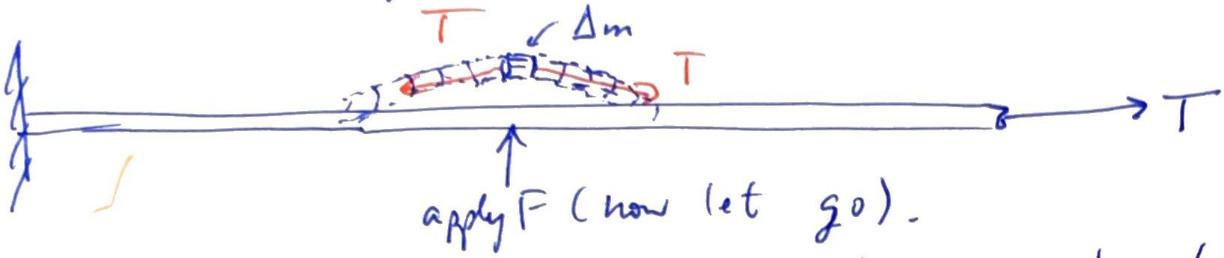
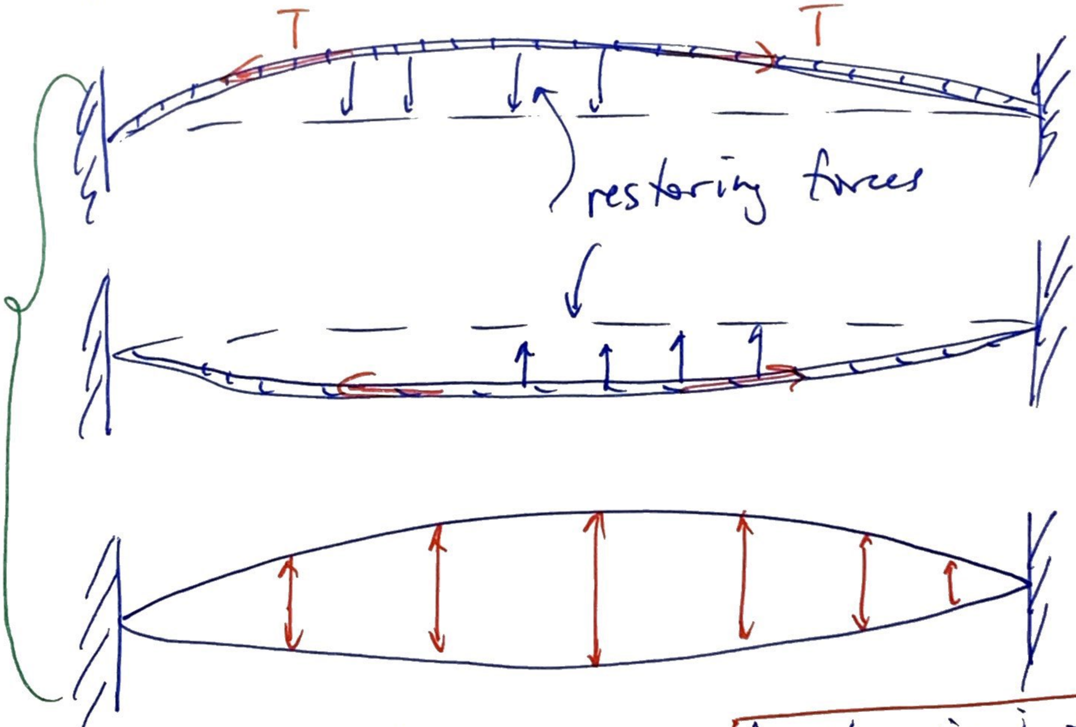
Chapter 11B Waves: on strings, in air, water

Consider a string, rope or a cable.



- The tension pulls the piece of mass back down
- But momentum carries the piece through the equilibrium line (position) but then swings to the other side where the tension slows the piece and pulls it back to repeate the cycle.

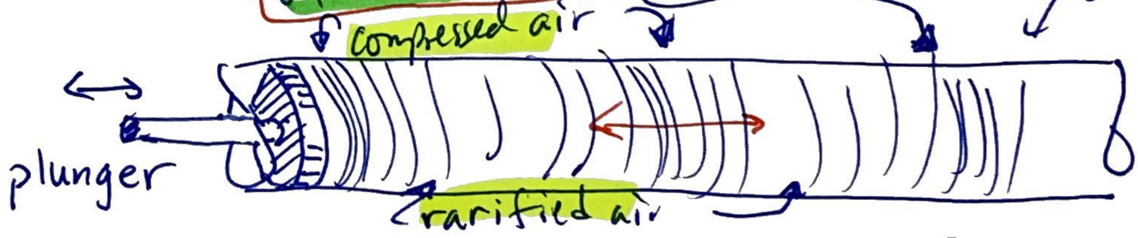
TRANSVERSE WAVE



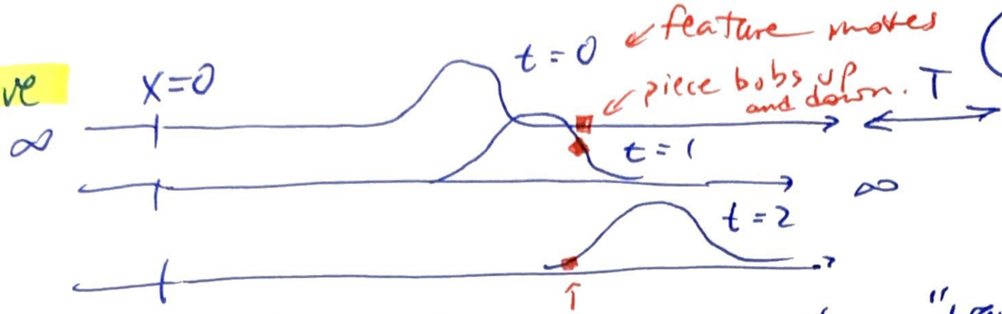
motion is $\perp$ to the displacement.

