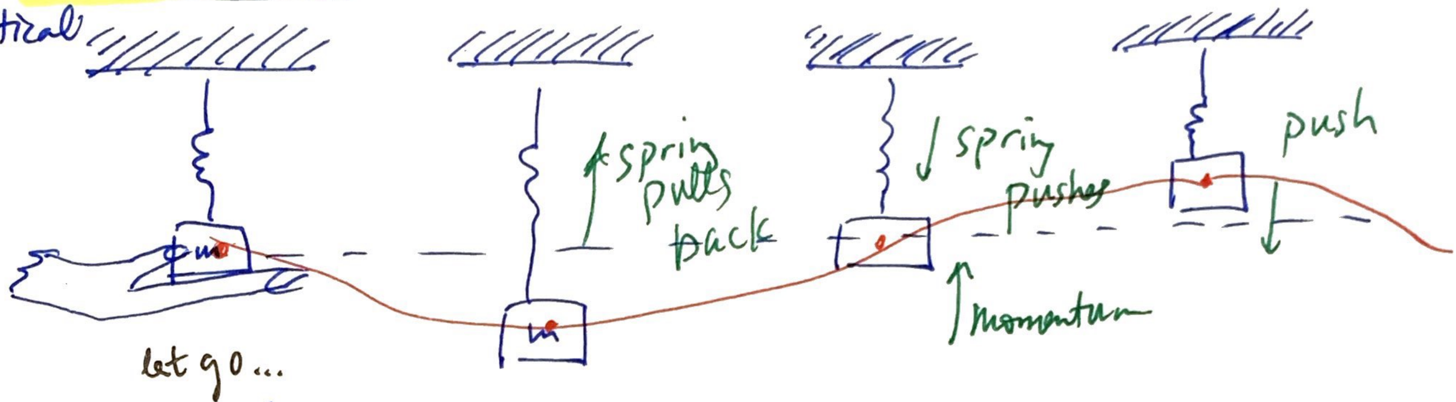
Chapter 11 A Oscillation

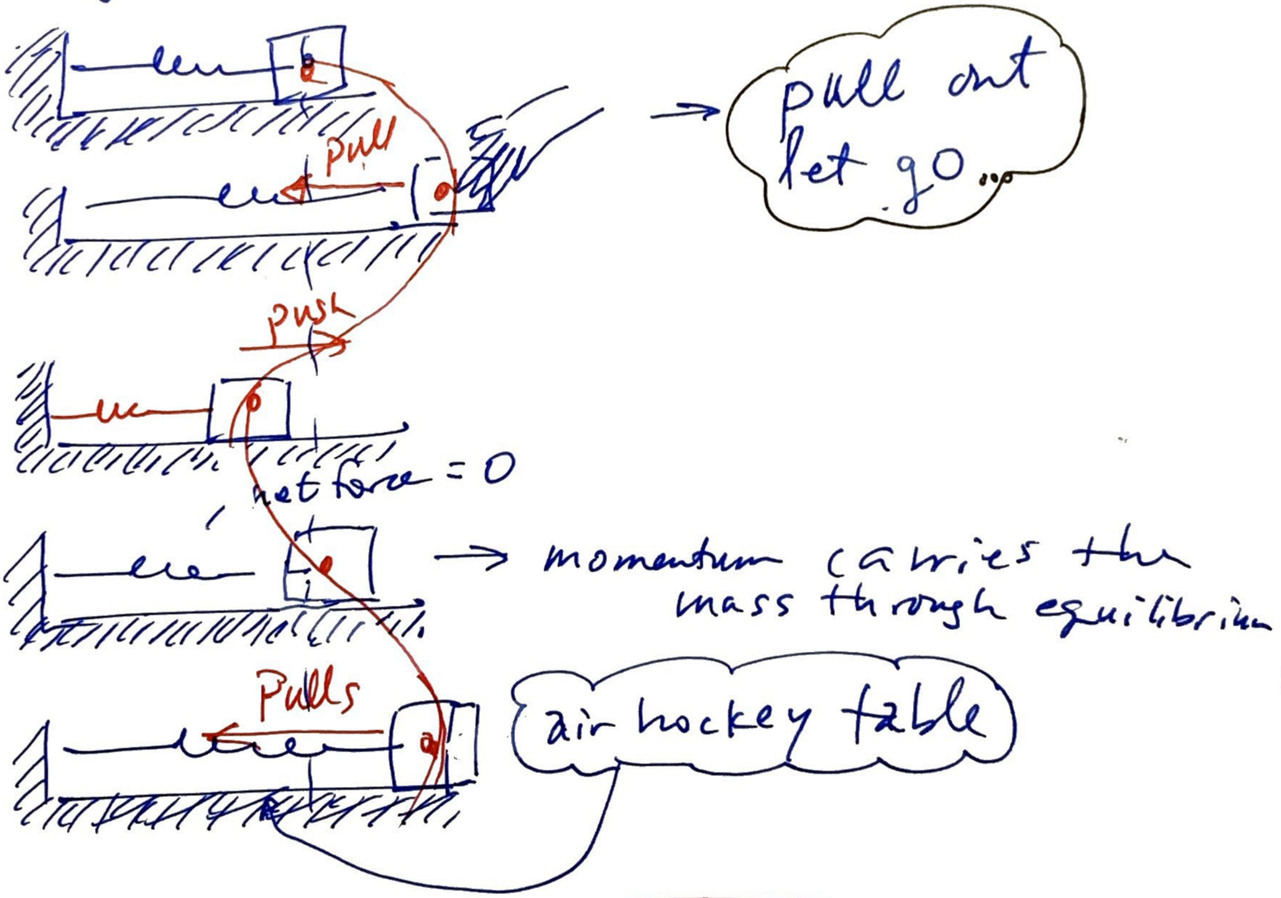
Constrained objects that move possess some degree of oscillation

