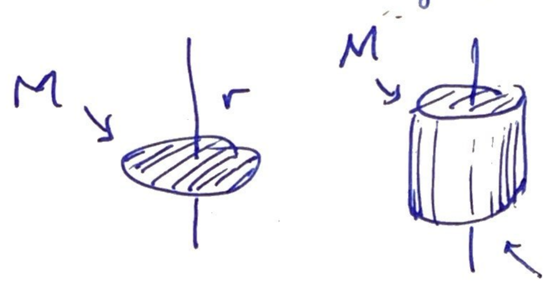
Table of Moments of Inertia

- Hoop or hollow cylinder



$\boxed{I = MR^2}$

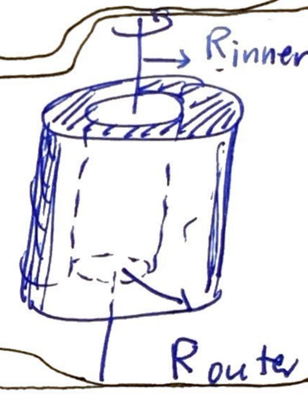
- Solid disk or hockey puck or solid cylinder



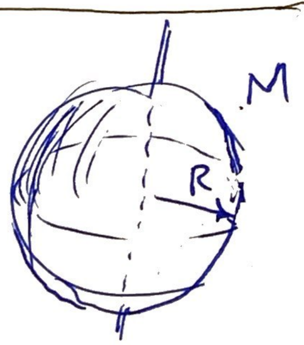
$\boxed{I = \frac{1}{2} MR^2}$

- Thick-walled hollow cylinder

$\boxed{I = \frac{1}{2} M \left(R_{in}^2 + R_{out}^2 \right)}$



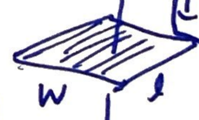
- Solid sphere



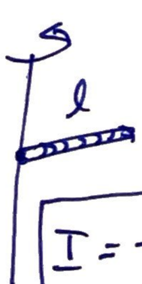
$\boxed{I = \frac{2}{5} MR^2}$

- plate

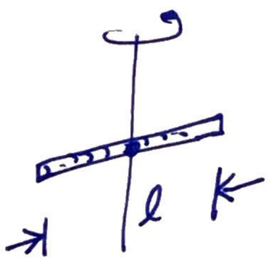
$\boxed{I = \frac{1}{12}(l^2 + w^2)}$



- Rods



$\boxed{I = \frac{1}{3} Ml^2}$

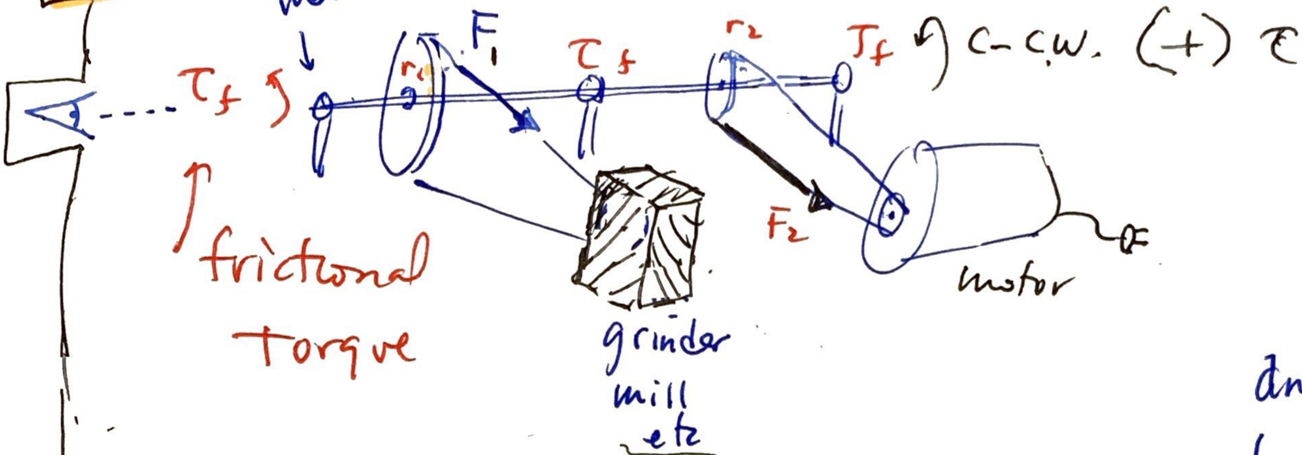


$\boxed{I = \frac{1}{12} Ml^2}$

⊛ Newton's Laws for rotation

- Linear $\sum F = ma$
- rotational $\boxed{\sum \tau = I\alpha}$ sum of torques = intertia · angular acc'ln