- Longitudinal waves: displacement is in the direction of the motion: pipe

disturbance travels down pipe.



- pulse wave



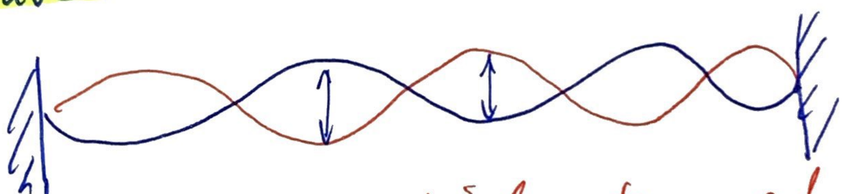
The pulse traves with a "wave" velocity V

- continuous waves

Feature move, but the "float" bobs up and down

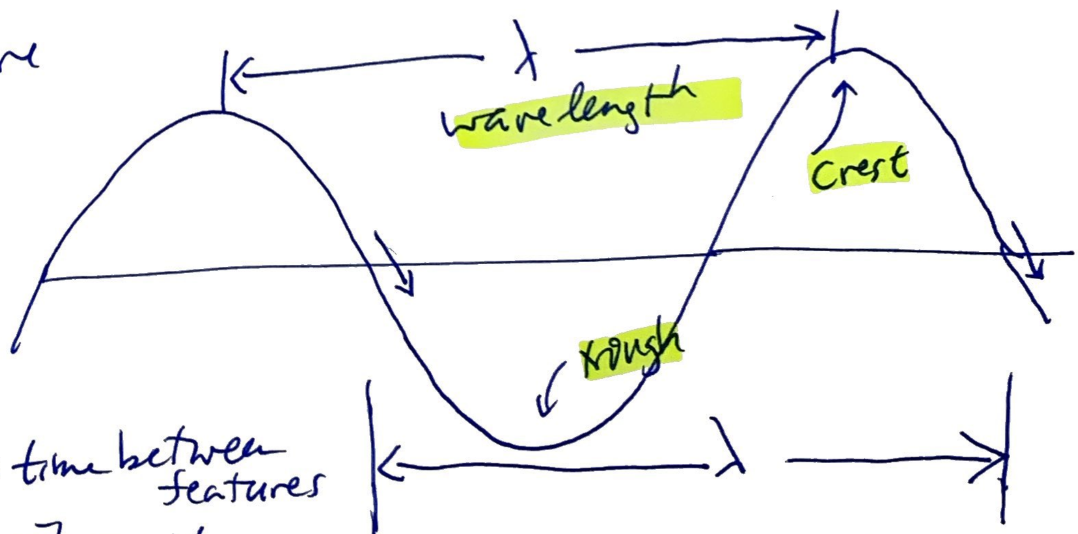
waves in water (stadium wave)

- Standing wave



No visible longitudinal motion, only transverse motion

* Nomenclature



- Period, T, = time between features
- wave velocity = V

speed of a feature, such as a trough.

$$\boxed{V = \frac{\lambda}{T}} = \lambda f \quad \text{since } f = \frac{1}{T}$$

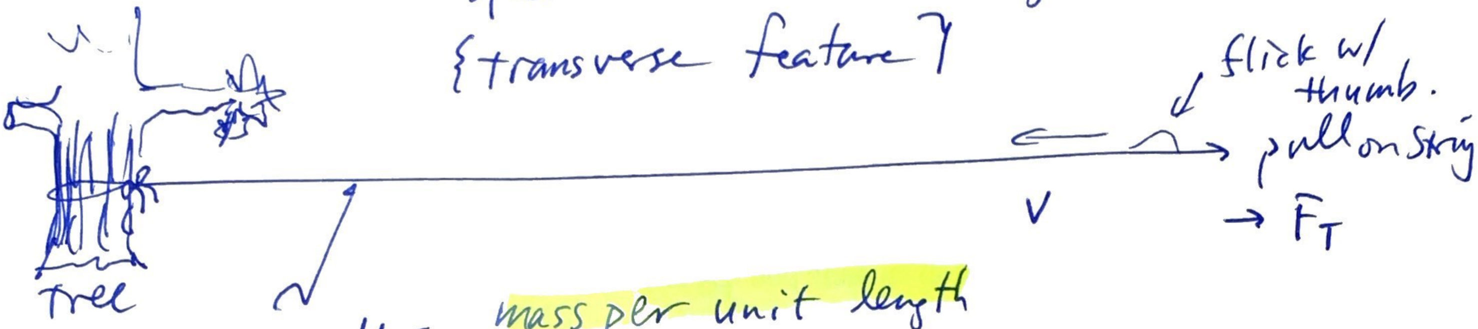
wave speed on a string (rope, wire) is

$$\boxed{V_T = \sqrt{\frac{F_T}{\mu}}}$$

F_T ← Tension on the rope

μ ← Linear mass density

V_T ← speed of a feature moving down the rope. {transverse feature}



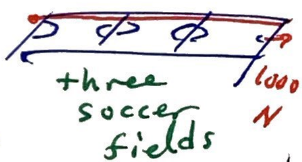
μ = mass per unit length

ex $\mu = m_{\text{whole string}} / \text{length of whole string}$

Ex A wave whose wavelength is 0.3 m is travelling down a 300 m long taught wire.