spring-mass

• vertical



• horizontally

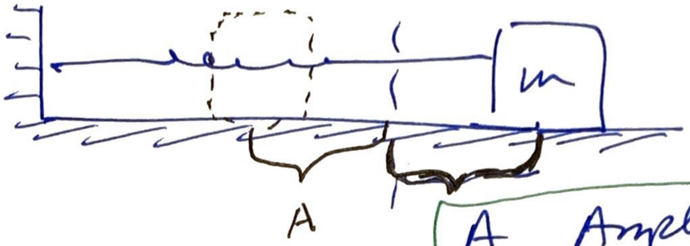


Hooke's Law: $\boxed{F_{sp} = -k\Delta x}$

2

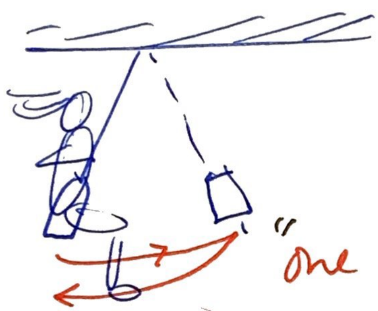
* Nomenclature

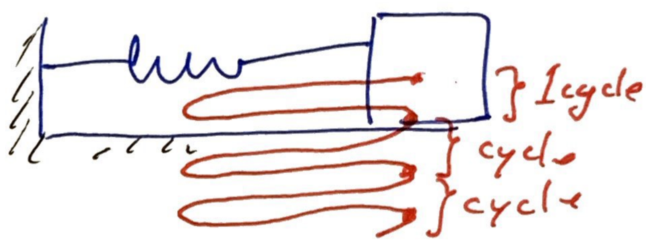
"equilibrium" (spring does not have a force) $F_{sp} = 0$



$\boxed{\text{A Amplitude}}$

- "cycle" when a system completes motion and starts over:





- Time to complete one cycle is called the $\boxed{\text{period of oscillation}}$ "T"

$[T] = \text{sec.}$

- "frequency": $\boxed{f \equiv \frac{1}{T}}$

$[f] = \text{per sec or} = \frac{1}{s}$

$= \text{Hertz}$

$= \frac{\text{one cycle}}{\text{sec.}}$

$f = \#\text{cycles}/\text{sec}$

EX music wave 262 Hz that is, the string on a guitar vibrates 262 cycles/sec.

Ex

girl on swing has 5 seconds/cycle

3

$T = 5 \text{ sec/cycle}$

$f = \frac{1}{5} \frac{\text{cycles}}{\text{sec}} = \boxed{0.2 \text{ Hz}}$ one fifth of a cycle every sec.

