Chpt 13 B   Gas Laws

⊗ Gas Laws

The attributes of a container of gas particles are

- Temperature ($\propto$ velocity of molecules)
- Pressure ($\propto$ momentum of the particles striking the container walls)
- Volume

Early scientist developed relations between these quantities

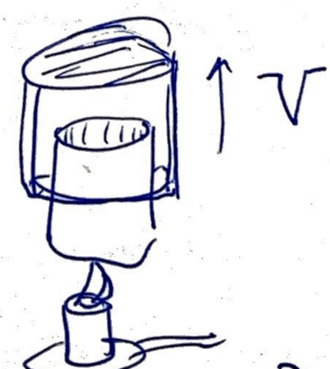
- Boyle's Law : $P \propto \frac{1}{V}$ Increase volume of container and pressure decreases

  $\Rightarrow \boxed{PV = \text{const.}}$ for the same temperature

- Charles's Law : $T \propto V \Rightarrow \boxed{\frac{V}{T} = \text{constant}}$

  Keep pressure the same

  $\Rightarrow$ heat a variable container and it expands



- Guy-Lussac's Law : $P \propto T$ $\boxed{\frac{P}{T} = \text{const}}$

  constant volume

All together : Ideal Gas Law

$$\boxed{PV = nRT}$$

Kelvins = Celcius + 273

- $n \equiv$ number of "moles" of gas (quantity)

$$\boxed{n = \frac{\text{mass of gas in grams}}{\text{the mass of the molecule of the Gas}}}$$

- R = called the universal gas constant

$$\boxed{8.314\ J/mol\cdot K}$$

Ex What is the volume of one mole of any gas at 20°C @ 1 atm. (Std. Atm)

$$PV = nRT$$

$$V = \frac{nRT}{P} = \frac{(1\,mol)(8.314\,\frac{J}{mol\cdot K})(273+20)K}{(1.013\times 10^5\ N/m^2)}$$

$$\boxed{1\,l = 10^{-3}\,m^3}$$

$$V = 22.4\times 10^{-3}\,m^3/mol = \boxed{22.4\,l/mol}$$

Ex How much does a He Party Balloon (2.2 water bottle) weigh? Let radius = 18 cm @ std. atm

- Strategy : get amount, n, in moles convert to gms.

$$n = \frac{PV}{RT} = \frac{(1.013\times 10^5\,N/m^2)(\frac{4}{3}\pi\,[0.18\,m]^3)}{(8.314\ J/mol\cdot K)(273+20)K} = \underline{1.066\,moles}$$

- He has an atomic number of 4 so it is 4 gm/mol

$$(1.066\ mol)(4\,g/mol) = \boxed{4.26\ g\ of\ He}$$

Definitions:

- monoatomic gas (one atom)
  He, Ne, Ar, Kr, Xe, Rn } Nobel Gases.
- diatomic gas: H_2, O_2, N_2 ...
- triatom gasses: ?

Ex Find the weight of air in a standar Living room @ STP if the room is 5m x 3m x 2.5m High

- The atmosphere is 20% O_2 & 80% N_2 so the atomic mass of air is

$$0.20\,(2\times 16u \text{ for } O_2) + 0.80\,(2\times 14u \text{ for } N_2)$$

O^{16}

$$= \boxed{28.8u \text{ for air}}$$

So lets use 0.029 kg for 1 mol of air.

- Now the number of mols in the Living Room is

$$n = \frac{PV}{RT} \qquad \boxed{n = \frac{V}{22.4\,l/mol}} \qquad b/c\ \frac{TR}{P}\text{ @ STP } = 22.4\,l/mol$$

- The weight of the air is

$$\text{weight} = \left(0.029\ \text{kg air}/\text{mol}\right)\frac{(5m \times 3m \times 2.5m)}{[22.4\ l/mol]}\left[\frac{10^3 l}{m^3}\right]$$

$$= 50\text{kg} \quad \{ * 9.8\,m/s^2 \} = 500N \text{ or } \underline{112\,lbs \text{ of air}}$$

Ex Fill a tire to 210 kPa (gauge Pressure) on a cold day (10°C). What is the tire pressure on a HOT day (40°C)?