Total mass of the string is 15 kg

If the string is under 1000 N tension



Q: If the string is under 1000 N tension what is the speed of a hammer strike on the string?

$$V_T = \sqrt{\frac{1000\,N}{(15\,kg/300\,m)}} = \sqrt{\frac{1000\,N}{0.05\,kg/m}} = \underline{\underline{140\,m/s}}$$

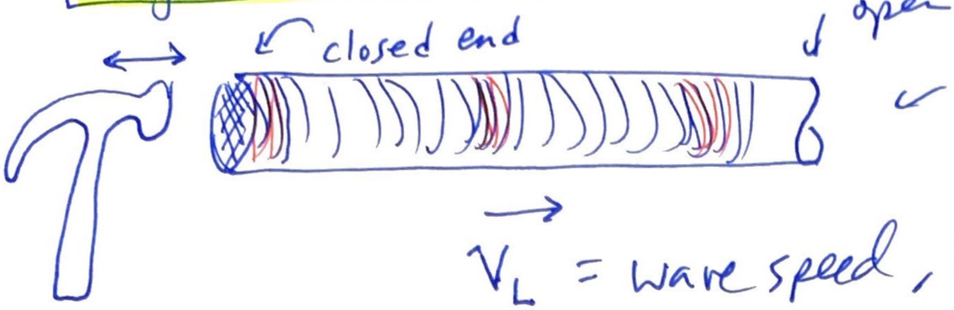
V_T ← trans-verse disturbance

BTW: If we pluck the string in the middle, and form a standing wave

$$f = \frac{V}{\lambda} = \frac{140\,m/s}{0.3\,m} = \underline{\underline{470\,Hz}}$$

"A" note is 440 Hz

* Longitudinal Waves



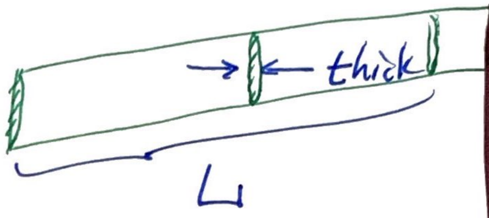
cable, air, column of water
brass or steel rod

$\vec{V}_L$ = wave speed, longitudinal

high-aspect ratio.

- For a 1-dimensional cable or rod

$$\boxed{V_L = \sqrt{\frac{Y}{\rho}}}$$



High-Aspect Ratio: $AR = \frac{L}{t}$ is "high"

- 3-dim medium

Bulk modulus

$$\boxed{V_L = \sqrt{\frac{B}{\rho}}}$$

EX Dolphins use echolocation

if $f = 100{,}000\,Hz$ {normal humans hear between 20 & 20,000 Hz}

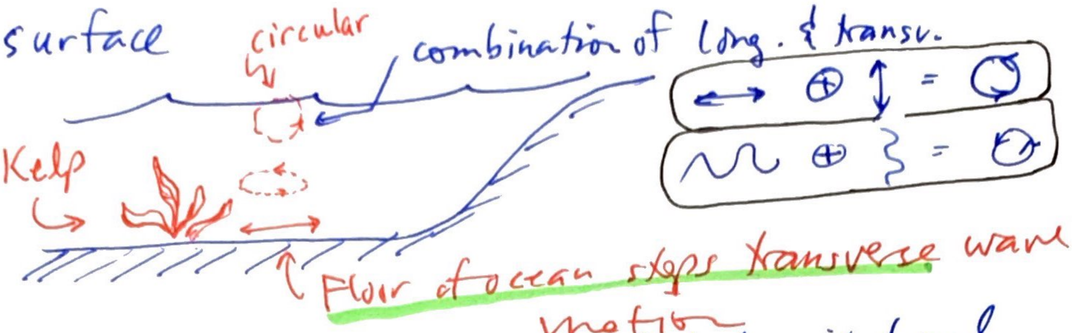
Q: what is the wave speed and wavelength

- $V_L = \sqrt{\frac{B_{water}}{\rho_{salt\ water}}} = \sqrt{\frac{2\times10^9\,N/m^2}{1.025\times10^3\,kg/m^3}} = \underline{1400\,m/s}$

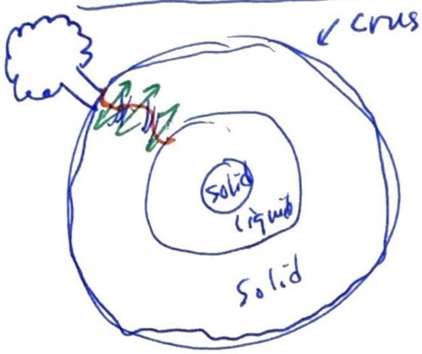
- $\lambda = \frac{V}{f} = \frac{1400\,m/s}{100{,}000\,cycles/sec} = 0.014\,m/cycle = \boxed{14\,mm \text{ wavelength}}$

* ocean waves are both longitudinal nor transverse.

near the surface



* earthquake waves are both transversal & longitudinal.