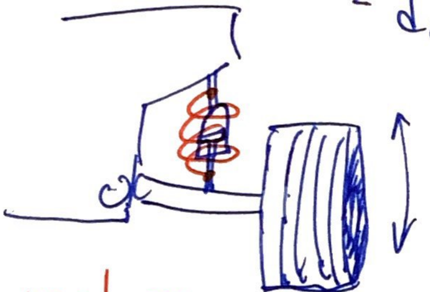
Ex

Car shock absorber

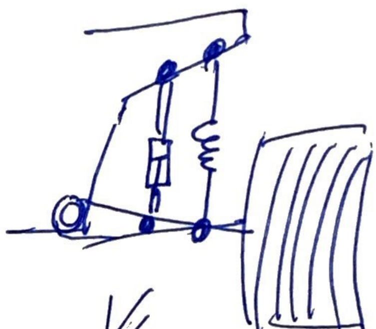
"strut" = shock absorber + spring

restores the tire to operating level

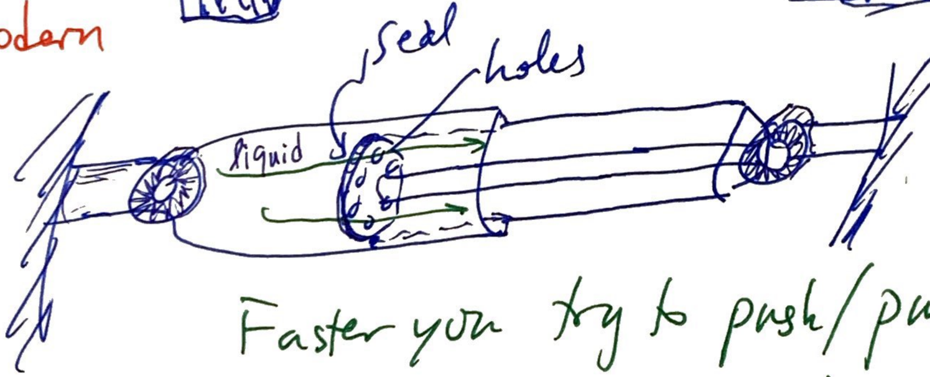
dampens the chuck hole in the road



older



modern



Faster you try to push/pull the more resistance you have.

(a) Four family members total 200kg in the family car. The 4 spring displace 3.0 cm each. Find the spring constant of all springs.

$k = \Delta F / \Delta x = \frac{(200\text{kg})(9.8\text{m/s}^2)}{(0.03\text{m})} = \boxed{65{,}000 \text{ N/m}}$

$\div 4$, $\boxed{k = 16{,}250 \frac{N}{m}}$ per spring.

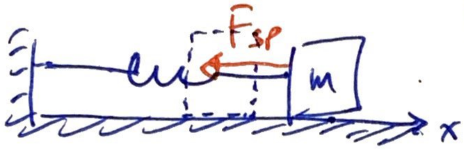
(b) How much lower will the car drop when your 100 kg cousin hops in?

$$F = k\Delta x \rightarrow \Delta x = \frac{F}{k} = \frac{(200+100)\,kg\,(9.8)}{65{,}000\ N/m} = 0.045\,m$$

or 4.5 cm total drop

add'tl Δx = 4.5 − 3.0 cm = 1.5 cm additional drop

Equations of Motion



$$x:\ \sum F_x = ma_x$$

$$-kx = ma_x$$

but $a = \frac{\Delta v}{\Delta t}$ but $v = \frac{\Delta x}{\Delta t}$ so $a = \frac{\Delta}{\Delta t}\left(\frac{\Delta x}{\Delta t}\right)$

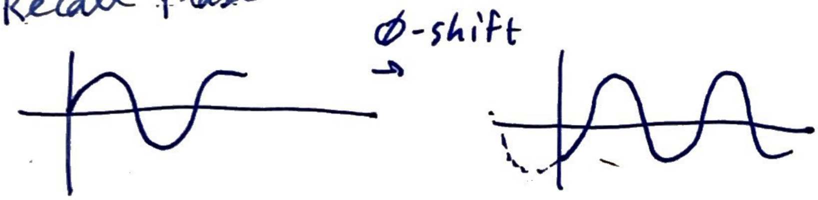
$$\Rightarrow (-kx) = \frac{\Delta^2 x}{\Delta t^2}\cdot m \rightarrow \boxed{m\frac{\Delta^2 x}{\Delta t^2} + kx = 0}$$ difference eqn. (differential eqns)

$$\Rightarrow \frac{\Delta}{\Delta t}\left(\frac{\Delta x}{\Delta t}\right) + \frac{k}{m}x = 0$$

- Solution: $\boxed{x(t) = A\cos(\omega t + \phi)}$ "phase"

where $\omega = \sqrt{k/m}$ = frequency, $\boxed{\omega = 2\pi f}$ angular freq. linear freq.

