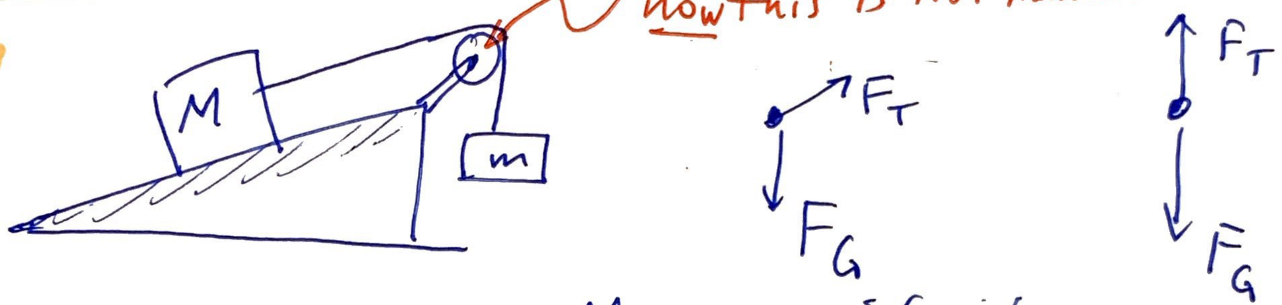
Chapter 8 8A Rotational Motion

← Kinematics

- We introduced rotational motion in Ch 5 when we studied circular motion and gravity
- In this chapter we explore all of Newton's Law in a rotational setting.

Ex

now this is NOT massless



we ignored the pulley ... if the rope sticks (a bit) to the pulley we need to include its mass also.

- We split chpt 8 into two pieces
  - → 8a : rotational Kinematics
  - → 8b : rotational dynamics

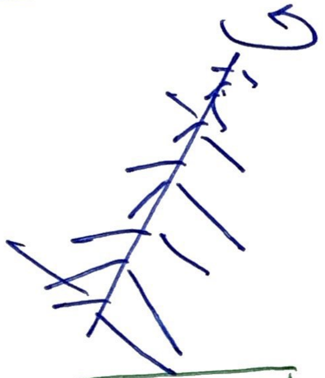
8a : Rotational Dynamics

Def: **Rigid Body** is a group of objects connected together such that their spatial orientation does not change.

Ex

artificial christmas tree

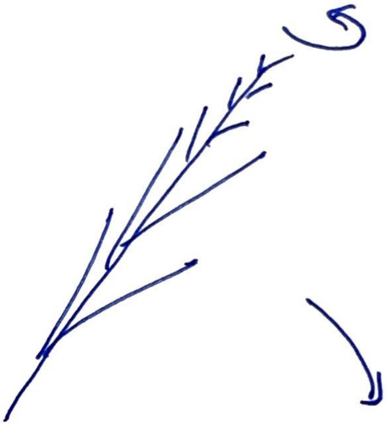
spin



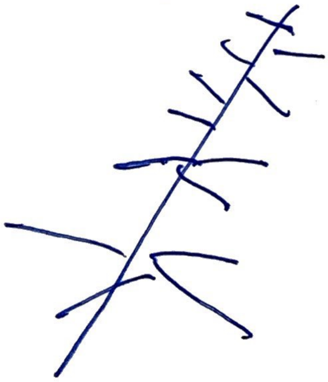
Rigid Body -

- Branches do not fold.
- will not lose orientation when spun

w/ folding branches



Branches open-up.

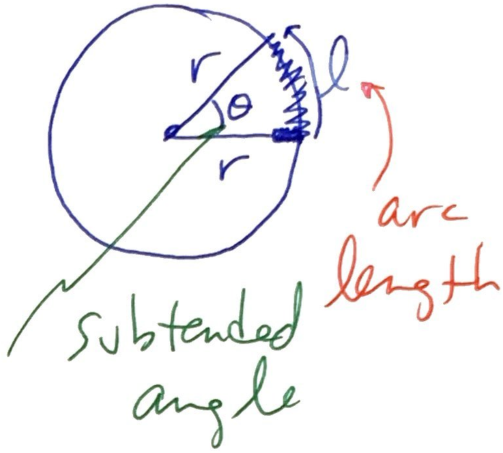


Non-Rigid Body

8A Angular Kinematics

Linear: x, v, a

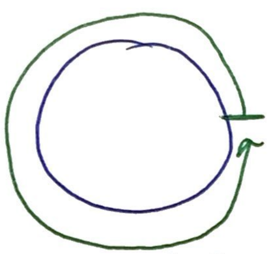
Rotational: θ, ω, α



- **Angular Displacement**

Arc length, l, is given by $\boxed{l = r\theta^r}$

All formulas require θ be in radians.