Ex



angular acc'ln of axel

$$\sum \tau : \boxed{-3\tau_f - \underbrace{r_1 F_1} + \underbrace{r_2 F_2} = I_{TOT}\,\alpha}$$

three axle mounts

friction torque

resistance torque by mill

torque driving the axel

total moment of inertia of pulleys & axel berings ((resistance to acc'ln))

Ex A flywheel has a mass of 50 kg and a diameter of 0.9m. What is its moment of inertia? (6)

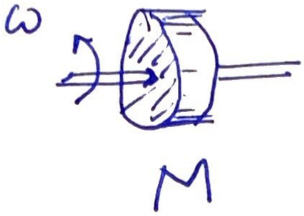


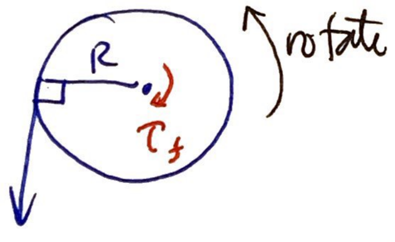
Chart : $I = \frac{1}{2} MR^2$ ← solid cylinder

$= \frac{1}{2}(50\,kg)\left(\frac{0.9m}{2}\right)^2$

$= \boxed{5\ kg \cdot m^2}$

Ex Find the moment of inertia for a non-standard shaped pully if all we know is that it takes 15 N of force applied tangentialy to acc'lt the pulley from 0 to 30 rad/sec in 3 sec's of time. The radius is 33cm & the beirings have $\tau_f = 1.1\,mN$.

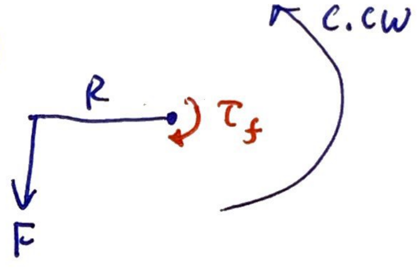
(i)



$F = 15N$

$\tau_B = RF$

(ii)



(iii) $\sum \tau = I\alpha$

$\boxed{-\tau_{friction} + \tau_{Belt} = I\alpha}$

(iv) Solve for I:

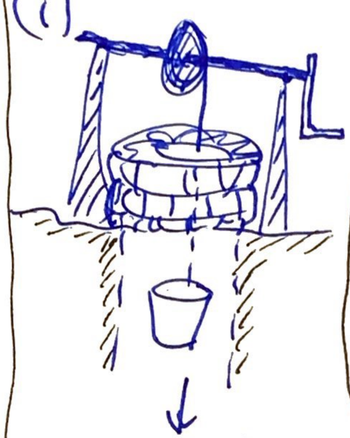
$I = \frac{\tau_{belt} - \tau_{friction}}{\alpha}$, $\alpha = \frac{\Delta\omega}{\Delta t} = \frac{30\,r/s}{3s} = \underline{\underline{10\,r/s^2}}$

$I = \frac{(0.33m)(15N) - 1.1\,mN}{10\,r/s^2} = \boxed{0.385\ kg\,m^2}$

EX {more challenging}

An empty bucket is released and it falls into a well. The pulley is no longer massless. Find the acc'ln of the bucket. Use "I" from the previous example

(i)



DATA

$m = 1.53\ kg$

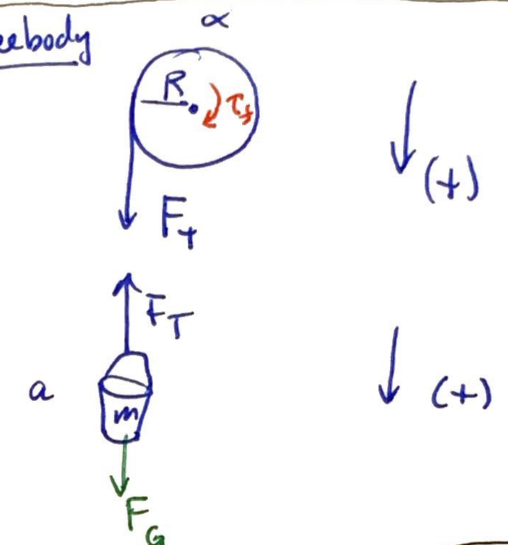
$I_p = 0.385\ kgm^2$

$R = 0.33 m$

$\tau_{fric} = 1.1 mN$

$a = ?$

(ii) freebody



(iii) equations of motions: $\sum \tau = I\alpha$, $\underline{\tau = rF}$, $\underline{a = R\alpha}$