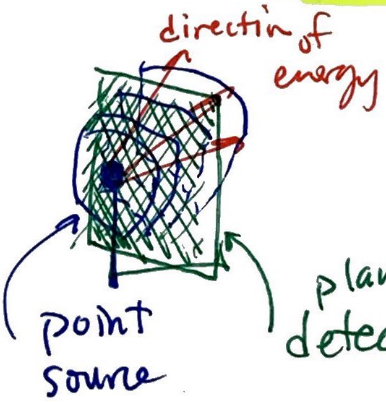
- Transverse wave (side-to-side) do not travel in liquids. (S-waves, shear)

- Longitudinal waves (forward then back) the travel in both solids and liquids (P-waves)

These two waves help geologists determine the interior of the earth. {youtube "How do geologists know what the center is}

(*) Energy Transport In the Stadium we saw "energy" (motion) occur and propagate - waves may not transfer the medium they live in, but they do transfer the motion which is energy.

- Intensity of a wave is $\equiv \frac{\text{Power}}{\text{Area}}$



planar detector = $\underbrace{\text{Joules/sec}}_{\text{watt}}$/area

$$\boxed{I = \frac{\text{Power}}{\text{Area}}} = \frac{\text{Watts}}{m^2}$$

We typically have three geometries that we utilize to model energy transport in.

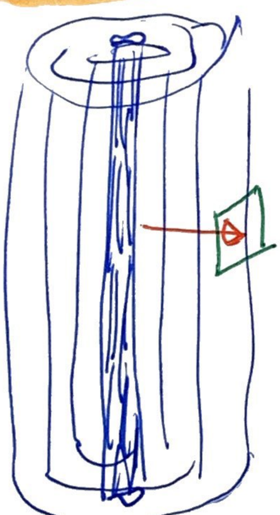
3-D (spherical)

$$I \propto \frac{1}{r^2}$$

Inverse square decay

point source

2-D linear (cylindrical)

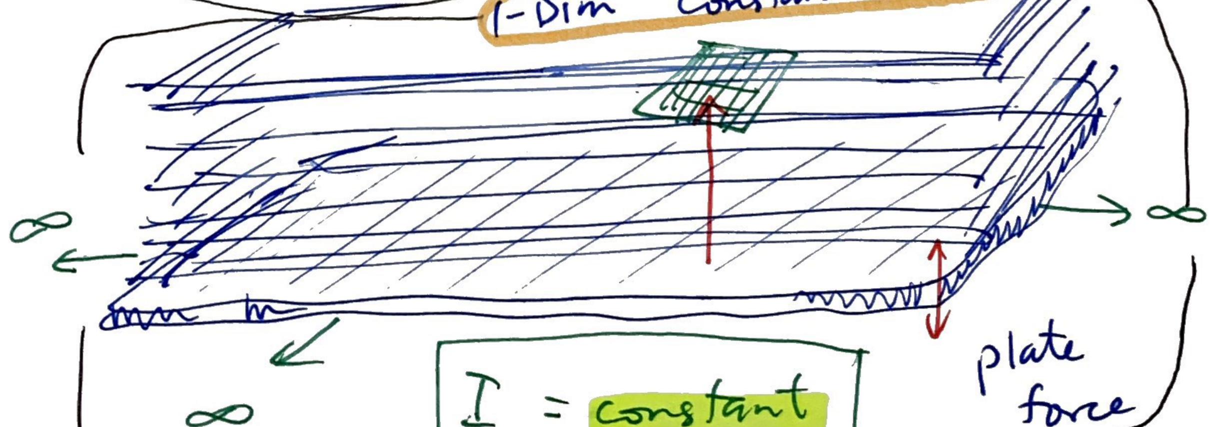


$$I \propto \frac{1}{r}$$

Inverse Decay

line source

1-Dim Constant front

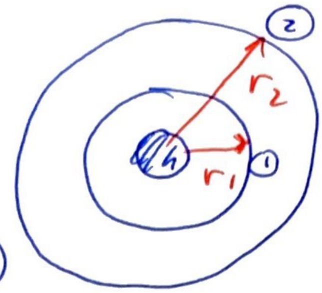


$$I = \text{constant}$$

plate force

3 Dim

$$I = \frac{\text{Power}}{\text{Area}} = \frac{P}{4\pi r^2}$$



So power $P_1 = I_1 4\pi r_1^2$ at station 1

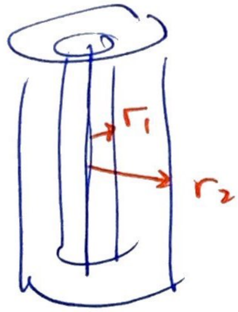
and at station 2 $P_2 = I_2 4\pi r_2^2$

But $\boxed{P_1 = P_2 \text{ due to cons. of energy}}$

$$\Rightarrow \quad I_1 4\pi r_1^2 = I_2 4\pi r_2^2 \quad \Rightarrow \quad \boxed{\frac{I_2}{I_1} = \left(\frac{r_1}{r_2}\right)^2}$$