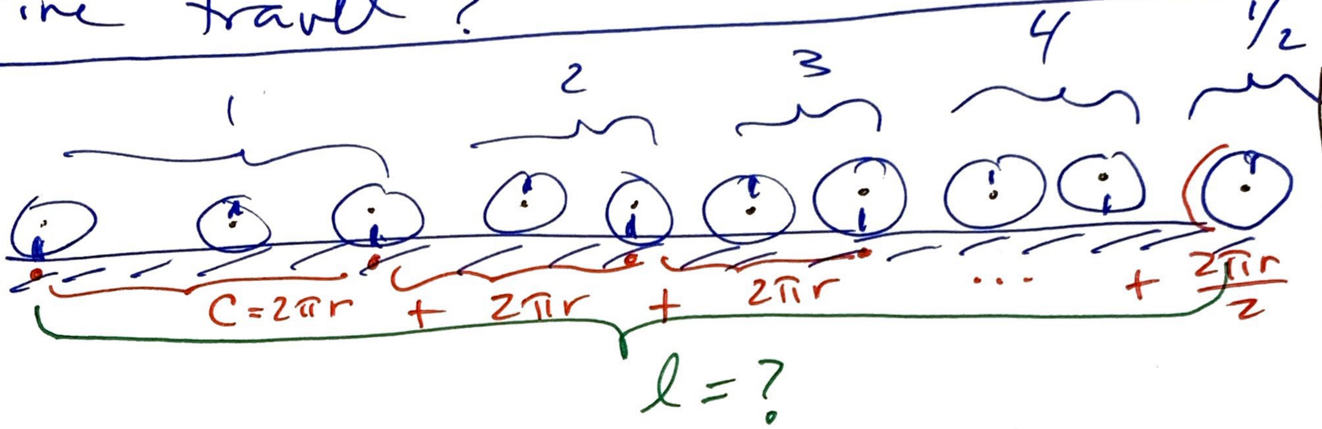
$\boxed{360° = 2\pi^r}$ $\xrightarrow{\text{OR}}$ $\boxed{180° = \pi^r}$

Also: $\boxed{1 \text{ revolution} = 360° = 2\pi^{rad}}$

EX A bicycle tire rotates $4\frac{1}{2}$ times. How many radians was subtended by the tire's valve stem?

$$\theta^{rad} = (4.5\text{ rev})\left(\frac{2\pi^{rad}}{1\text{ rev}}\right) = 9\pi^r \approx \boxed{28.27^{rad}}$$

Ex In the previous example if $r = 13''$ then how far down the road did the tire travel?



Formula : $\boxed{l = r\theta}$

$(2\pi r) \cdot 4\frac{1}{2} = 2\pi 13'' \frac{9}{2}$

$= 117\pi$

$= 367.57''$

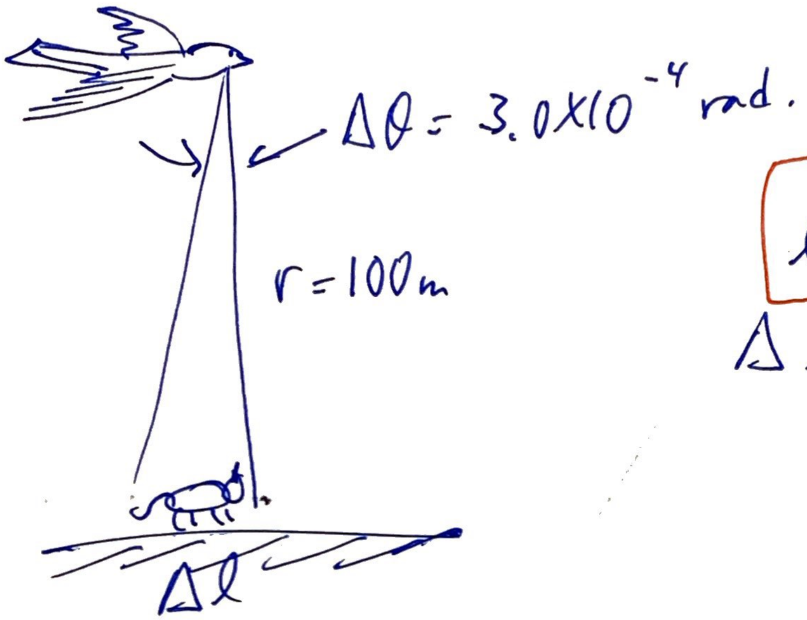
$= 13''(28.27^{rad})$

$= \underline{367.5''}$ or $30.6ft$

about 10 yards

Ex A Hawk can see, and just distinguish, a rat on the ground if the subtended angle is 3×10⁻⁴ radians or larger