Recall phase

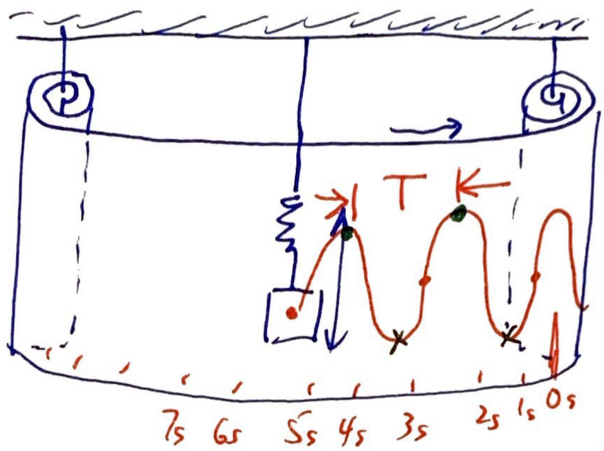


$[\omega]$ = rad/sec

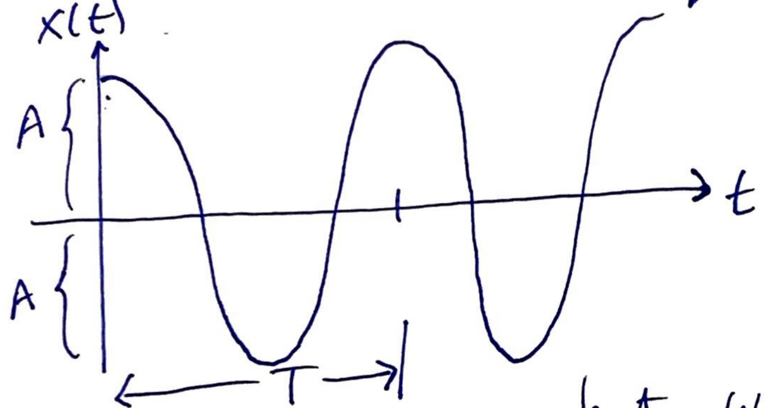
$[f]$ = cycles/sec

* $(2\pi\ \text{rad/cycle}) = \omega$

plotting oscillations



strip chart recorder



$$\boxed{x(t) = A\cos(\omega t)}$$

$\phi = 0$ since we start @ top.

$$\boxed{\omega = \sqrt{\frac{k}{m}} \text{ for springs}}$$

angular freq.

but $\omega = 2\pi f$

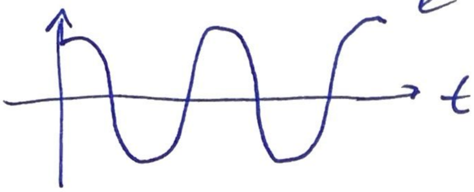
so $\boxed{f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}}$ but $\boxed{T = \frac{1}{f} = 2\pi\sqrt{\frac{m}{k}}}$

linear freq period

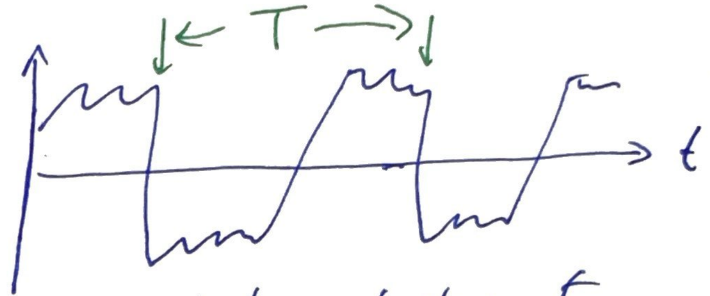
⊛ energy

- SHO = Simple Harmonic Oscillation

cosine or sine wave

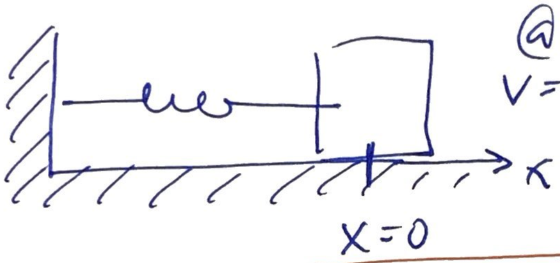


- non SHO ⟹



periodic, but not simple.

