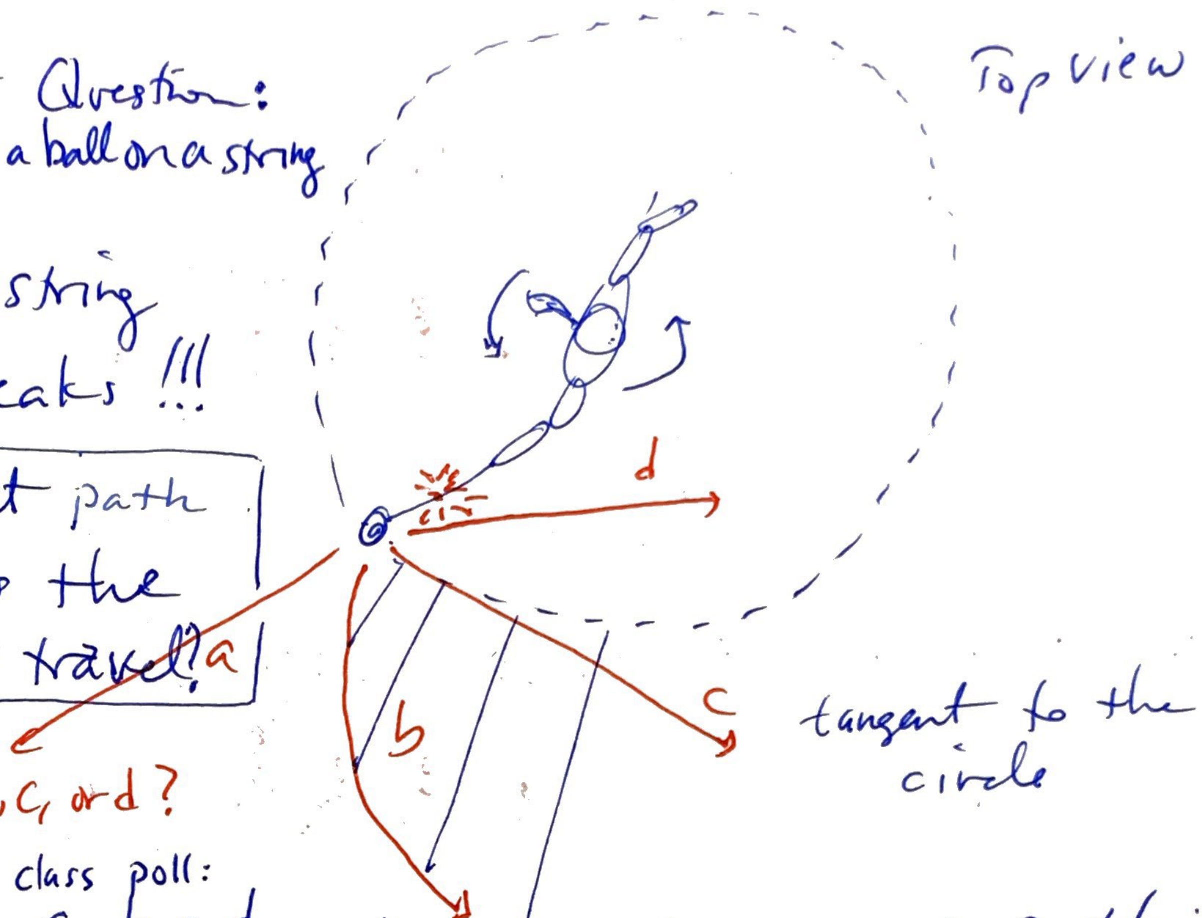
Chapter 5 Circular Motion

{ A : Circ. Motion
B : Gravity

Concept Question:
swing a ball on a string



- The string Breaks !!!

Q: what path does the mass travel?

a, b, c, or d?

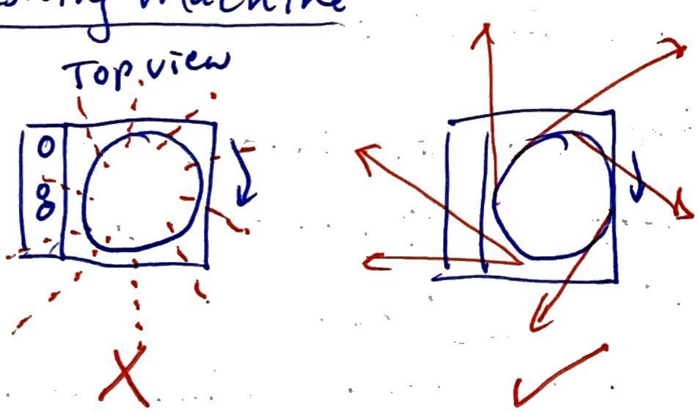
Ans: class poll:

a	b	c	d
1		1111	1

"c" is the correct path:

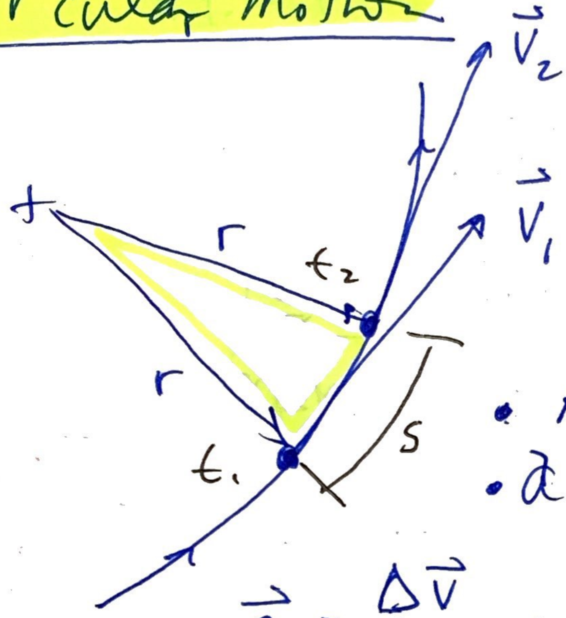
- There is no longer a force altering the path of the object & it travels in a straight line

- Washing Machine



BTW Newton asked: Why is the earth going around the SUN instead of in a straight line past the SUN? He conceived that there must be an invisible force between the two. ⇒ Law of Gravity (4th Law)

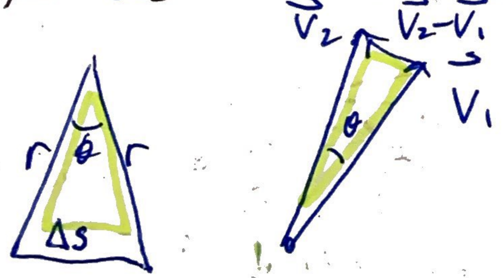
A. Circular motion



- $\|\vec{v}_1\| = \|\vec{V}_2\|$ but they point in different directions.
- Assume const. speed,
- assume circular motion
- $\vec{a} = \frac{\Delta \vec{V}}{\Delta t}$, we need not only change the magnitude, but we can also change only the direction

⊛ we get acc'ln due to the change of direction

let ΔS = arc length between t_1 & t_2



use similar Δ's:

$\boxed{\|\vec{V}_1\| \text{ is to } \|V_2 - V_1\| \text{ as } r \text{ is to } \Delta S}$

use similar triangles

Cont ⇒

• proportional triangles:

Δs is to r AS $\Delta \vec{v}$ is to $\vec{v}$

$$\boxed{\frac{\Delta s}{r} = \frac{\Delta v}{v}} \quad \div \Delta t$$

$$\frac{\Delta s}{r\,\Delta t} = \frac{\Delta v}{\Delta t}\frac{1}{v}$$

solve for $\frac{\Delta v}{\Delta t}$

$$\frac{\Delta v}{\Delta t} = v \cdot \left(\frac{\Delta s}{\Delta t}\right) \cdot \frac{1}{r}$$

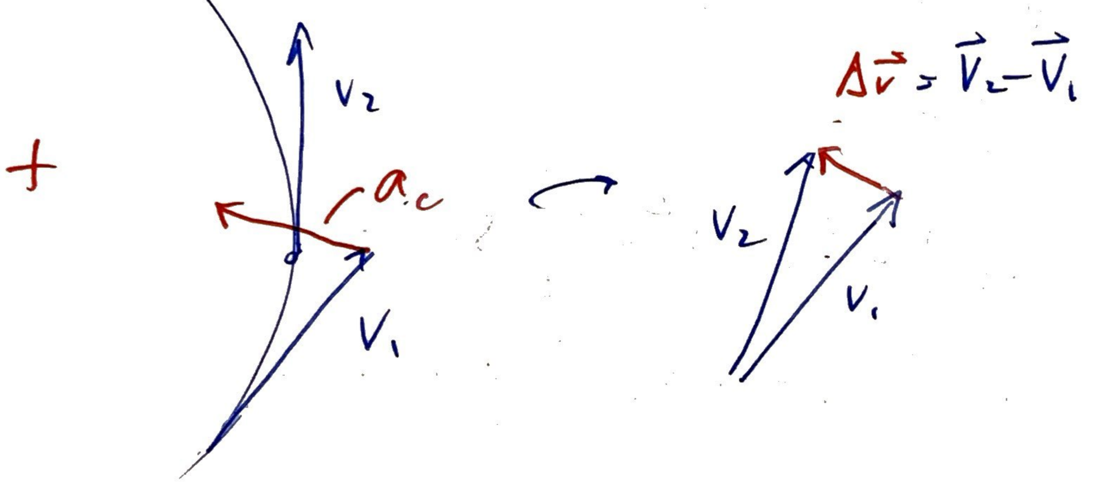
but $\frac{\Delta \vec{v}}{\Delta t} \equiv \vec{a}$ and $\frac{\Delta s}{\Delta t} = v$

$$\Rightarrow \quad \boxed{a_c = \frac{v^2}{r}}$$

acc'n due to uniform circular motion

centripetal force

• The direction is towards the center of the circle



⊛ Frequency and Period

T = to be the period = time for one complete revolution about the circle.