Q: If the Hawk is 100m overhead how small can the rat be to not be observed by the Hawk?



$$\boxed{l = r\theta}$$

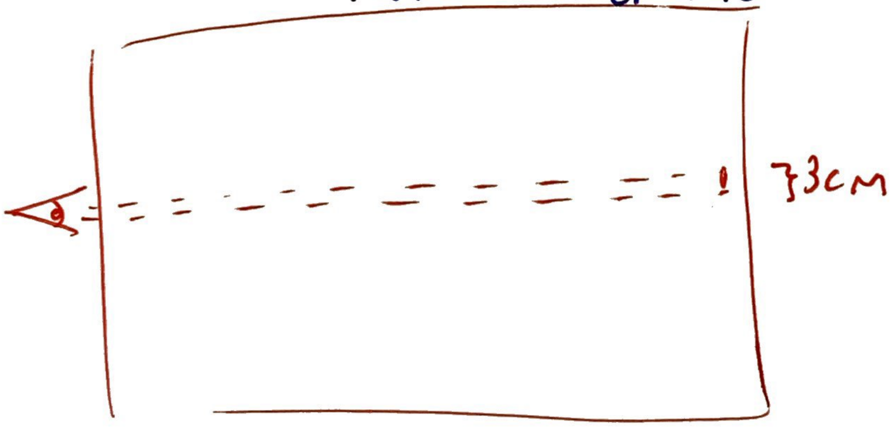
$$\Delta l = r\,\Delta\theta$$

$$= (100\,\text{m})(0.0003\,\text{rad})$$

$$= 0.03\,\text{m}$$

$$= \boxed{3\,\text{cm}} \text{ or less.}$$

100m Soccer Field



- angular velocity "ω" $\begin{cases} \Omega \text{ big omega} \\ \omega \text{ little omega} \end{cases}$

average angular velocity

$$\bar{\omega} \equiv \frac{\Delta\theta}{\Delta t}$$

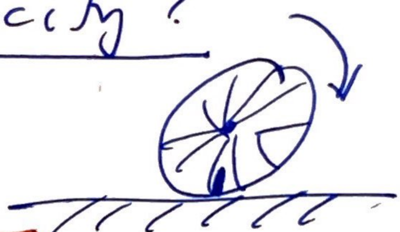
$[\omega]$ = radians/sec

$\omega = \frac{d\theta}{dt}$ calculus

- rpm = rev. per min

Ex A 13" bicycle rotates 4.5 times in 7 secs

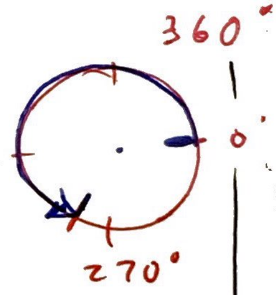
what is the angular velocity?



$$\omega = \frac{\Delta\theta}{\Delta t}$$

$$= \frac{4.5\,\text{rev}}{7\,\text{sec}} = \boxed{0.643\ \text{rev/sec}}$$

$$= \frac{0.643\,\text{rev}}{\text{sec}}\left(\frac{360^\circ}{\text{rev}}\right)$$

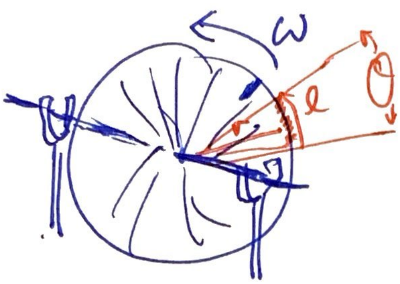


$$= \boxed{231.43^\circ/\text{sec}}\left(\frac{\pi\,\text{rad}}{180^\circ}\right)$$

$$= \boxed{4.04\ \text{rad/sec}}\left(\frac{60\,\text{sec}}{\text{min}}\right)\left(\frac{1\,\text{rev}}{2\pi\,\text{rad}}\right)$$