$$\boxed{RF_T - \tau_{friction} = I\alpha}$$

$$\underbrace{\sum F = ma}_{\text{Bucket}} \Rightarrow \boxed{-F_T + F_g = ma}$$

(iv) math

$$\begin{cases} F_T = F_g - m\boxed{a} \\ F_T = mg - m\boxed{(R\alpha)} \end{cases}$$

$$R(mg - mR\alpha) - \tau_{fric} = I\alpha$$

want

want

$$Rmg - m\alpha R^2 - \tau_f = I\alpha$$

combine α: $\alpha\left[I + mR^2\right] = Rmg - \tau_f$

$$\boxed{\alpha = \frac{Rmg - \tau_f}{I + mR^2}}$$

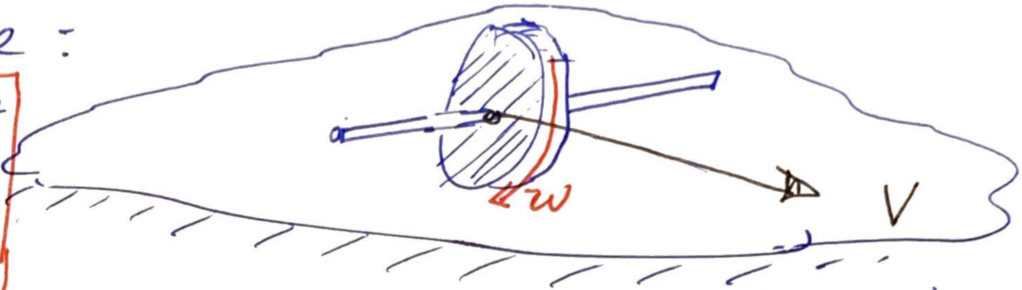
$$\alpha = \frac{(0.33 m)(1.53 kg)(9.8\ m/s^2) - 1.1 mN}{0.385\ kg\cdot m^2 + (1.53\ kg)(0.33 m)}$$

$$\boxed{\alpha = 6.98\ r/s^2} \xrightarrow{a = R\alpha} 2.3\ m/s^2 \text{ of Bucket}$$

⊛ KE in Rotation

Consider a wheel rotating and traveling on a surface:

- $K_{linear} = \frac{1}{2} m v^2$
- $K_{rot} = \frac{1}{2} I \omega^2$



we have both translational (linear) KE and rotational KE.

So total $\boxed{K_{TOT} = K_{rot} + K_{linear}}$

Ex Let $R = 50cm$, $m = 40\,kg$, $V = 10\,m/s$
if $I = 30\,kg \cdot m^2$ What is the total KE?

$K_{TOT} = K_{linear} + K_{rot}$ of the wheel above

$$= \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2 \qquad \text{but } v = r\omega$$

$$= \frac{1}{2} m v^2 + \frac{1}{2} I \left(\frac{v}{r}\right)^2$$

$$= \frac{1}{2} v^2 \left[m + \frac{I}{r^2} \right]$$

$$= \frac{1}{2} (10\,m/s)^2 \left[50\,kg + \frac{30\,kg \cdot m^2}{(0.5\,m)^2} \right]$$

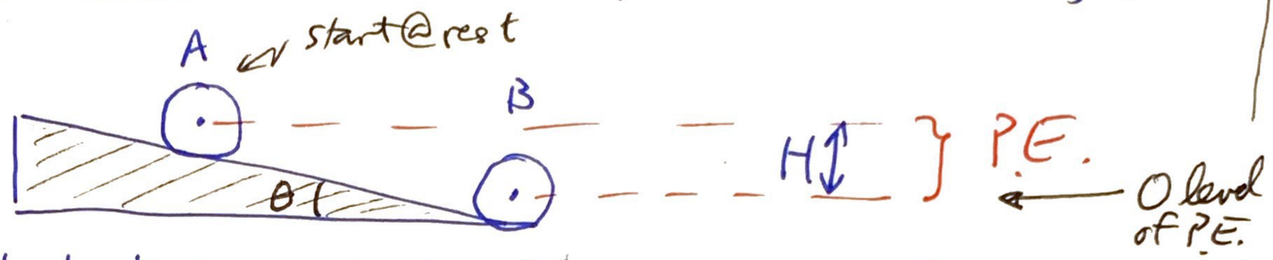
$$= \boxed{6200\,J}$$

Breakdown: $\begin{cases} 200\,J \text{ of linear KE} \\ 6000\,J \text{ of rotational KE} \end{cases}$

EX Find a formula for the speed of a round object rolling down a ramp.