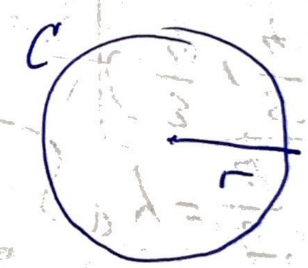
f = the frequency of the motion is the number of complete circuits in a fix time, 1 sec.

- **Dimensions:**

$[T] = \text{sec}/\text{cycle}$, $[f] = \text{cycles}/\text{sec}$

$$\boxed{T = \frac{1}{f} \quad \text{or} \quad f = \frac{1}{T}}$$

- circumerence $C = 2\pi r$



⇒ speed of object $v = \dfrac{\text{Circumference}}{\text{period}}$

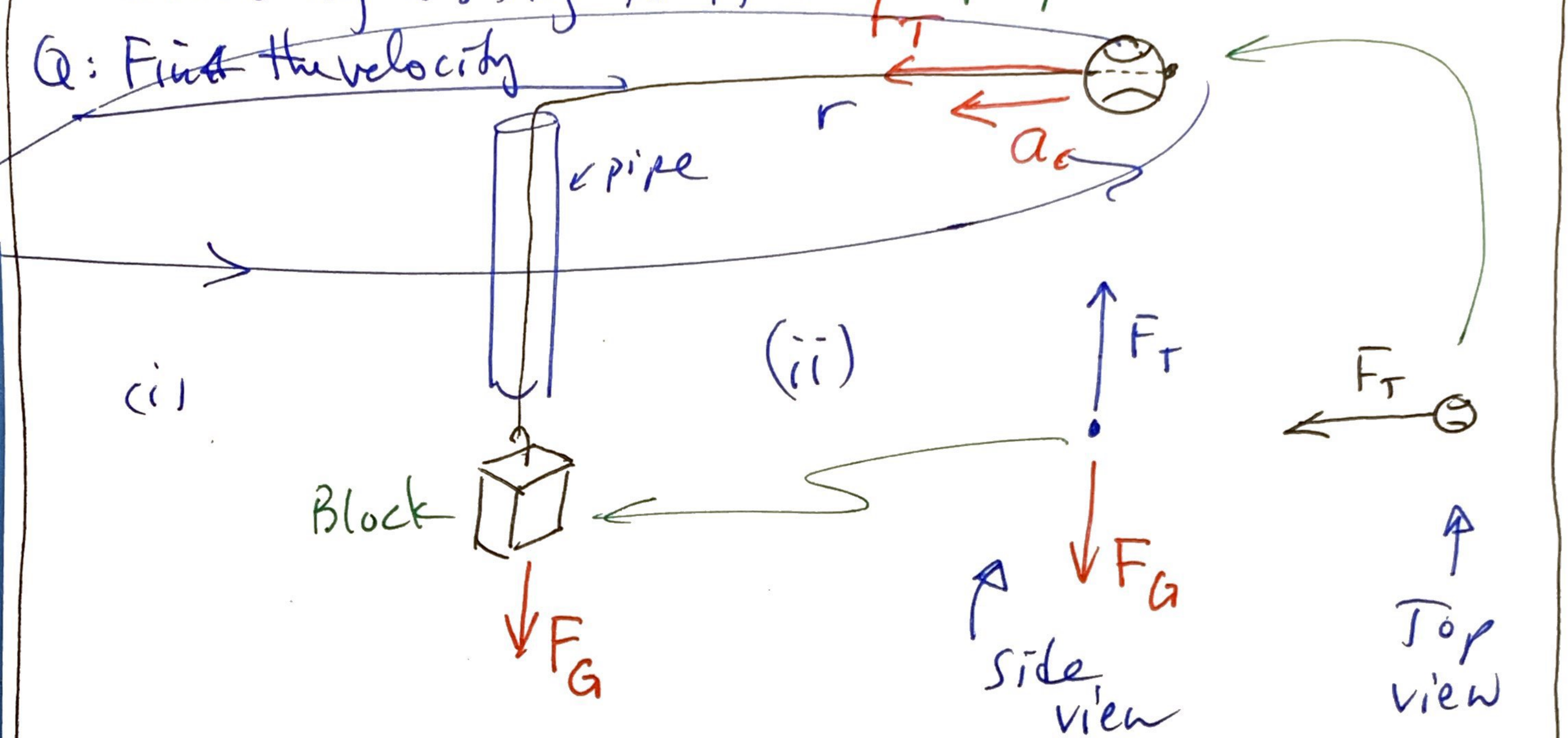
$$\boxed{v = \frac{2\pi r}{T}}$$ speed of object with period "T" in radius of circle "r"

—OR—

$$\boxed{v = 2\pi r f}$$

EX (a) you swing a Baseball in a circle by attaching a string to it. Keep the block motionless

Q: Find the velocity



(iii) eqns for circular motion

radial: $\boxed{\sum F_{radial} = m a_c}$ $\quad a_c$ centripetal acc'n

$$\boxed{a_c = \frac{v^2}{r}}$$

• radial: $F_T = m\left(\frac{v^2}{r}\right)$

• vertical: $F_T - F_G = 0$ ← no vertical motion of the hanging block

(iv) Solve $\quad m_{Block}\, g = m_{Baseball}\left(\frac{v^2}{r}\right)$

$\Rightarrow \quad r\, m_{Block}\, g = m_{Baseball} \cdot v^2$

$\Rightarrow \quad \boxed{v = \sqrt{\frac{r g\, m_{Block}}{m_{Baseball}}}}$ If hanging mass is not moving then v is given here