$$= \boxed{38.6\ \text{rpm}}$$

⊛ Angular velocity vs. Linear velocity

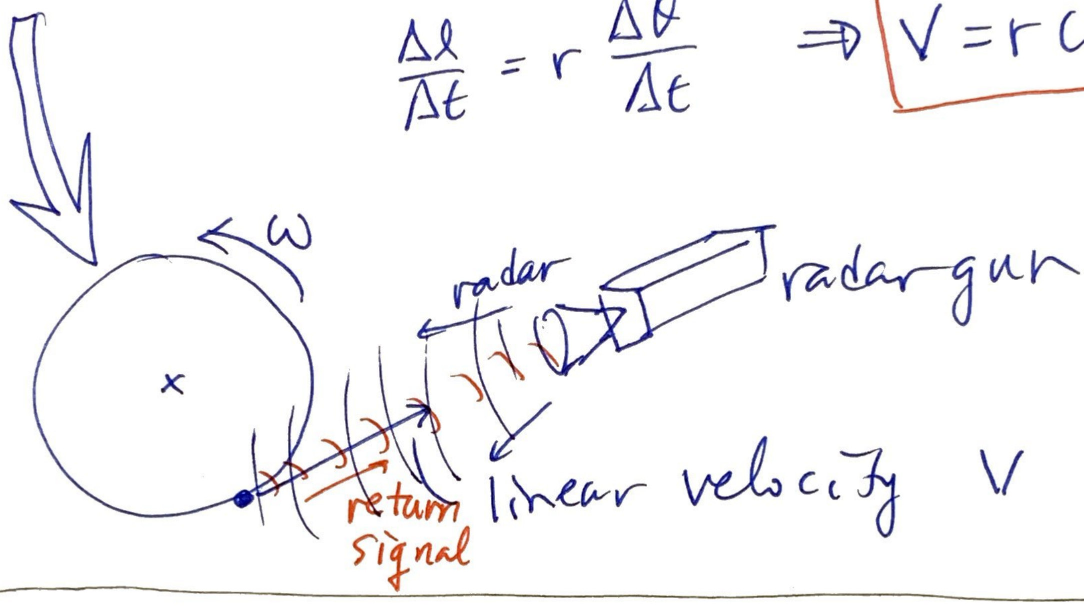


- Recall

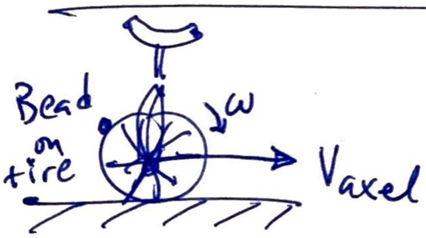
$$l = r\theta$$

$$\frac{\Delta l = r\Delta\theta}{\Delta t}$$

$$\frac{\Delta l}{\Delta t} = r\frac{\Delta\theta}{\Delta t} \Rightarrow \boxed{V = r\omega}$$



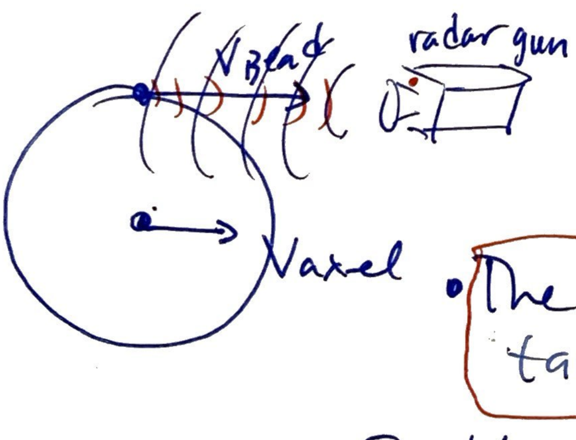
EX For the tire in the previous example: How fast is it moving down the road?



$$V_{Bead} = r\omega$$

$$= 13'' \left[4.04 \,{}^{r}/_{sec}\right]$$

$$= \underline{\underline{52.52 \,''/sec}}$$



- The axel speed is $\frac{1}{2}$ of the tangential velocity, V_{Bead}

So $V_{axel} = \frac{52.52}{2} = \boxed{26.26 \,''/s} = \boxed{2.188 \frac{ft}{s}}$

EX A carousel rotates at 1 rotation per 6 sec.