Use Guy-Lussac's Law $\frac{P}{T} = \text{const.}$

$$\Rightarrow \quad \frac{P_1}{T_1} = \frac{P_2}{T_2}$$

need absolute pressure

$$\Rightarrow \quad P_2 = \left(\frac{T_2}{T_1}\right) P_1$$

gauge atmospheric

$$= \left(\frac{273+40}{273+10}\right)\left(210\times10^3\ \text{Pa} + 1.013\times10^5\ \text{Pa}\right)$$

$$= 344\ \text{kPa}$$

subtract one atm $\Rightarrow$ $\boxed{243\ \text{kPa on the gauge}}$

This is a 16% increase in tire pressure.

Avogadro's Number

$$\boxed{N_A = 6.02\times10^{23}\ \text{molecules/mol}}$$

Then we can convert R to one based on N_A:

$$n = \frac{N}{N_A} \begin{array}{l} \leftarrow \text{molecules in the gas} \\ \leftarrow \text{molecules in a mol} \end{array}$$

The gas law become $PV = nRT = \left(\frac{N}{N_A}\right)RT$

k

$$\boxed{PV = NkT}$$

k = Boltzmann's Const

$$\boxed{k = 1.38\times10^{-23}\ \text{J/K}}$$

Ex Find the mass of a Hydrogen Atom if we Know it to be $1.008u$, u = atomic units

$$m = \frac{\text{weight of 1 mol of H gas}}{N_A}$$

$$= \frac{(1.008u)(10^{-3} kg/g)}{6.02 \times 10^{23} \text{ molecules or atoms per mol}}$$

$$= \boxed{1.67 \times 10^{-27} kg} / \text{H gas molecule}$$

$\{ u/g = \frac{1}{mol}$ or $mol = gm/u \}$?

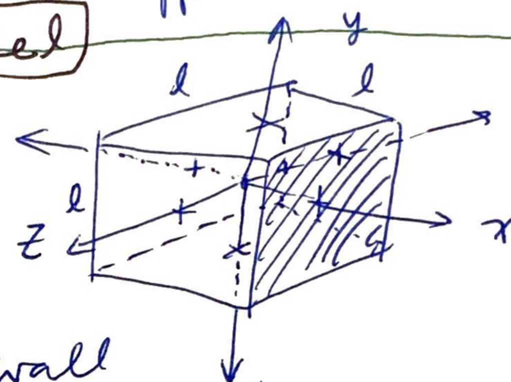
⊛ Statistical Physics {Kinetic Theory of Gas} ⑦

An ideal gas needs

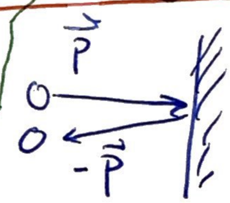
- large numbers
- large distances between atoms/or molec.
- 100% ellastic collisions
- exist in a setting where classical mechanics can be applied {not Quantum Mech}

$\boxed{\text{Start of building out a model}}$

- Consider a box :



- collisions off of a wall experiences twice the momentum change



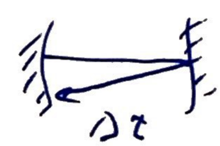
100% ellastic

$$\Delta p = \vec{p} - (-\vec{p}) = 2p$$

- Newtons Law

$$\Delta F = \frac{\Delta P}{\Delta t}$$

- let Δt = time to travel down/across from one side and back



$$\Rightarrow \Delta F_x = \frac{\Delta mv}{\Delta t} = \frac{2mv_x}{2l/v_x} = m\frac{v_x^2}{l}$$

$t = d/v$

average change of force on the x-wall/molecule

$\boxed{\text{BTW}}$: $\Delta F_x = m\frac{V_x^2}{l}$ shows that if

$l \to l/2$ cut in half then $\underline{\Delta F \text{ doubles}}$

ie $1/_{1/2} = 2$

(This confirms Boyle's Law: $P_0 = \frac{1}{V_0}$

the $P = 1/(V_0/2) = \underline{2P_0}$)

- Now sum all the particles contributions to the ΔF_x @ wall $\perp$ to x-axis

$$\Delta F_x = \frac{m}{l}\left(V_{1x}^2 + V_{2x}^2 + \cdots (V_{N_A})_x^2\right)$$

let $N = N_A$ = one mole of gas

$$\boxed{\Delta F_x = \frac{m}{l} N_A \overline{V_x^2}} \quad \text{where } \overline{V_x^2} = \frac{\sum V_{ix}^2}{N_A}$$

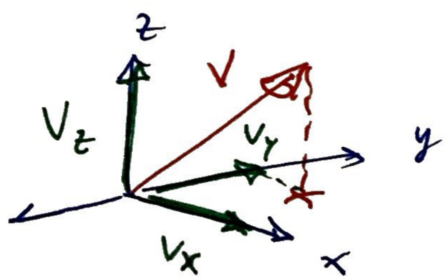
change of force on wall (+) x.