$\boxed{E_{TOT}}$



@ equilibrium
$v=0,\ x=0$

$$E = P + K \quad , \quad \boxed{P = \frac{1}{2}kx^2} \quad , \quad \boxed{K = \frac{1}{2}mv^2}$$

$$E_{TOT} = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$$

No motion, no stretch, $0 = E_{TOT}$

@ Equilibrium $E_{TOT} = 0 \Rightarrow \underline{P = -K}$

Now... Pull it out

@ max stretch $\begin{cases} v = 0 \text{ so } K = 0 \\ x = A \text{ so } P = \max = \frac{1}{2}kA^2 \end{cases}$

$$\boxed{E_{TOT} = \frac{1}{2}kA^2}$$

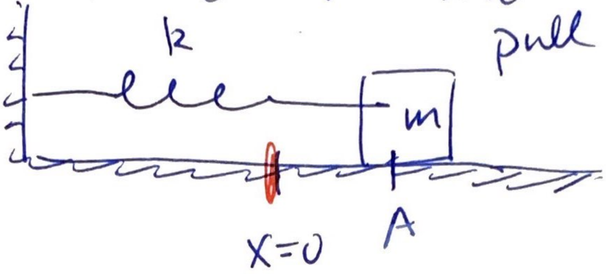
we then let go... oscillation occurs.

@ Inbetween eq. & max stretch $E_T = P + K$

$$\boxed{\frac{1}{2}kA^2 = \frac{1}{2}kx^2 + \frac{1}{2}mv^2}$$

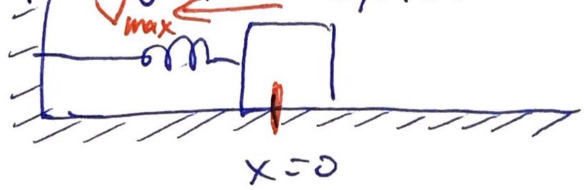
- Stretch to value, A cm

pull to $x = A$, let go



$$\boxed{E_{TOT} = \frac{1}{2} k A^2}$$ defines total energy

- passing thru equilibrium: max. velocity

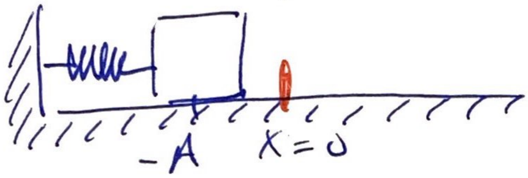


$E_{TOT} = \frac{1}{2} m V_{max}^2$, No P.E.

$\frac{1}{2} k A^2 = \frac{1}{2} m V_{max}^2$ solve for V_{max}

$$\boxed{V_{max} = \sqrt{\frac{k}{m}}\, A}$$

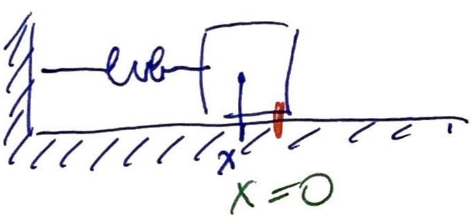
- total compression



$V = 0$, max. stretch/compression

$E_{TOT} = \frac{1}{2} k (-A)^2$

- Inbetween



$$\boxed{\frac{1}{2} k A^2 = \frac{1}{2} m v^2 + \frac{1}{2} k x^2}$$

Solve for V

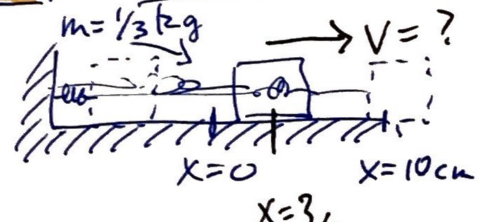
$\frac{1}{2} m v^2 = \frac{1}{2} k A^2 - \frac{1}{2} k x^2$ $\quad \times 2$ $\quad \div m$

$$v = \sqrt{\frac{k}{m}(A^2 - x^2)}$$

velocity at any position x

$$\boxed{v(x) = A \sqrt{\frac{k}{m}\left(1 - \left(\frac{x}{A}\right)^2\right)}}$$

EX if $k = 1600\ N/m$, $A = 10cm$, what is V when $x = 3cm$



$$v(3cm) = 10cm \sqrt{\frac{1600\ N/m}{0.33kg}\left(1 - \left[\frac{3cm}{10cm}\right]^2\right)}$$

$v(@3cm) = 660.9\ cm/s = \boxed{6.6\ m/s\ @_{x=3cm}}$

Lets build out formulas...