Top view

(a) If the outer horse is 2m from the center of rotation how fast will the person travel if they jump off the horse and hit the ground?

$$\boxed{V = r\omega}$$

$$= (2m)\left(\frac{1 \text{ rot}}{6 \sec}\right)\left(\frac{2\pi \text{ rad}}{\text{rot}}\right)$$

$$= \boxed{2.09 \text{ m/s}}$$

(b) If the lion is 1m out from the center, how fast is it clocked by the cop and her radar gun?

$$\boxed{V = r\omega}$$

$$= (1m)\left(\frac{2\pi \text{ rad}}{6 \sec}\right) = \boxed{1.05 \text{ m/s}}$$

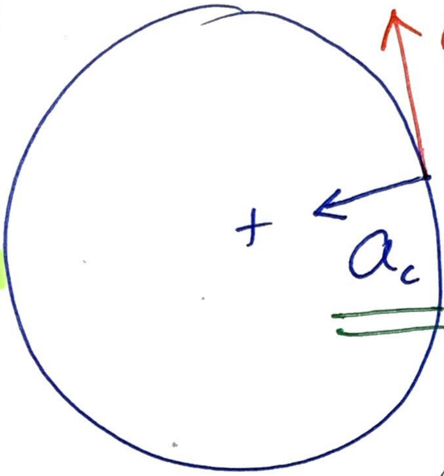
⊛ angular acc'n

$$\boxed{\alpha \equiv \frac{\Delta \omega}{\Delta t}} \longrightarrow \boxed{a_{TAN} = r\alpha}$$

- Linear tangent to circle

In Chpt 5:

- radial ⊥ to circle



a_{TAN}

$V = r\omega$

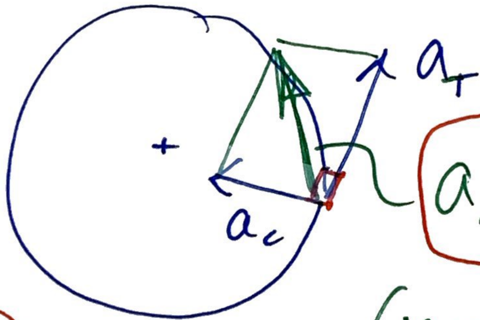
$$a_c = \frac{v^2}{r} = \frac{r^2\omega^2}{r} = r\omega^2$$

- Net acc'n

$V \neq$ const.

$\alpha \neq$ const.

therefore $\omega \neq$ const



- vector magnitude

$$\boxed{a_{Net} = \sqrt{a_c^2 + a_T^2}}$$

(non-uniform motion)

- vector addition

$$\boxed{\vec{a}_{net} = \vec{a}_c + \vec{a}_{Tan}}$$

Ex A merry-go-round is initially at rest while kids get on board. At $t=0$ the platform starts to rotate with an angular acc'l'n of $\boxed{\alpha = 0.06\,r/s^2}$ This acc'l'n lasts for 8 sec until the desired speed is acheived.

(a) What is the angular velocity at 8 sec.

Since $\boxed{\alpha = \frac{\Delta\omega}{\Delta t}}$ mult by Δt $\Rightarrow \alpha \cdot \Delta t = \omega_f - \omega_o$

or $\omega_f = \alpha \cdot \Delta t$ here $\omega_f = (0.06\,r/s^2)(8s)$

$\Rightarrow$ $\boxed{\omega = 0.48\,r/s}$

(b) For a chied at 1.5 m from the center what is its linear speed? (radar gun)

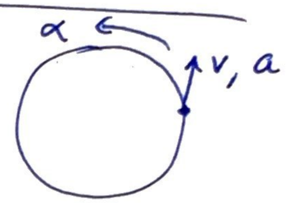
$\boxed{V = r\omega}$

$V = (1.5\,m)(0.48\,rad/s)$

$\underline{V = 0.72\,m/s}$

(c) what is the child's tangential acc'n during start up?

$$\boxed{a_{Tan} = r\alpha}$$



$$= (1.5\,m)(0.06\,r/s^2)$$

$$a_T = 0.09\ m/s^2$$

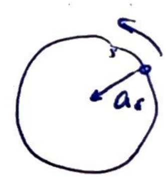
$$\frac{9\ cm/s}{sec}$$

9cm/s every sec

(d) what is the centripetal acc'n @ t=8s

$$\boxed{a_c = \frac{v^2}{r}} = r\omega^2 = (1.5\,m)(0.48\,r/s)$$

$$a_c = 0.72\ m/s^2$$



(e) what is the net acc'n @ t=8 sec

$$\vec{a}_{net} = \vec{a}_c + \vec{a}_{Tan}$$

$$\boxed{\|\vec{a}_{net}\| = \sqrt{a_c^2 + a_{Tan}^2}}$$

$$= \sqrt{(0.72\ m/s^2) + (0.09\ m/s^2)^2} = 0.73\ m/s^2$$

Angular vs. Linear Kinematics (Chpt 2)