$\boxed{I = kMR^2}$ $\quad k=1$ for hoop, $k=\frac{1}{2}$ disk, $k=\frac{2}{5}$ sphere

(i)



(ii) freebody not usefull (energy Problem)

(iii) eqns: $\boxed{PE_A = KE_B}$ $\qquad P_1 + K_1 = P_2 + K_2 + W_{ork}$

(iv) math: $MgH = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$ $\qquad v = R\omega$

$$\cancel{M}gH = \frac{1}{2}\cancel{M}v^2 + \frac{1}{2}(k\cancel{M}R^2)\left(\frac{v}{R}\right)^2$$

$$2gH = v^2 + kv^2$$

$$\boxed{v = \sqrt{\frac{2gH}{1+k}}}$$

- No mass in final formula
- No Radius of object

- The larger k is the slower "v".

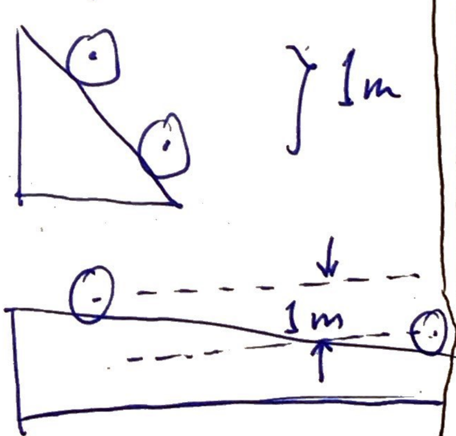
EX How fast is a solid sphere moving after descending 1m vertically down a ramp?

$$v = \sqrt{\frac{2(9.8\,m/s^2)(1m)}{1+2/5}}$$

$$= \sqrt{\frac{2(9.8)}{7/5}}$$

$$= \sqrt{\frac{10}{7}(9.8)}$$

$$= \boxed{3.74\,m/s}$$



Drop It!

$mgh = \frac{1}{2}mv^2$

$\sqrt{2gh} = v$

k = 0, No spin!

(*) Work done by torque

- linear: $W = F \cdot d$
- rotational: $\boxed{W = \tau\theta}$ work needed to twist an object

(*) Power

$$P = \boxed{\frac{W}{\Delta t}} = \frac{\tau \Delta\theta}{\Delta t} = \boxed{\tau\omega}$$ • rotational

$$P = F \cdot v$$ • linear

(*) Angular Momentum

- linear: $p = mv$
- rotational: $\boxed{L = I\omega}$ angular momentum

- linear: $F = \frac{\Delta p}{\Delta t}$
- rotational: $\boxed{\tau = \frac{\Delta L}{\Delta t}}$ Newton's Law recast in rotational dynamics.

(*) Conservation of angular momentum of a system

"The total angular momentum of a rotating system remains constant if the net torque acting on it is zero"

(*) Angular Momentum

Just as we saw linear momentum being conserved, their is a rotational counter-part called angular momentum also conserved.

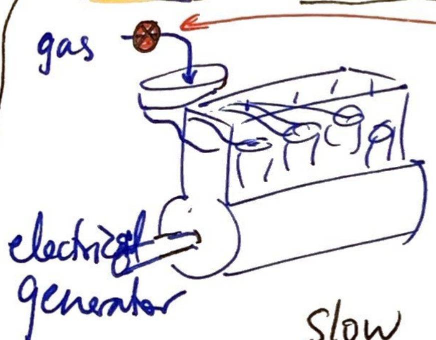
· Linear	$p = mv$	
· Angular	$L = I\omega$	

L is called the angular momentum.