spherical decay

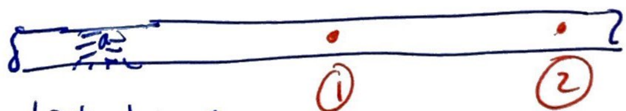
2-Dim



$$\boxed{\frac{I_2}{I_1} = \frac{r_1}{r_2}}$$

cylindrical decay

1-Dim

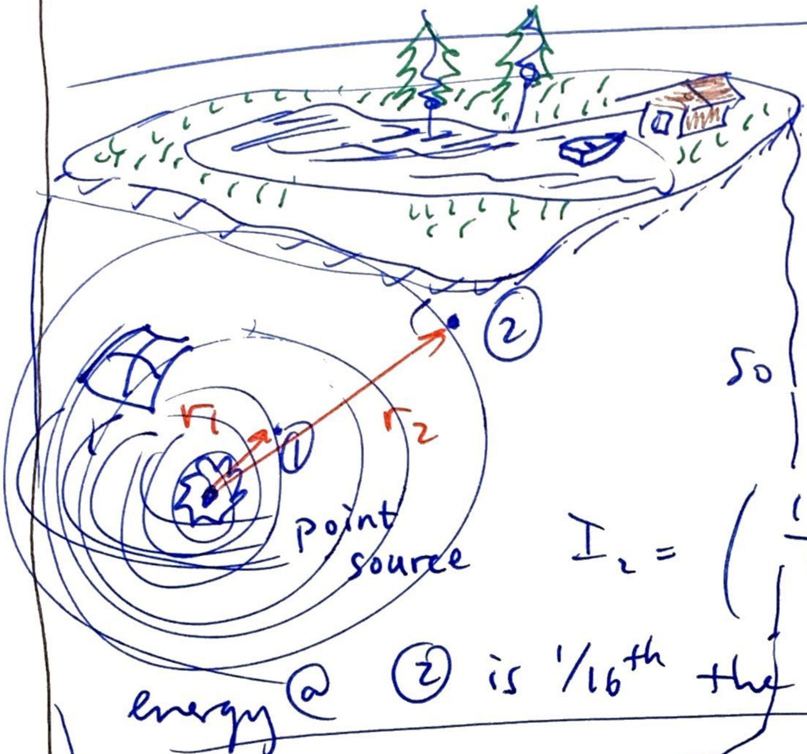


$$\boxed{I_1 = I_2}$$

planar decay

all energy is contained in the pipe or rod or air column and cannot spread out.

EX A pressure wave (P-wave) tra ' with in the earth due to a powerful, terr earthquake. What is the intensity of the disturba when a detector is 400km away if, at 100 km from the disturbance the Intensity is measured @ $1.0E6 \frac{W}{m^2}$



$$\frac{I_2}{I_1} = \left(\frac{r_1}{r_2}\right)^2$$

so

$$I_2 = \left(\frac{r_1}{r_2}\right)^2 I_1$$

$$I_2 = \left(\frac{100\,km}{400\,km}\right)^2 \left(1.0\times10^{6}\,\frac{W}{m^2}\right) = \boxed{6.3\times10^{4}\,\frac{W}{m^2}}$$

energy @ 2 is 1/16th the energy at 1.

⊛ Energy Transport for SHO.

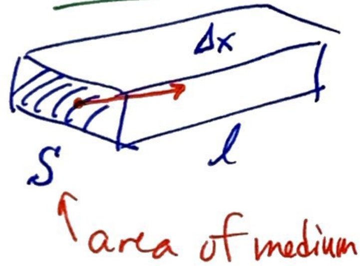
- Spring-Mass $\boxed{E = \frac{1}{2}kA^2}$, $T = 2\pi\sqrt{\frac{m}{k}}$ or $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

Solve the latter form for "k"

$$k = 4\pi^2 m f^2$$

Place into $E = \frac{1}{2}kA^2$ to get $E = \frac{1}{2}(4\pi^2 m f^2)A^2$

- use the fact that $\boxed{\rho V = m}$ & $\underset{}{V}^{\swarrow \text{volume}} = S \cdot l$, & $l = \overset{\swarrow \text{velocity}}{v}t$



$$\Rightarrow \boxed{E = 2\pi^2 \rho S v t f^2 A^2}$$

- use $\boxed{\text{Power} = E/t}$

$$\Rightarrow P = 2\pi^2 \rho S v f^2 A^2$$

amplitude of disturbance

- use $\boxed{I = \frac{P}{\text{Area}}}$

$$\Rightarrow \boxed{I = 2\pi^2 \rho v f^2 A^2}$$

wave speed

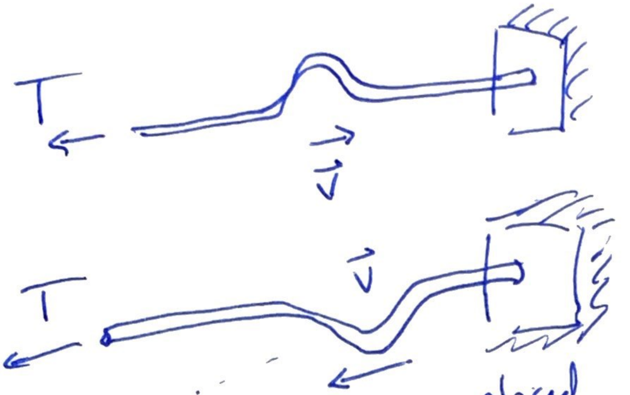
frequency

amplitude of wave.