Ex (b) We time 10 complete circuits to take 25 s

((10 revolutions in 25 seconds))

Find the centripetal acc'l

(iii) eqns: $a_c = \frac{v^2}{r} = \left(\frac{2\pi r}{T}\right)^2 \cdot \frac{1}{r}$

$v = \frac{2\pi r}{T}$

$\Rightarrow \boxed{a_c = \frac{4\pi^2 r}{T^2}}$ let $r = 20\,cm$

$f = \frac{10 \text{ cycles}}{25 \text{ sec}} = 0.4\, c/s$

$T = \frac{1}{f} = \frac{1}{0.4\,Hz} = \underline{\underline{2.5\,s}}$

BTW

cycle/sec = Hertz

$[f] = Hz$

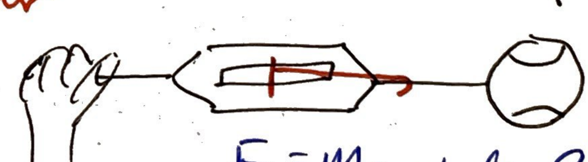
$\Rightarrow a_c = \frac{4\pi^2 (0.2\,m)}{(2.5\,s)^2}$

$\boxed{a_c = 1.26\, m/s^2}$

towards the center

Concept:

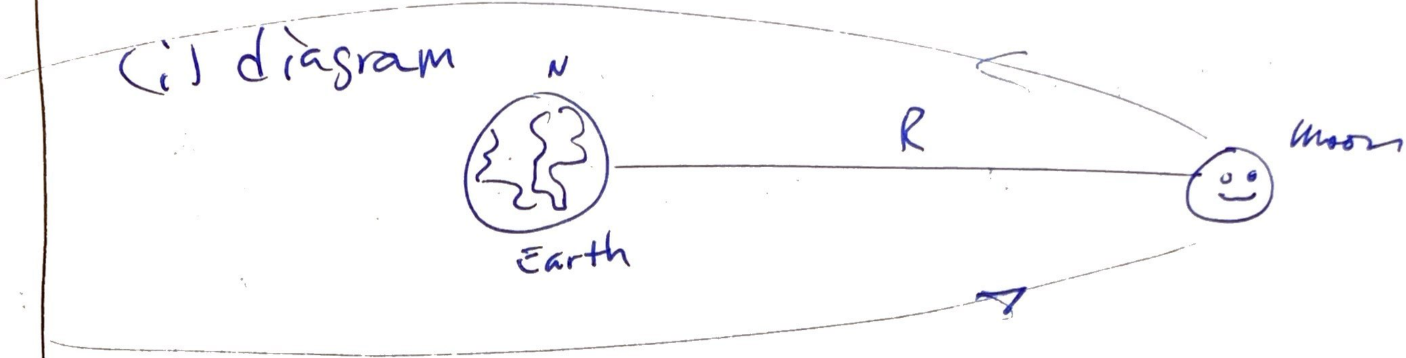
$\leftarrow \underline{\underline{F_T = m_{Baseball} \cdot g}}$



$\underline{\underline{F_T = m_{Baseball}\, a_c}}$

Ex: Find the moon's centipetal acc'ln

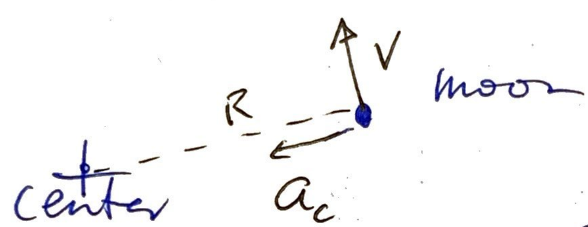
(i) diagram



$T = 27.3 \text{ days}$

$R = 384{,}000 \text{ km}$

(ii)



(iii) eqns: $V = 2\pi r f$, $\boxed{a_c = \frac{V^2}{R}}$, $f = \frac{1}{T}$

(iv) Do the math:

$V_{moon} = \frac{2\pi(384{,}000{,}000 \text{ m})}{T}$, $T = 27.3 \text{d} \left(\frac{24 \text{h}}{\text{d}}\right)\left(\frac{3600 \text{s}}{\text{h}}\right) = \underline{2{,}358{,}720 \text{ s}}$