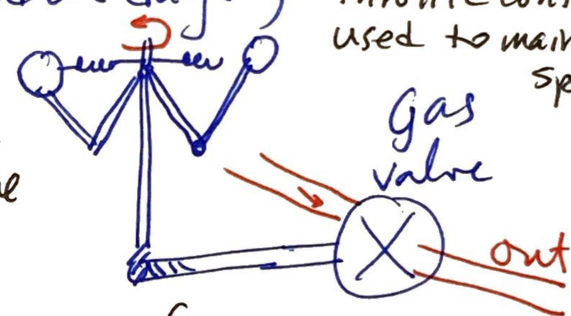
• Conservation formula:

$$\boxed{L_{Before} = L_{after}}$$

Ex Governor (of olden days) is a mechanical throttle control used to maintain speed

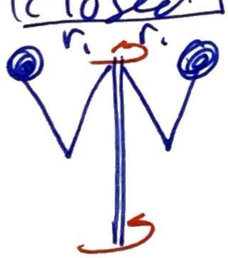


the governor goes on the fuelline



slow

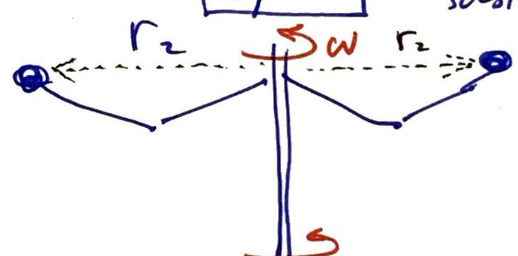
closed = slow, need to apply more gas.