Linear	Connectors *	Angular
$V = V_0 + at$	$V = r\omega$	$\omega = \omega_0 + \alpha t$
$x = x_0 + V_0 t + \frac{1}{2}at^2$	$x = r\theta$	$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$
$V_f^2 = V_0^2 + 2a\Delta x$		$\omega_f^2 = \omega_0^2 + 2\alpha\Delta\theta$
$a = \frac{\Delta v}{\Delta t}$	$a = r\alpha$	$\alpha = \frac{\Delta\omega}{\Delta t}$
$V = \frac{\Delta x}{\Delta t}$		$\omega = \frac{\Delta\theta}{\Delta t}$

Also usefull: $\omega = 2\pi f$ and $f = \frac{1}{T}$

* we assume that wheels/pulleys roll w/o slipping

Ex A centrifuge acc'lts a blood sample from rest to 20,000 rpm in 30 seconds. Assume constant acc'ln (12)

(a) Find the angular acceleration, α

$$\omega_0 = 0, \quad \omega_f = 20000\ \text{rpm}, \quad t = 30\ \text{s}$$

$$\boxed{\alpha = \frac{\Delta\omega}{\Delta t}} = \frac{\omega_f - \omega_0}{t} = \frac{\left(20000\ \frac{\text{rot}}{\text{min}}\right)\left(\frac{2\pi^r}{\text{rot}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right) - 0}{30\ \text{s}}$$

$$\boxed{\alpha = 69.81\ ^{\text{rad}}/\text{s}^2}$$

(b) Through how many revolutions did the centrifuge travel to get up to speed?

$$\boxed{\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2}$$

$$= 0 + 0\cdot t + \frac{1}{2}(69.81\ ^r/\text{s}^2)(30\ \text{s})^2 = \underline{\underline{31{,}416}}\ \text{radians subtended}$$

$$= (31416\ \text{rad})\left(\frac{1\ \text{rev}}{2\pi^r}\right)$$

$$= \boxed{5000\ \text{rotations}}$$

Alt formula: $\boxed{\omega_f^2 = \omega_0^2 + 2\alpha\cdot\theta}$

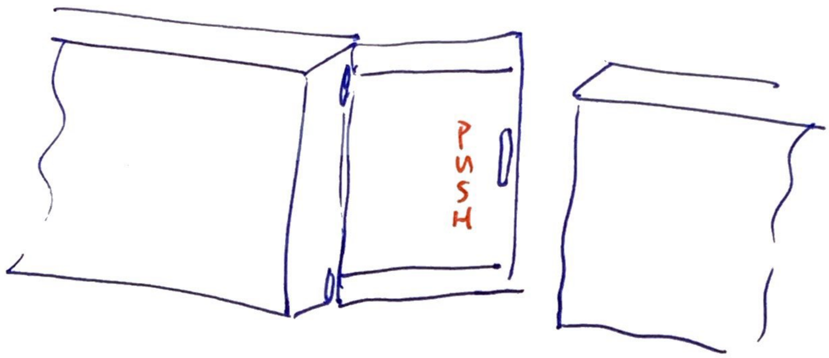
$$\theta = \frac{\omega_f^2 - \omega_0^2}{2\alpha}$$

$$\theta = \frac{\left[20000\left(2\pi\ ^r/\text{rev}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right)\right]^2 - 0^2}{2(69.81\ ^r/\text{s}^2)}$$

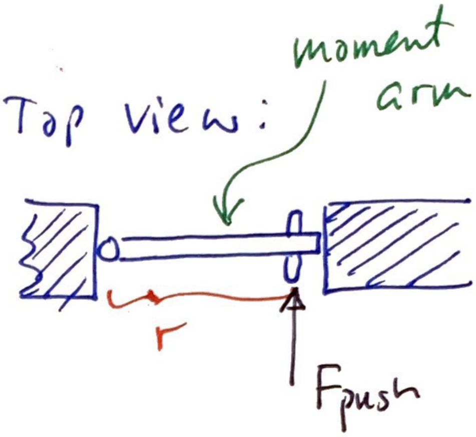
$\theta = \underline{31417}$ radians

⊛ Angular Force [Torque]

"Twisting Force"