density of medium

* Reflections

(a) anchored connection

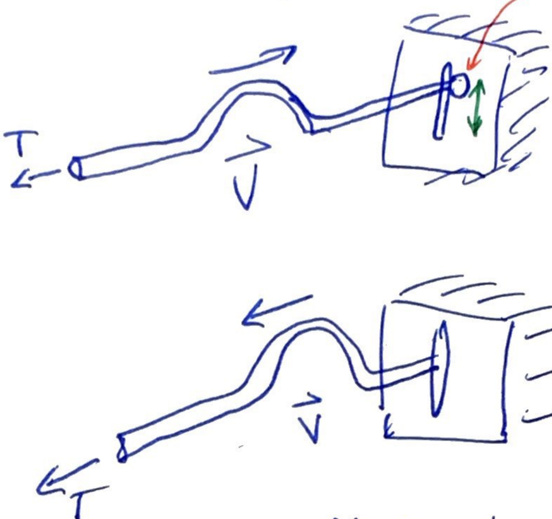


closed boundary

Disturbance is inverted

(b) slotted.

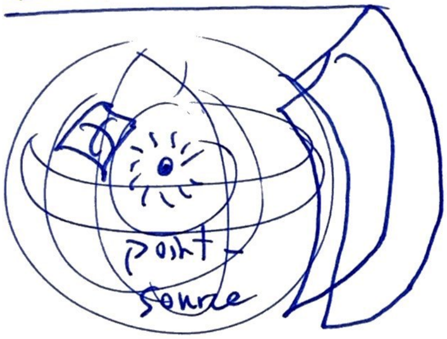
Bead keeping the rope attached.



open boundary

disturbance is NOT inverted.

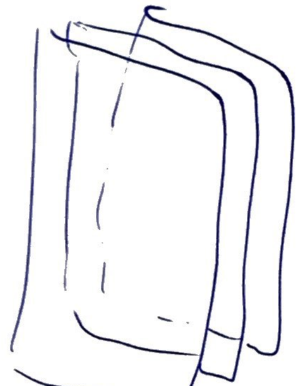
⊛ Near and Far field



spherical waves

"Nearfield"

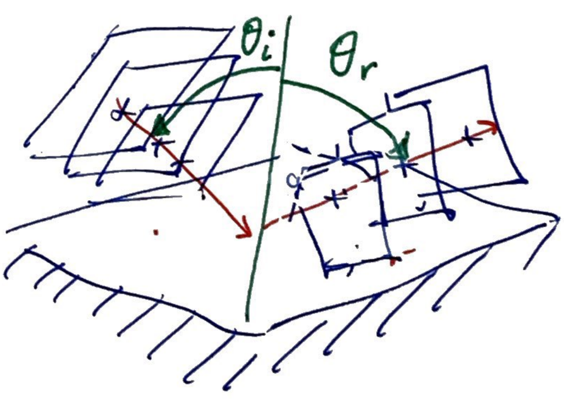
long distances



plane waves

"Far field"

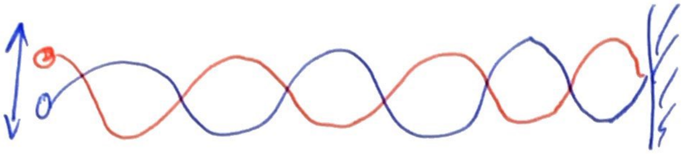
⊛ planar-reflections



$$\boxed{\theta_{incident} = \theta_{reflected}}$$

* Standing Waves

move up and down

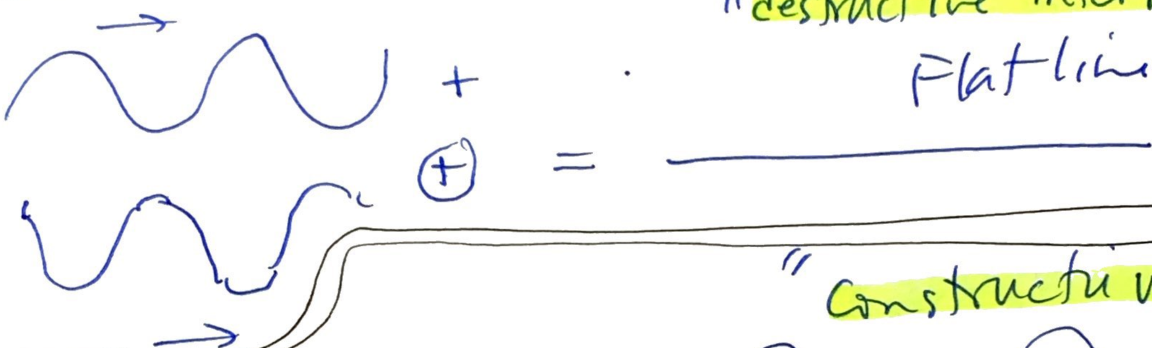


The reflection adds into the incident wave and together the set a wave that does not show motion