- Add in the other two directions:

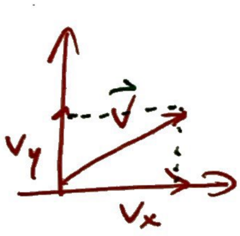
assume $\overline{V_x^2} = \overline{V_y^2} = \overline{V_z^2} \Rightarrow \overline{V_x^2} + \overline{V_y^2} + \overline{V_z^2}$

$= 3\overline{V_x}$

lets just call it $\boxed{\overline{V^2} = 3\overline{V_x^2}}$



{ 2 - D



$V^2 = V_x^2 + V_y^2$
$= 2V_x^2$ }

- So we can replace $\overline{V_x^2}$ in $\Delta F_x = \frac{m}{l} N_A \overline{V_x^2}$ with $\overline{V^2}/3$

$$\Rightarrow \quad \Delta F_x = \frac{m}{l} N_A \left(\frac{\overline{V^2}}{3}\right)$$

- Divide by area of a wall (x-wall) $A = l^2$

$$P = \frac{F}{A} = \frac{\frac{m N \overline{V^2}/3}{l}}{A} = \frac{m N \overline{V^2}/3}{A \cdot l}$$

volume

$$\boxed{P = \frac{1}{3} \frac{N_A m \overline{V^2}}{V}}$$

- But $PV = NkT$ and $KE = \frac{1}{2} m\overline{V^2}$ so this equation becomes

÷ and × by 2

$$PV = \frac{1}{3} N m \overline{V^2}$$

$$= \frac{1}{3} N \left(\frac{m\overline{V^2}}{2}\right) \cdot 2$$

$$= \frac{1}{2} N (KE) \cdot 2$$

$$= \frac{2}{3} N \cdot (KE) \quad \text{but}$$

$$\Rightarrow \quad \frac{3}{2} PV = N \cdot (KE) \qquad \text{but } N = \frac{PV}{kT}$$

$$\Rightarrow \quad \boxed{KE = \frac{3}{2} kT}$$

KE per particle in a gas of temperature T kelvins

Bottom Line $\boxed{KE \propto \text{Temperature}}$

• So lets put $\frac{1}{2}mV^2$ in for KE

$$\Rightarrow \sqrt{\overline{V^2}} = \sqrt{\frac{3kT}{m}}$$

Root mean square

$\sqrt{V^2} = V$

$$\boxed{V_{rms} = \sqrt{\frac{3kT}{m}}}$$ Velocity

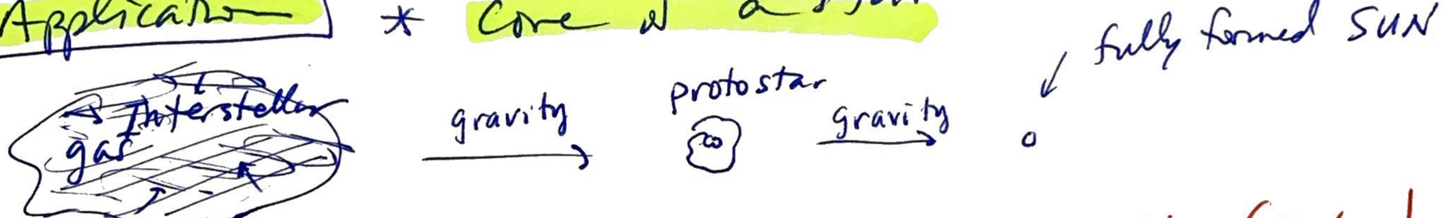
Notice : • If m = big particle (atom) the for a fixed Temperature the speed decreases.