- max velocity can be computed from $\boxed{A,\ k \& m}$

$$\boxed{V = A\sqrt{\frac{k}{m}\left(1-\left(\frac{x}{A}\right)^2\right)}}$$ factor out

$$= A\sqrt{\frac{k}{m}}\left[\sqrt{1-\left(\frac{x}{A}\right)^2}\right]$$

@ $x=0$ we have $V = V_{max}$ as the mass flys through the origin

$$V_{max} = A\sqrt{\frac{k}{m}}\underbrace{\sqrt{1-\left(\frac{0}{A}\right)^2}}_{1}$$

so $$\boxed{V_{max} = A\sqrt{k/m}}$$

same

this lets us write $$\boxed{V(x) = V_m\sqrt{1-\left(\frac{x}{A}\right)^2}}$$

Now Since $\boxed{x(t) = A\cos(\omega t)}$

$c^2+s^2=1$

plug into $V(t) = V_{max}\sqrt{1-\frac{A^2\cos^2(\omega t)}{A^2}} = V_{max}\sqrt{1-\cos^2(\omega t)}$

so $$\boxed{V(t) = V_{max}\sin(\omega t)}$$

Q: what about $\underline{\text{acc'ln}}$? $F = ma \rightarrow a = \frac{F}{m}$

but $F = -kx$ so $a = -\frac{k}{m}x \rightarrow$ @ $x=A$ $\boxed{a_{max} = -\frac{kA}{m}}$

$$F_{sp}(t) = -k\boxed{[A\cos(\omega t)]}$$

$\div m$ $$\boxed{a(t) = -a_{max}\cos(\omega t)}$$

Formulas for General SHO

- $\boxed{x(t) = A\cos(2\pi f t)}$ (A = x_{max})

- $v(t) = -v_{max}\sin(2\pi f t)$ but $\boxed{v_{max} = A\sqrt{\frac{k}{m}}} = \underline{A\omega}$

  $v(t) = -A\omega\sin(2\pi f t)$

  $\boxed{v(t) = -A\,2\pi f\sin(2\pi f t)}$

  $f = \frac{\omega}{2\pi}$, $\boxed{\omega = \sqrt{\frac{k}{m}}}$

  $\Rightarrow$ $\boxed{f = \frac{1}{2\pi}\sqrt{k/m}}$

  $\boxed{T = 2\pi\sqrt{m/k}}$

- $a(t) = -a_{max}\cos(2\pi f t)$

  $a(t) = -\frac{kA}{m}\cos(2\pi f t)$

  $\boxed{a(t) = -A\,4\pi^2 f^2\cos(2\pi f t)}$

  $\boxed{a_{max} = -\frac{kA}{m}}$

  $a_m = -(\omega)^2 A$

  $a_m = -(2\pi f)^2 A$

- Adapt these to our spring mass system: $\boxed{A, k, m}$

$\omega = \sqrt{\frac{k}{m}}$

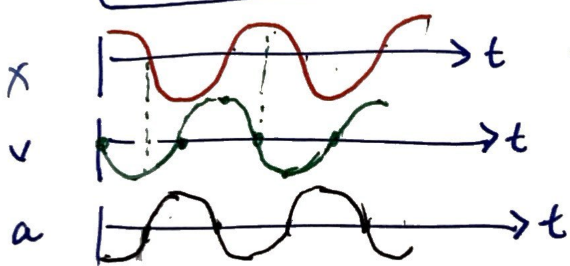
$\boxed{x(t) = A\cos\left(\sqrt{\frac{k}{m}}\,t\right)}$

$\boxed{v(t) = -A\sqrt{\frac{k}{m}}\sin\left(\sqrt{\frac{k}{m}}\,t\right)}$; $\boxed{v(x) = A\sqrt{\frac{k}{m}\left(1-\left(\frac{x}{A}\right)^2\right)}}$

$\boxed{a(t) = -A\frac{k}{m}\cos\left(\sqrt{\frac{k}{m}}\,t\right)}$ $\leftarrow$ $a = -\frac{kx}{m}$

energy: $\boxed{\frac{1}{2}kA^2 = \frac{1}{2}mv^2 + \frac{1}{2}kx^2}$, $E_{TOT} = \frac{1}{2}kA^2$