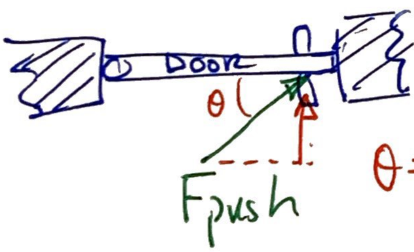
Top view:



$$\boxed{\overrightarrow{\text{Torque}} \equiv r\,\overrightarrow{F}_{\perp}}$$

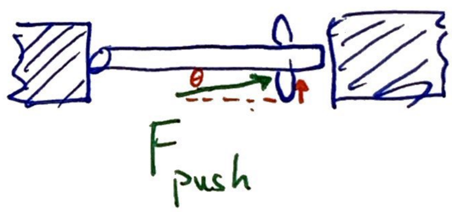
$F_{\perp}$ = Force perpendicular to moment arm

- max torque is acheived when applied force is $\perp$ to the moment arm
- other angle are less efficient

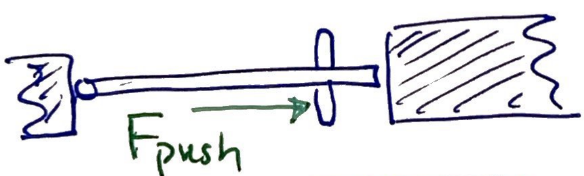


$\theta = 45°$

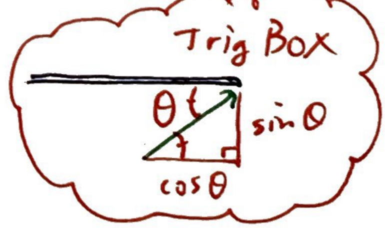
-OR-



$\theta = 20°$



$\theta = 0°$

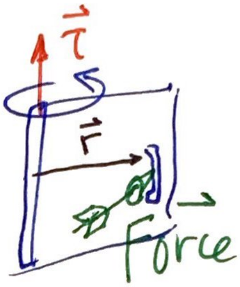


magnitude: $\boxed{\tau = rF\sin\theta}$

- Dimensions

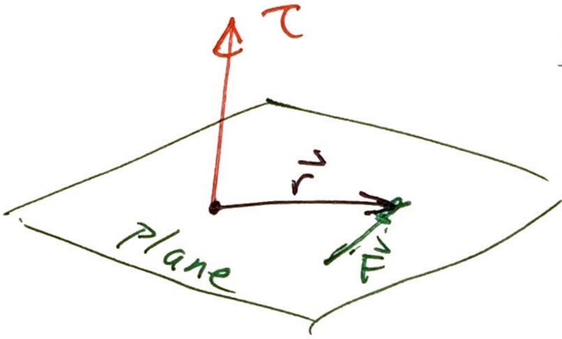
$$[\tau] = m \cdot N \qquad \{\text{wait... this is Work} = F \cdot d\}$$