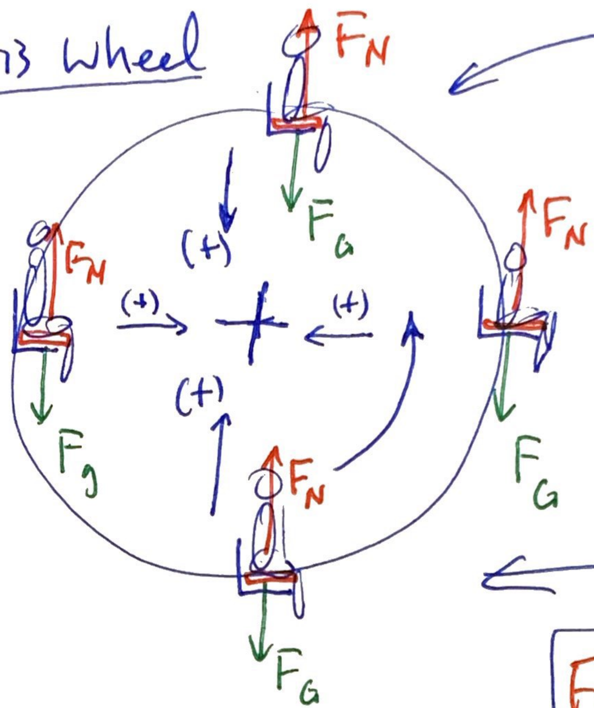
$V = \frac{2\pi(3.84 \times 10^{+8} \text{ m})}{2358720 \text{ sec}}$

$\boxed{V = 1022.9 \text{ m/s}} = \underline{2{,}300 \text{ mi/hr}} = \text{mach } 3$

So $a_c = \frac{V^2}{R} = \frac{(1022.9 \text{ m/s})^2}{3.84 \times 10^8 \text{ m}} = \boxed{0.0027 \text{ m/s}^2 \text{ towards the earth}}$

Vertical Circles: Ferris Wheel, pilot in a jet, etc.

* Ferris Wheel



TOP

$$\boxed{F_G - F_N = m\frac{v^2}{r}}$$

$F_N = F_G$

$F_N = F_G$

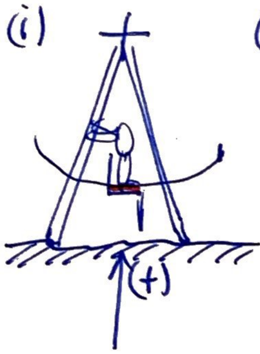
$$\sum F_{radial} = m a_c$$

$$\boxed{F_N - F_G = m\frac{v^2}{r}}$$

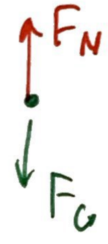
BOT

EX How much does a bathroom scale read out when sat upon by a 70kg person at the bottom of the swing of a swing with radius 3m? Assume the speed is 2.2 m/s horizontally

(i)



(ii)



(iii) $F_N - F_G = m\frac{v^2}{r}$

(iv) $F_N = F_G + \frac{mv^2}{r}$

$$F_N = mg + m\frac{v^2}{r} = m\left(g + \frac{v^2}{r}\right)$$

$$F_N = 70\text{kg}\left(9.8\,\text{m/s}^2 + \frac{(2.2\,\text{m/s})^2}{3\text{m}}\right)$$

$$F_N = 70\text{kg}\,(11.4\,\text{m/s}^2) = \boxed{798.9\text{N}}$$

$\div 9.8 = \boxed{1.2\,g}$

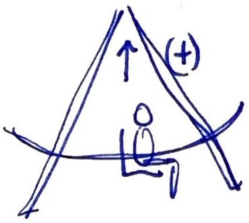
• effective mass (weight)

"F_G" $= mg$

$m = \frac{798.8\text{N}}{9.8} = 81.5\text{kg}$ Like

14% heavier

EX Swing set: How much does a 70 kg person "weigh" at the bottom of a swing if the swing has a radius of 3.0 m and your speed is 2.2 m/s at the bottom



$$\Sigma F = ma$$

$$F_N - F_G = m\frac{v^2}{r}$$

$$\boxed{F_N = F_G + \frac{mv^2}{r}}$$

$$= mg + \frac{mv^2}{r}$$

$$= m\left[g + \frac{v^2}{r}\right]$$

$$= 70\,kg\left[9.8\,\frac{m}{s^2} + \frac{(2.2\,m/s)^2}{3\,m}\right]$$

$$= 70\,k\left[11.41\,m/s^2\right]$$

$$= \underline{\underline{798.9\,N}}$$

effective "weight"