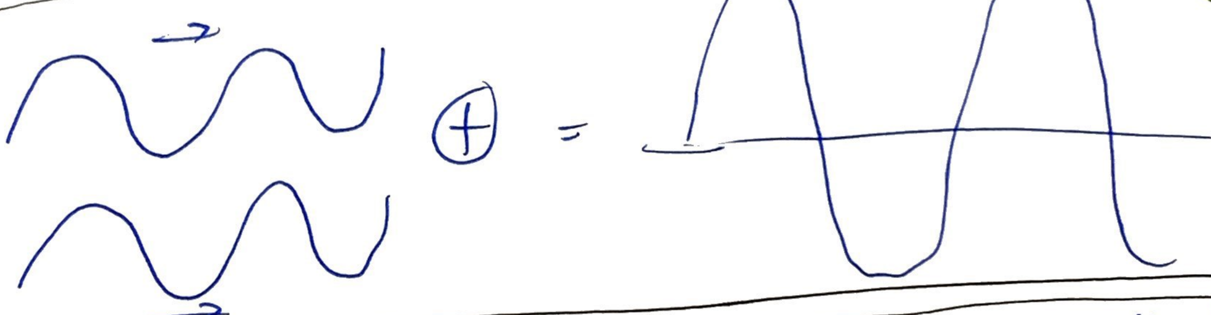
• Interferences

"destructive interference"

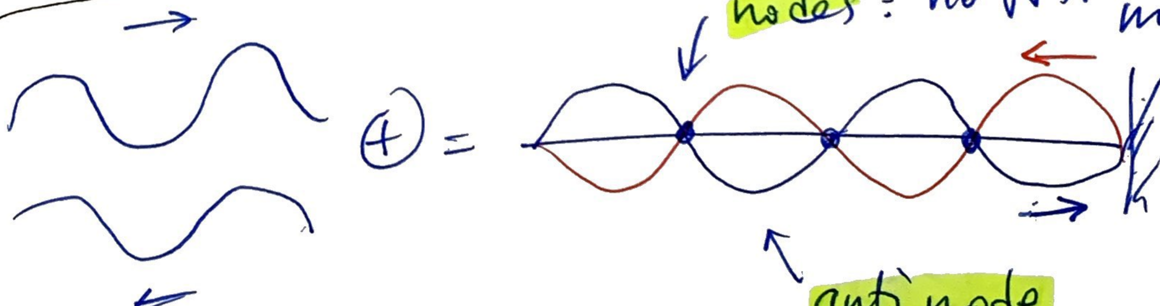
Flat line



"constructive interference"



nodes : no vertical motion

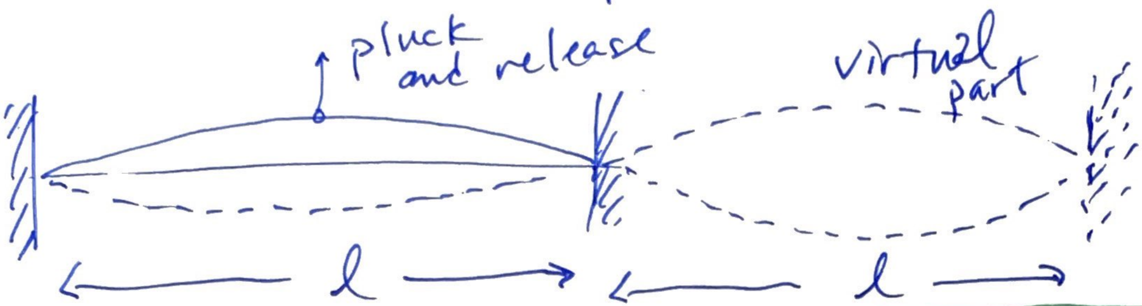


anti node

guitar string

Fundamental Frequencies

It turns that wavelengths of a standing wave comes at certain predictable frequencies.



$n=1$

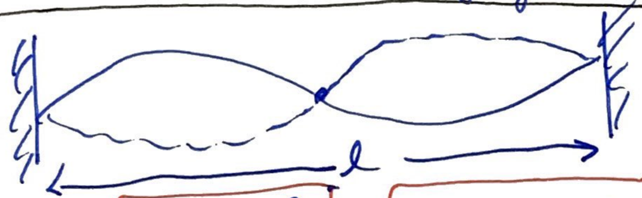
$n=1$, Fundamental mode, 1st harmonic

$$\boxed{\lambda_1 = 2l}\ ,\quad \boxed{f_1 = \frac{1}{\lambda_1} v = \frac{1}{\lambda}\sqrt{\frac{F_T}{\mu}}}$$

determined by geom.

determined by the length and Tension and μ

$n=2$

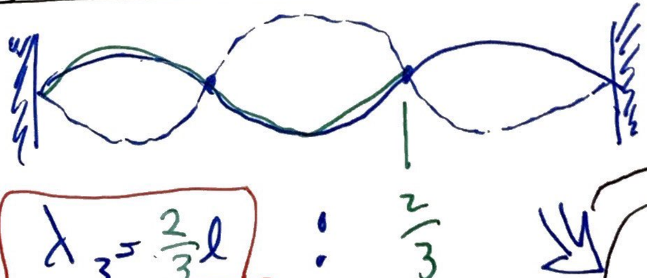


1st over tone

2nd harmonic

$$\boxed{\lambda_2 = l} \quad \boxed{f_2 = 2f_1}$$

$n=3$



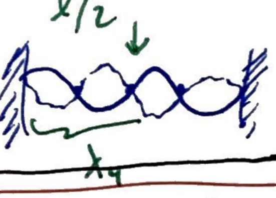
2nd overtone

3rd harmonic

$$\boxed{\lambda_3 = \frac{2}{3}l} \quad \vdots$$

$$\boxed{f_3 = 3f_1}$$

$n=4$



$\sqrt{F_T/\mu}$

nth harmonic

$$\boxed{\lambda_n = \frac{2l}{n}} \quad \boxed{f_n = \frac{v}{\lambda_n} = \frac{nl}{2}v = nf_1}$$