ω_1

fast

Open = fast, need to slow



ω_2 slower

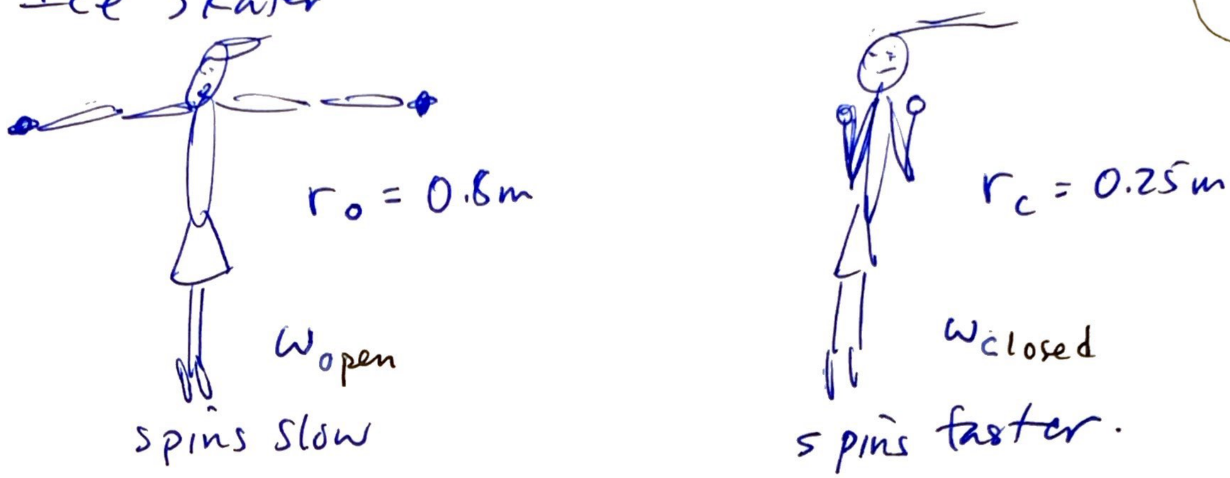
$$L_1 = L_2$$

$$I_1\omega_1 = I_2\omega_2$$

$$2(r_1^2 m)\omega_1 = 2(r_2^2 m)\omega_2$$

$$\Rightarrow \boxed{\omega_{open} = \left(r_1/r_2\right)^2 \omega_{closed}}$$

EX Ice Skater



$$\omega_o < \omega_c$$

$$\omega_o = \left(\frac{r_c}{r_o}\right)^2 \omega_c$$

$$\omega_o \left(\frac{r_o}{r_c}\right)^2 = \omega_c$$

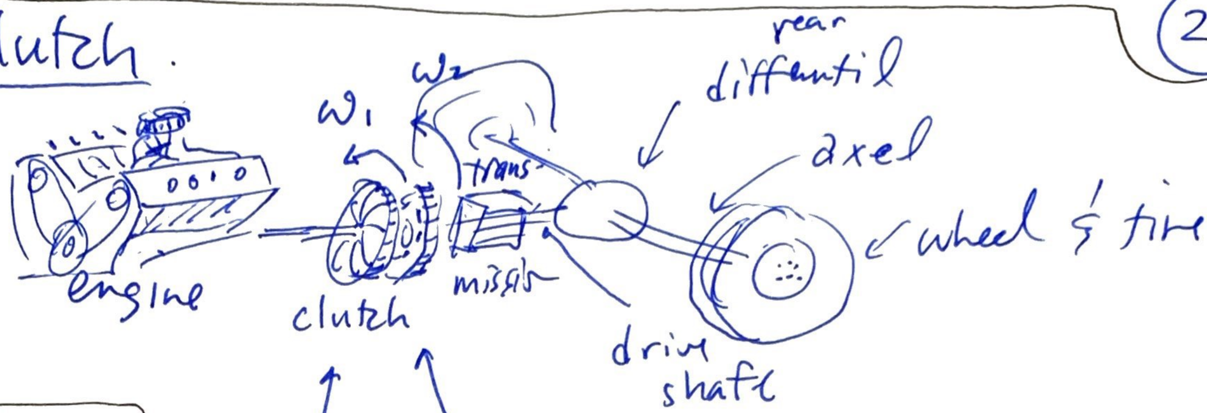
$$\omega_c = \left(\frac{0.8}{0.25}\right)^2 \omega_o$$

$$\boxed{\omega_c = 10.24\, \omega_o}$$

factor of 10 faster!

Ex Clutch

(i)



DATA

$$M_1 = 6\,\text{kg} \qquad M_2 = 9\,\text{kg}$$

$$R_1 = R_2 = 0.60\,\text{cm}$$

Before: $\omega_1 = 7.2\,\text{r/s}$ $\qquad \omega_2 = 0$

After: $\omega_1 = \omega_2 = ?$

Q: what is the combined rotational velocities if the plates were unattached to the drive shaft.

(iii) Before = After

$$\boxed{I_1\omega_1 + I_2\omega_2 = I_1\omega_1' + I_2\omega_2'}$$

after: $\omega_1' = \omega_2'$

(iv)

$$(M_1R_1^2)\omega_1 + 0 = (M_1R_1^2 + M_2R_2^2)\omega'$$

$R_1 = R_2 = R$

$$M_1\cancel{R^2}\omega_1 = (M_1 + M_2)\cancel{R^2}\omega'$$

$$\omega' = \left(\frac{M_1}{M_1 + M_2}\right)\omega_1$$

$$\omega' = \left[\frac{6.0\,\text{kg}}{(6\,\text{kg} + 9\,\text{kg})}\right](7.2\,\text{rad/sec})$$

$$\boxed{\omega' = 2.9\,\text{rad/sec}}$$ combined system

Ch 8c - Part III "Mixture of Angular and Linear Mom"

- angular mom. $L = I\omega$
- linear mom. $p = mv$

$$\begin{cases} x = r\theta \\ v = r\omega \\ a = r\alpha \end{cases}$$

connectors between angular & linear

(*) Even objects moving in a straight line have angular momentum: {w.r.t. a fixed point} this is important $\boxed{L = rp}$

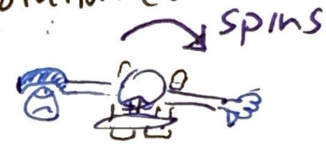
Ex Before Impact

straight line side ↓

$\boxed{rp = L}$

After Impact

rotational side



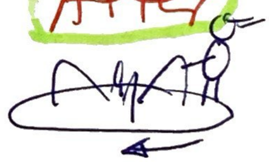
Catcher is not spinning

Catcher Is spinning

Ex merry go round (MGR)