$$m = \frac{F}{g} = \frac{798.9\,N}{9.8}$$

$$= \boxed{81.5\,kg}$$

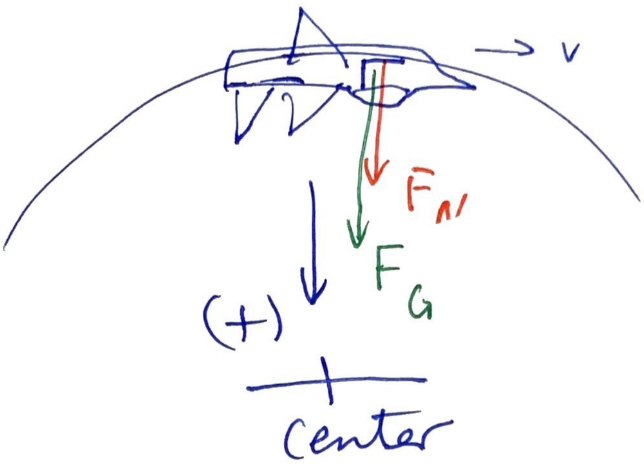
$$\frac{81.5 - 70}{70} = 0.16$$

or 16% heavier

$$\frac{11.41\,m/s}{9.8} = \boxed{1.2\,g\text{'s}}$$

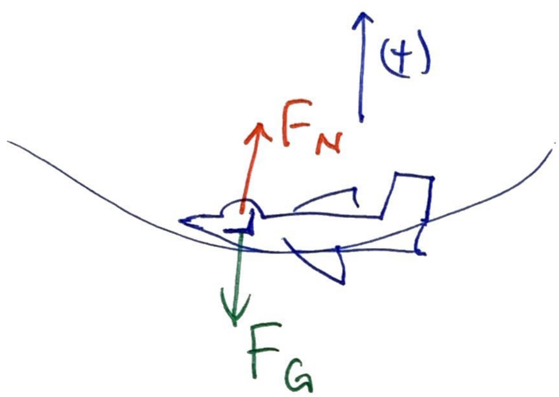
you feel 1.2 "g" but weigh one "g"

Inverted verticle circular motion



$$\sum F = ma_c$$

$$\boxed{F_N + F_G = m\frac{v^2}{r}}$$



$$\sum F = ma_c$$

$$\boxed{F_N - F_G = m\frac{v^2}{r}}$$

Ex (a) A fighter does an inverted vertical turn @ 50 m/s. If the radius is 275 m. what does the pilot feel? let the pilot weigh

$$F_N + F_G = m\frac{v^2}{r}$$

$$F_N = \frac{mv^2}{r} - F_G$$

$$F_N = m\left[\frac{v^2}{r} - g\right]$$

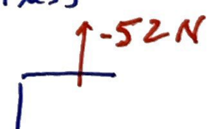
$$F_N = 73\,kg\left[\frac{(50\,m/s)^2}{275\,m} - 9.8\right]$$

$$F_N = 73\,kg\,[9.09 - 9.81]$$

$$F_N = 73\,kg\,[-0.71]$$

$= -51.76\,N$

weightless



"pilot falls off of seat"

(b) at what speed will the pilot just barely stay in his seat?

(iv)
$$F_N = m\left[\frac{v^2}{r} - g\right]$$
$$0 = m\left[\frac{v^2}{r} - g\right] \Rightarrow \boxed{v = \sqrt{gr}}$$

$$v = \sqrt{(9.8\,m/s^2)(275\,m)}$$
$$\boxed{v = 51.9\,m/s}$$

(c) Next attempt the pilot speeds up to 65 m/s. Now what does he feel.

$$F_N = m\left[\frac{v^2}{r} - g\right]$$

$$F_N = 73\,kg\left[\frac{(65\,m/s)^2}{275\,m} - 9.8\right]$$

$$F_N = 73\,kg\,[15.36 - 9.8]$$

$$F_N = 73\,kg\,[5.56\,m/s^2]$$

$\boxed{F_N = 406.1\,N}$ Bathroom scale

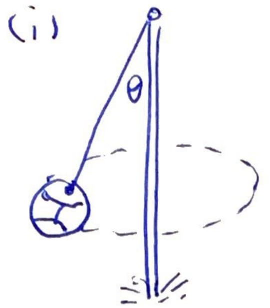
- effective "weight"

$$m = \frac{F_N}{g} = \text{"}41.4\,kg\text{"}$$

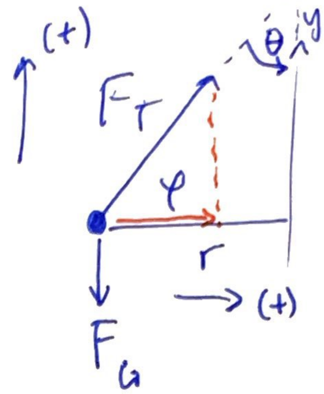
ilot feels $\frac{41.4-73}{73} = 0.$ his weight $\boxed{43\% \text{ less weight}}$

* Conical Pendulum {a form of Horiz. motion}

(i)



(ii)



(iii)

- radial:

$$\sum F = m\frac{v^2}{r}$$

$$\boxed{F_T \cos\varphi = m\frac{v^2}{r}}$$

- verticle:

$$\sum F = m\cancel{a_y}^{\,0}$$

$$\boxed{F_T \sin\varphi - F_G = 0}$$

- Since $\varphi = 90 - \theta$

$$\sin(\varphi) = \sin(90 - \theta) = \cos\theta$$
$$\cos(\varphi) = \cos(90 - \theta) = \sin\theta$$

conical pend.