• If T goes up so does V_{rms}

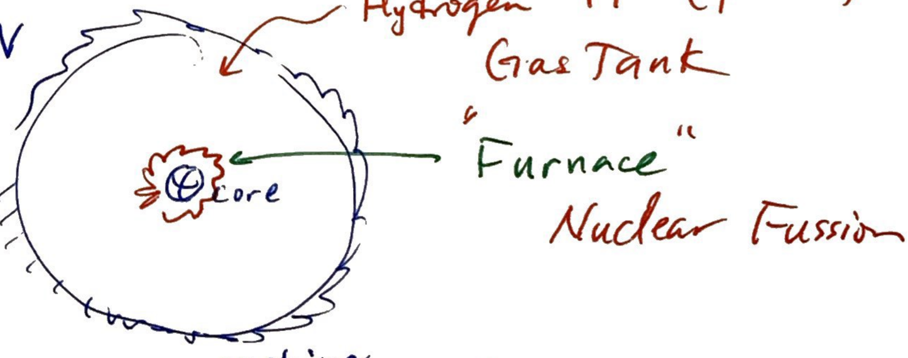
$T \propto V^2$

End Model Build Out

Application * Core of a star



• Interior of the SUN



Just as much pressure pushing outward, $PV = NkT$, we have as gravity sucking inward: $F = G\frac{Mm}{R^2}$

EX what is the average translational K. energy of a gas? let $T = 37°C$

$KE = KE_{trans} + KE_{rot}$

$\boxed{KE = \frac{3}{2}kT} = \frac{3}{2}(1.38 \times 10^{-23} J/K)(273 + 37°C)$

$= \boxed{6.42 \times 10^{-21} J \text{ /molecule}}$

· per mol gas?

$KE_{TOT} = \left(6.42 \times 10^{-21} \frac{J}{molec}\right)\left(6.023 \times 10^{23} \frac{molec.}{mol}\right)$

$= \boxed{3860 \text{ J /mol gas}}$ * Intermediate Beach Ball of gas

EX what is the speed of molecules of a gas at room temperature?

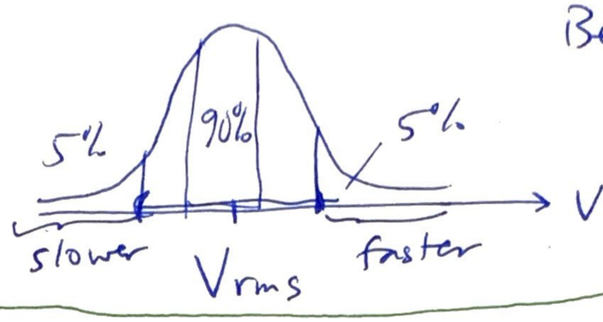
$V_{rms} = \sqrt{\frac{3kT}{m_{N_2+O_2}}} = \sqrt{\frac{3(1.38 \times 10^{-23} J/K)(293 K)}{[(28.8u)(1.66 \times 10^{-27} kg/u)]}}$

$\boxed{V_{rms} = 503.7 \text{ m/s}}$ Faster than the speed of sound.

* This is the energy of a 1 kg mass moving at 90 m/s

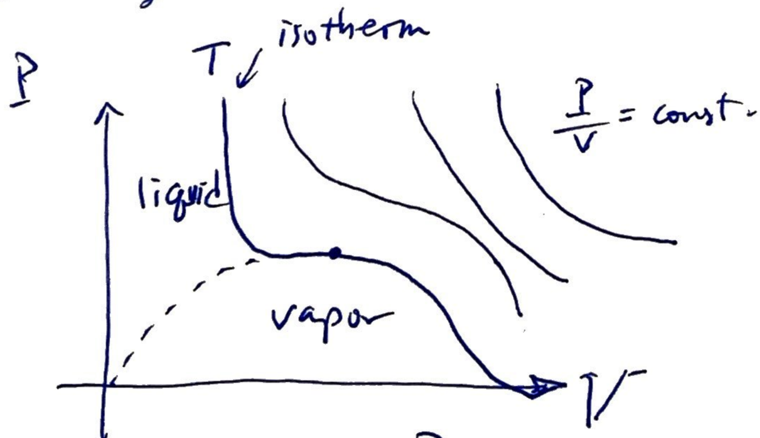
70 mph

* Speeds in a gas follow the Normal distribution "Bell Curve"