- Direction

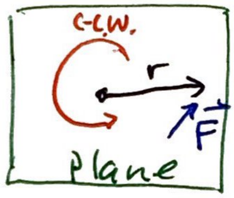


$\vec{\tau} \perp \vec{r}$ & $\vec{F}$ vectors

$$\left[\vec{\tau} = \vec{r} \times \vec{F}\right]$$
Calculus

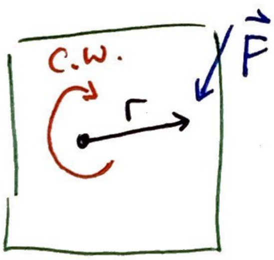


- Positive Torque is torque that tends to evolk counter-clockwise motion



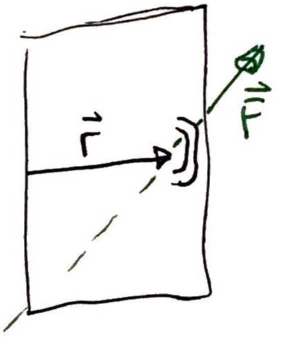
counter clock wise so

$$\tau > 0$$



$$\tau < 0$$

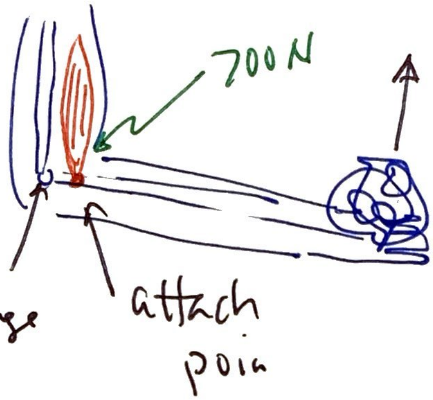
- Right Hand Rule



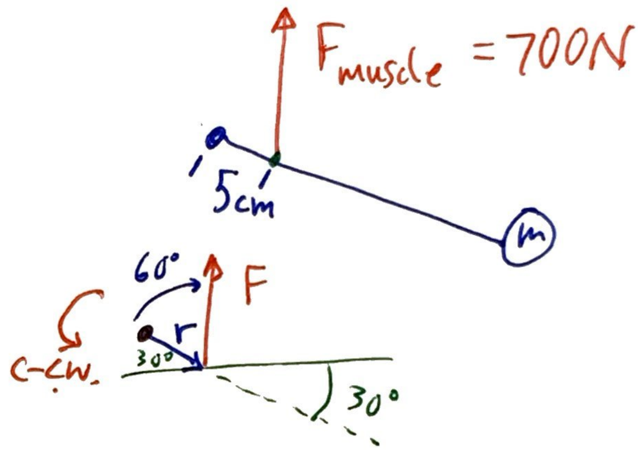
curl towards $\vec{F}$

Ex If your forearm is at 30° below your the horizontal and your Bicepts apply 700 N to lift a mass up, what is the torque at the elbow if the Bicepts is attached at 5cm from the hinge point of your elbow?

(i)



(ii)



(iii)

$$\tau = rF\sin\theta$$

(iv)

$$\tau = (0.05m)(700N)\sin 60°$$

$$\boxed{\tau = 30.3\ mN}$$

U.S. Torque wrench

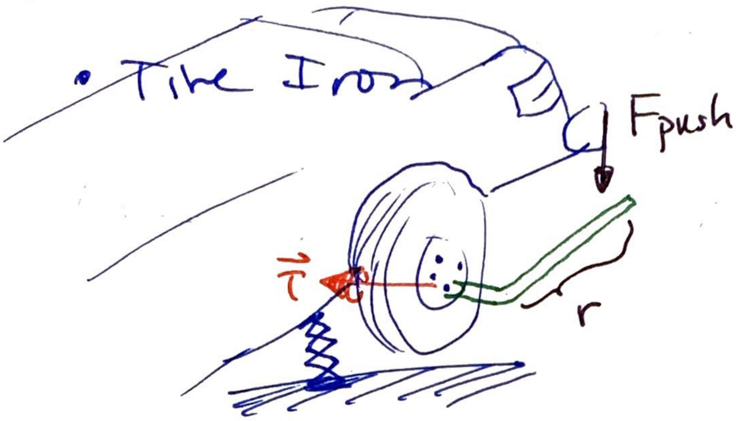
ft-lbs

dyne · cm

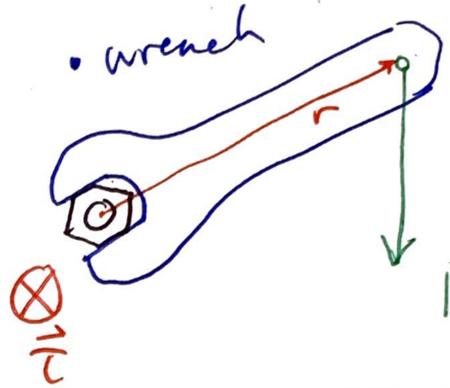
N · cm

Applications

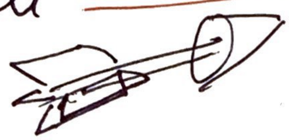
• Tire Iron



• wrench



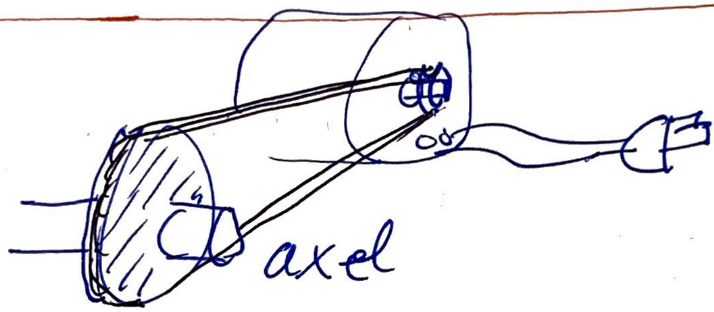
vector bubble

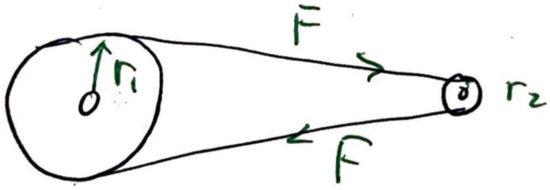


Front view (out of page) $\odot$

Rear view (into page) $\oplus$

• motors





more torque is being applied to the larger pulley