$\Longrightarrow$ Newtons Law:

$$\boxed{\begin{aligned} r &: \quad F_T \sin\theta = m\frac{v^2}{r} \\ y &: \quad F_T \cos\theta = mg \end{aligned}}$$

(iv) Some math

Math Trick: divide one eqn by the other

$$\frac{F_T \sin\theta}{F_T \cos\theta} = \frac{mv^2/r}{mg}$$

$$\boxed{\tan\theta = \frac{v^2}{gr}}$$

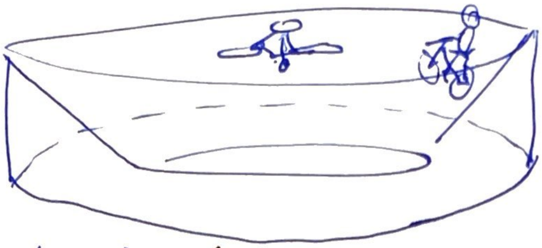
conical pendulum eqn

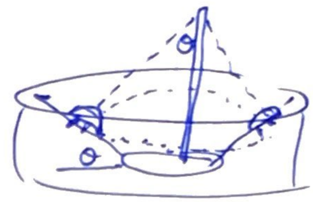
Ex You play tether ball and keep it moving at $\theta = 30°$. If $r = 1m$ and $m = 2kg$ Find v

$$v = \sqrt{gr\tan\theta} = \sqrt{(9.8)(1m)(\tan 30°)} = \boxed{2.38\ m/s}$$

The Banked Curve (velodrome)

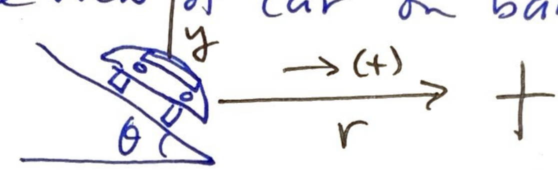
(i)



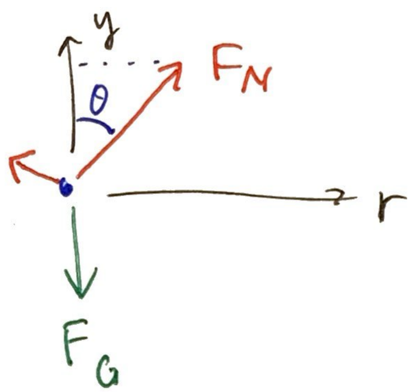


side view of car on banked curve.

(i)



(ii)



0 for now

(iii) • radial direction

$$\sum F = m\frac{v^2}{r}$$

$$\boxed{F_N \sin\theta = m\frac{v^2}{r}}$$

• vertically:

$$\boxed{F_N \cos\theta - F_G = 0}$$

no motion off the circular plane

(iv) Some math:

$$F_N \sin\theta = mv^2/r$$
$$F_N \cos\theta = mg$$

$$\boxed{\tan\theta = \frac{v^2}{rg}}$$

Banked curve

ideal banke curve speed.

Comments: If we let $F_f = 0$ then if the car goes at any speed but if $\boxed{v \neq \sqrt{rg\tan\theta}}$ the car will slip into the center arena, OR, if the car slip up over the edge depending on if its speed is too slow or too fast respectively.

EX A 1000 kg Car rounds a curve on a road having a radius of 50 m. The car is moving at a speed of 54 km/hr

Q: If the road is dry will the car slip off the curve if $\mu_s = 0.6$ {rubber on cement}

(a) Lets compare the speed to the design speed:

$$V_{des} = \sqrt{R g \mu_s}$$

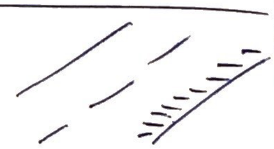
$$= \sqrt{(50\,m)(9.8\,m/s^2)(0.6)}$$

$$= 17.15\,m/s \left(\frac{1\,km}{1000\,m}\right)\left(\frac{3600\,s}{hr}\right)$$

$$= \boxed{61.7\,km/hr}$$ design ...

Since $54 < 61.7$ we do NOT slide off the road.

(b) If there is an icy (black ice) region on the curve with $\mu_s = 0.25$ {rubber on ice} will the car stay on the curve?



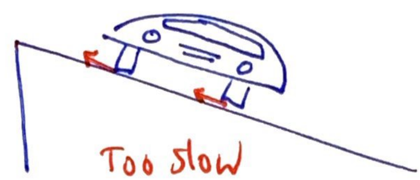
$$V_{ice} = \sqrt{(50\,m)(9.8\,m/s^2)(0.25)}$$

$$= \boxed{39.8\,km/hr}$$ max before slidding.

⊛ Car will slide off road ⊛

Banked curve with friction

(i)



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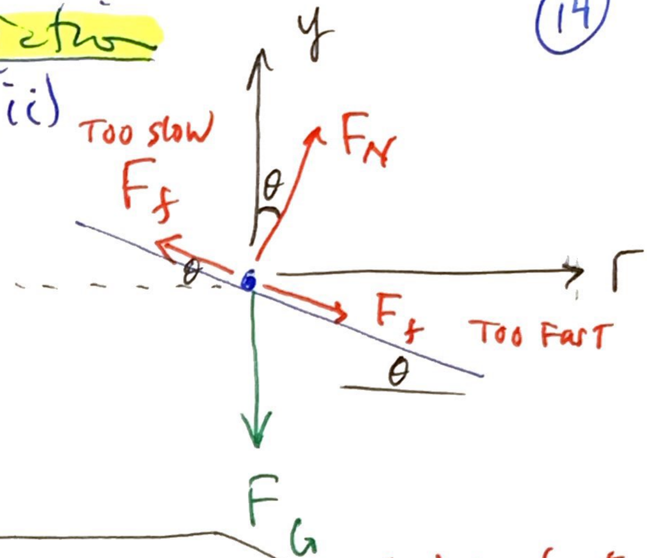
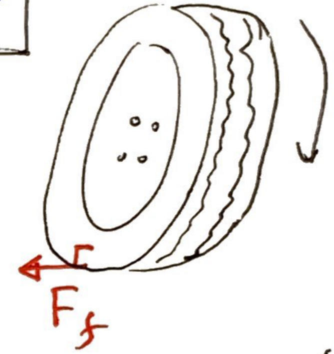