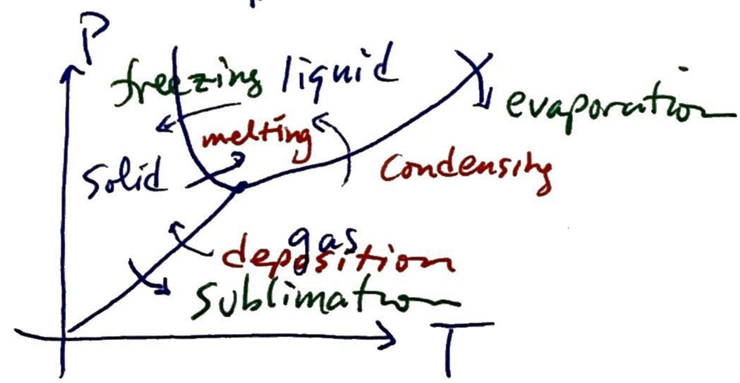
Phase Changes

- matter has multiple phase in which to exist
  - solids ⇄ phase change
  - liquids
  - gas
  - plasma
- we use $\boxed{\text{phase diagrams}}$ to denote phase changes

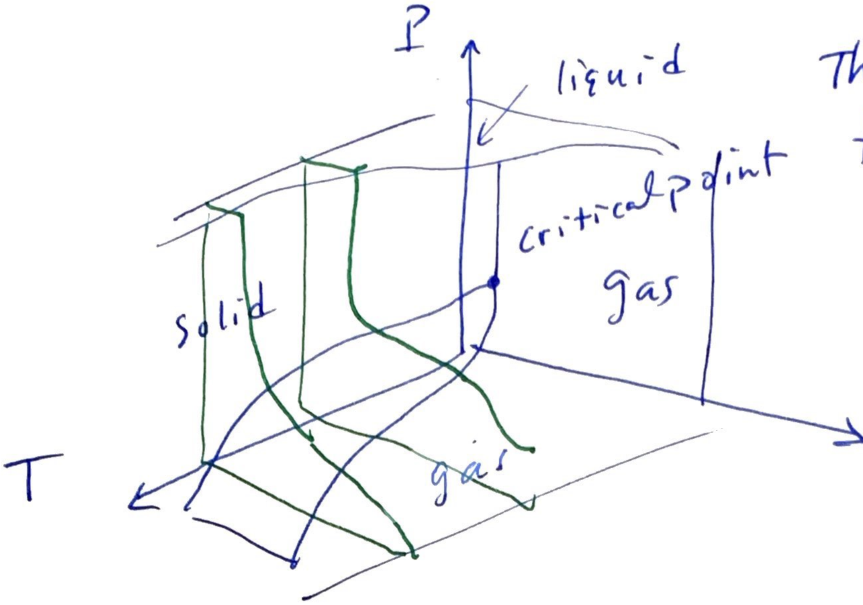


← $\boxed{P \text{ vs } V}$

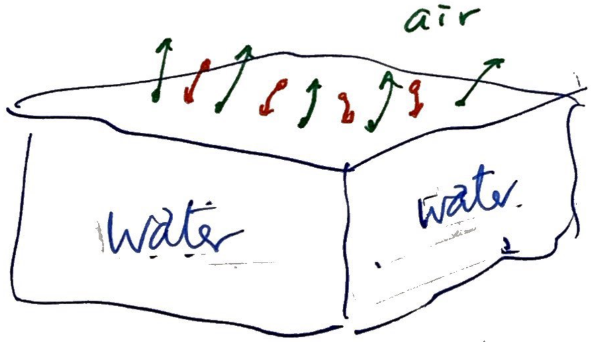
$\boxed{P \text{ vs } T}$ →



These 2-D cuts come from a 3-D plot: (13)



- At the interface between a liquid and a gas there is a constant exchange of molecules. Depending on the temperature there will be more evaporation than condensations, for example.
- When these are equal we have (phase equilibrium)



landings leavings

- similar in solid-liquid and solid-gas.

Relative Humidity (RH)

RH measures water's landing rates vs. leaving rates

- RH < 100% landing rates < leaving rates (steam predominates)
- RH > 100% landing rates > leaving rates (condensation predominates - aka Rain)

{ RH = 100% → raining or misting }

- RH depends on the density and temperature of the vapor (steam)

ex warm water and air means more leaving rates due to higher KE of interface molecules

EX Breathing warm moist air from lungs into cold air results in "steam", you can see your breath.
⟹ Temperature ↓, Volume ↓ vapor densities therefor go ↑ and condensation predominates

EX If you run in a RH near 100% the water barely evaporates, air feels damp, Perspiration does NOT cool you off.

EX Mothballs (Napthalene - solid) in your closet sublimate and shrink due to there being a lower RH of napthalene in the surrounding air.