Before

After



MGR is not spinning

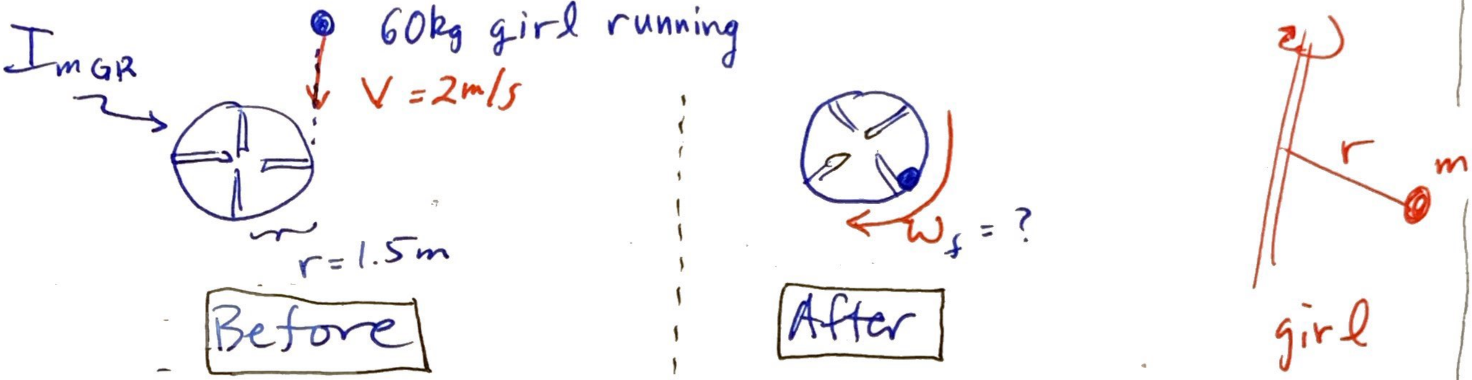
MGR is Spinning

Top view

Top View

Ex A MGRound has an $I_{MGR} = 112\,kg\,m^2$ and a radius of 1.5m. A 60kg girl runs tangent to the non-moving MGR and grabs on.

Q: What is the combined rotational speed after attachment?



- Conservation of angular momentum

$$\boxed{L_{Before} = L_{after}}$$

$$L_{mgr} + L_{girl} = L_{both}$$

$$I_{MGR}\cancelto{0}{\omega_0} + rp = I_{combined} \cdot \omega_f$$

$$r(mv) = (I_{mgr} + \underbrace{I_{girl}}_{mr^2}) \cdot \omega_f$$

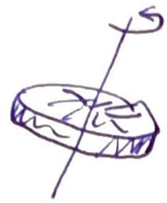
- solve

$$\Rightarrow \boxed{\omega_f = \frac{rmv}{I_{mgr} + mr^2}}$$

$$\omega_f = \frac{(1.5m)(60kg)(2m/s)}{112\,kg\,m^2 + (60kg)(1.5m)^2}$$

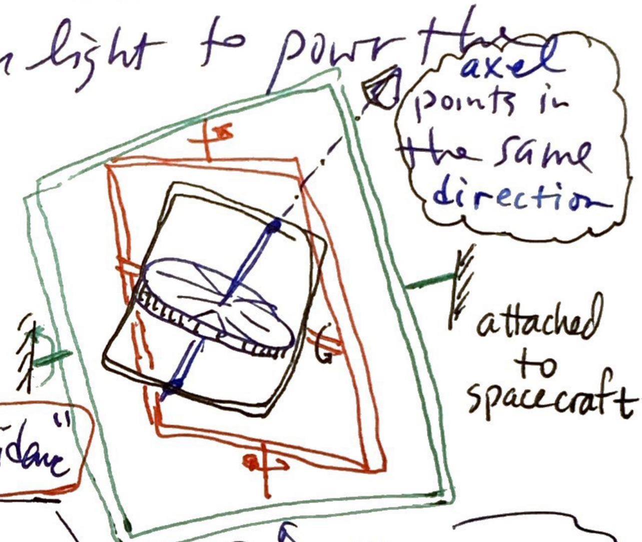
$$\boxed{\omega_f = 0.73\,rad/sec} \cdot \left(\frac{1\,rot}{2\pi\,rad}\right) = \underline{7\,rpm}$$

⊛ gyroscopes {a bit too complicated for Phys Ø6}



A spinning wheel in empty space will always point to the same far away star. These systems is what space telescopes, the ISS, and aircraft use to keep their orientation.

- The I.S.S. uses 4 such. Instead of using little rocket engines (thrusting) the station uses gyros to point it always towards the SUN so the Solar arrays gain max. sunlight to power the station.



* Basic gyro has 3-gimbles

- Demo: "Walter Lewin Inertial Guidance"

sensors record the diversion from reference to display the orientation

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* Gyros can control motion:

- Demo: person with bicycle on rotating platform "Walter Lewin Bicycle wheel Gyroscope"

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* Finally torque can cause precession of a gyro:

- Demo: "Walter Lewin Wheel Momentum"