(iii) eqns

$$\boxed{\begin{aligned} \text{radial: } & F_N \sin\theta \pm F_f \cos\theta = m\frac{v^2}{r} \\ \text{vertical: } & -F_G + F_N \cos\theta \mp F_f \sin\theta = 0 \end{aligned}}$$

(+) too fast
(−) too slow

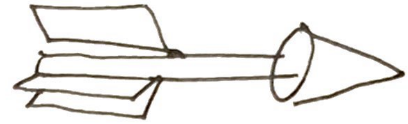


If we are not "skidding sideways"

$F_f = F_{static}$

but friction is applied si

* Flat Curve



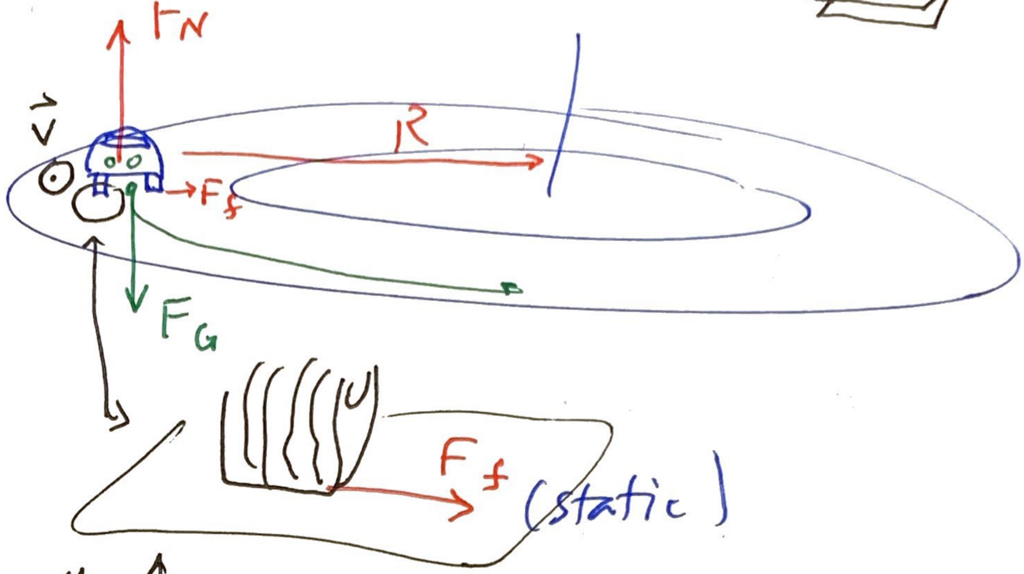
Front view

$\odot$

Rear view

$\oplus$

(i)



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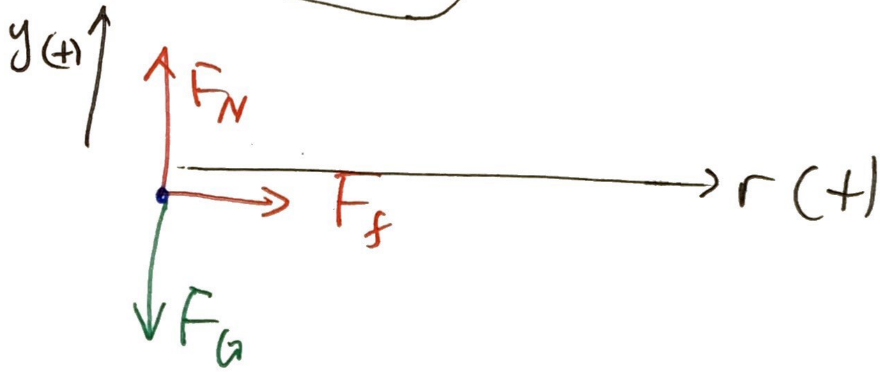

(iii)

$$r: \quad F_f = m a_c$$

$$y: \quad F_N - F_G = 0$$

$$f: \quad F_f = \mu_s F_N$$

static

$$a_c = \frac{v^2}{R}$$

$$F_G = mg$$

$$\boxed{v = \sqrt{R g \mu_s}}$$

$$\mu_s F_G = m\left(\frac{v^2}{R}\right)$$

$$\mu_s \cancel{m} g = \frac{\cancel{m} v^2}{R} \rightarrow \boxed{R = \frac{v^2}{\mu_s g}}$$

* Design radius for desired speed & friction