EX
- Cold with low RH:

  freezer burn, freeze-dried foods

  ~~frozen~~ water puddles sublimate away in cold.
- Cold but high RH:

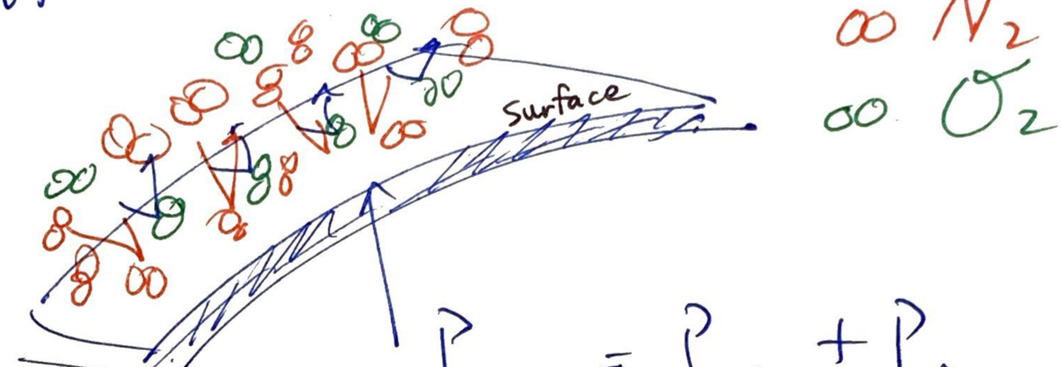
  frost on windows, dew on grass (~~frost~~ if cold enough)
- Cold with RH > 100% : snow
- Warm with RH > 100% : rain

ssure

⊛ Partial Pressures

An gas that is made up of different constituent gasses can be "decomposed" into partial pressures from each type of gas contributing to the full pressure of the mixture

• Air:



$$P_{TOT} = P_{N_2} + P_{O_2}$$

$\boxed{\text{partial pressures}}$

$\boxed{EX}$ air in our atmosphere is 78% N_2 and 21% O_2

so at 1 atm : $\begin{cases} P_{N_2} = 0.78 \text{ atm} \\ P_{O_2} = 0.21 \text{ atm} \end{cases}$

Vapor Pressures

Normal air contains water molecules.
The saturated vapor pressure is a measure of pressure needed, in a chamber, to allow the liquid to "rain out" of the air.

This allows us to redefine RH:

$$\boxed{\text{Rel. Humidity} = \frac{\text{partial pressure of } H_2O}{\text{saturated vapor pressure of } H_2O}}$$

Table (moist air):

T °C	Vapor Pressure of H_2O
-50 °C	0.03 Torrecellis (vs. pascal)
-10 °C	2.0 Torr
0 °C (freezing)	6.6 torr
20 °C	17.5 torr
70 °C	234 torr
100 °C (boiling)	760 torr

Here torr = # of millimeters in Hg thermometer

Ex: On a hot 30 °C day we measure the partial pressure of H_2O to be 21.0 torr

Q: What is the RH?

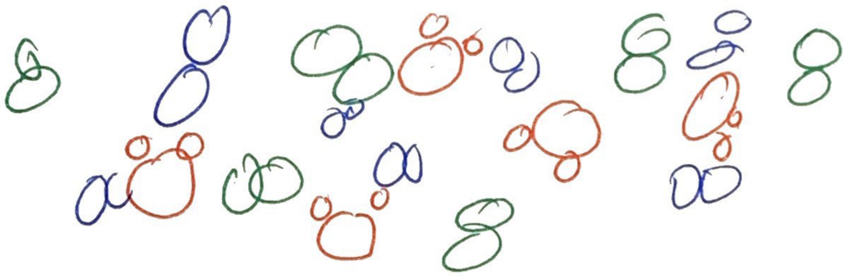
$$RH = \frac{\text{partial press. of } H_2O}{\text{saturate press of } H_2O} = \frac{21.0 \text{ torr}}{31.8 \text{ torr}} = 0.66$$

66°

Humans feel best comfort when RH is between 40% to 50%

Dew Point

The dew point is the amount of water the air can hold inbetween the air molecules



N_2 = OO
O_2 = OO
Oo = H_2O

Some meteoroligists are switch from RH to D.P. since the dew point more correctly expresses human comfort, best around a D.P. of 50%

Ex