EX Piano String is $l = 1.10$ m long

It's mass is $m = 9.00$ gm

What tension is needed to get a fundamental tone of 131 Hz? $\{n=1\}$

$$v = \sqrt{\frac{F_T}{\mu}} \quad \text{but} \quad f = \frac{v}{\lambda} \Rightarrow \boxed{\lambda f = \sqrt{\frac{F_T}{\mu}}}$$

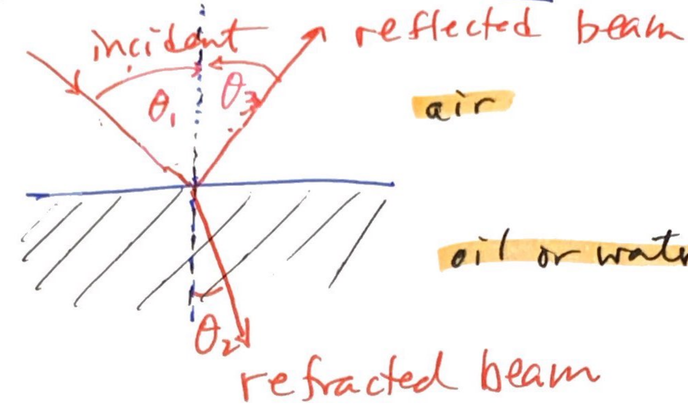
Solve for F_T : $\lambda^2 f^2 = \frac{F_T}{\mu}$

So $\boxed{F_T = \mu \lambda^2 f^2} = \left(\frac{0.009\,\text{kg}}{1.10\,\text{m}}\right)(1.10\,\text{m})^2\left(131\,\frac{\text{cycles}}{\text{sec}}\right)^2$

$$\boxed{F_T = 680\,N}$$

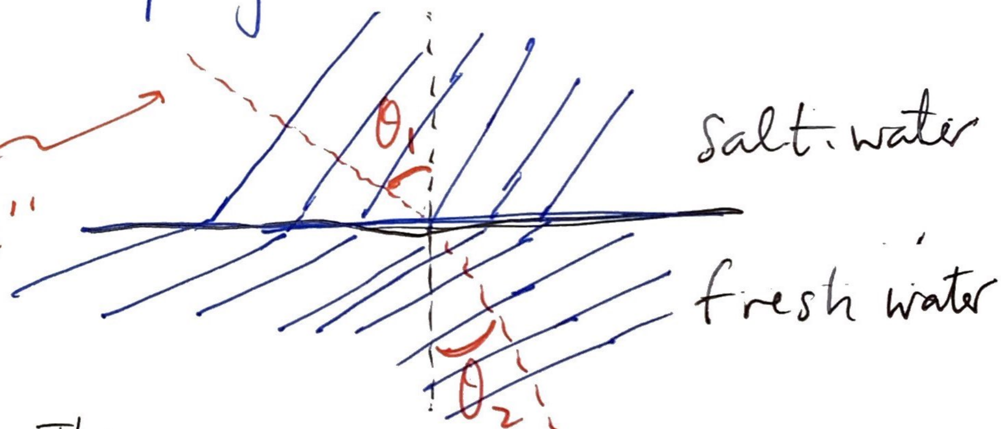
⊛ Refraction and Reflection

- Nomenclature
use Laser Beams

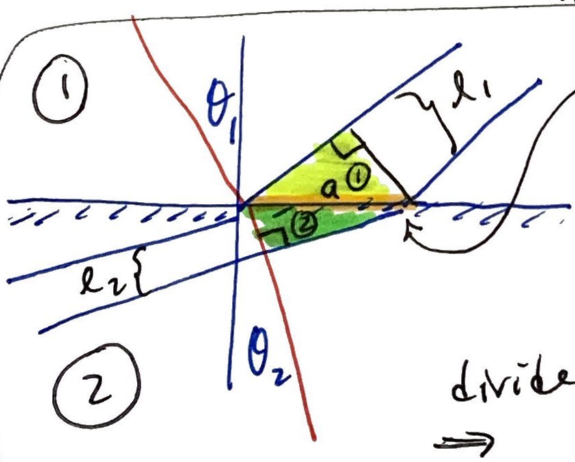


Vibrational disturbances incident on an interface formed between fresh water and salt water, or between water and oil will display reflected and refracted waves.

"normal to the wave"



Theory



The triangles share a hypotenuse

$\sin\theta = \dfrac{l}{\boxed{a}}$ $\quad l = vt$

$$\sin\theta = \frac{vt}{\boxed{a}} \;\rightarrow\; ①\ \sin\theta_1 = \frac{v_1 t}{a}, \quad ②\ \sin\theta_2 = \frac{v_2 t}{a}$$

divide $\Rightarrow$ $\boxed{\dfrac{\sin\theta_1}{\sin\theta_2} = \dfrac{V_1}{V_2}}$ Law of refraction

Chpt 11 is done