- plots

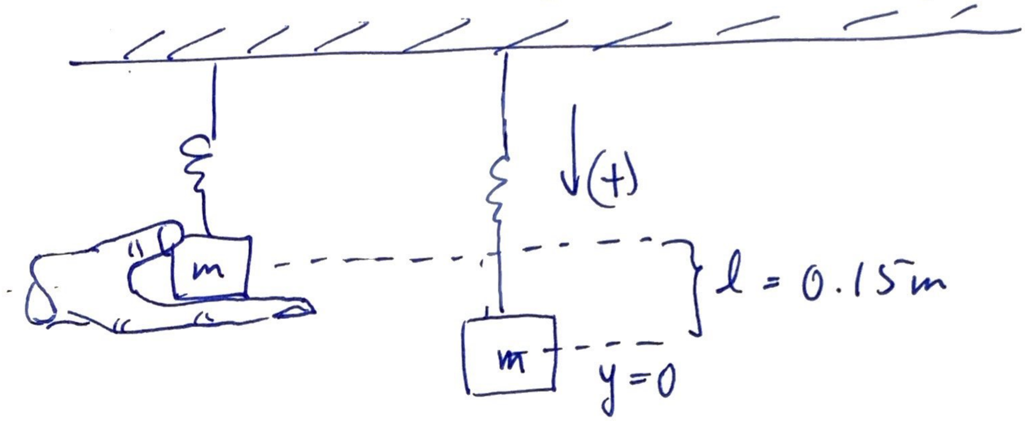


"90° out of phase"

"180° out of phase"

Ex Vertical hanging spring is supported at its natural length. When released it stretches 0.15 m if a 0.3 kg mass is suspended by the spring.

(a)

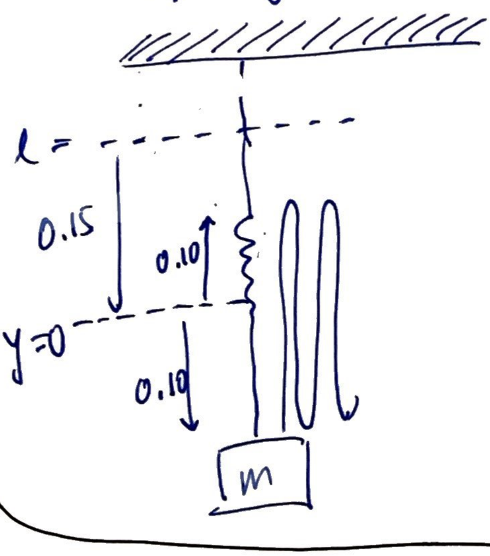


Find the spring constant, k ?

$$F = kx \rightarrow k = \frac{F}{l} = \frac{mg}{l} = \frac{(0.3\,kg)(9.8\,m/s^2)}{0.15\,m}$$

So $\boxed{k = 19.6\ N/m}$

(b) We pull the spring down an add'l 10 cm. Find max velocity once we release the spring.



$$V_{max} = A\sqrt{\frac{k}{m}} = 0.10\sqrt{\frac{19.6\,N/m}{0.3\,kgm}}$$

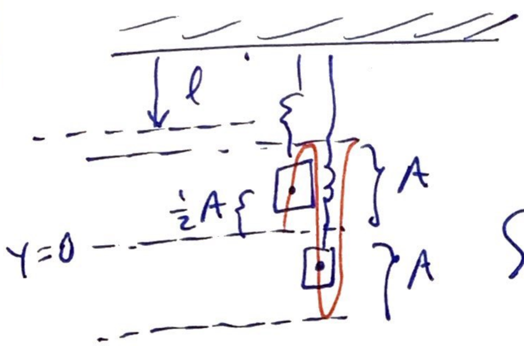
$\boxed{V_{max} = 0.81\ m/s}$ as the block passes through the equilibrium position

(c) What is the maximum acc'ln?

$$a_{max} = -\frac{kA}{m} = -\frac{(19.6\,\frac{N}{m})(0.10\,m)}{0.3\,kg} = \boxed{6.53\,m/s^2}$$

this occurs at max discplacement ($\ell + 0.10\,m$ or $\ell - 0.10\,m$ i.e. $\underline{\underline{0.25\,m}}$ or $\underline{\underline{0.05\,m}}$

(d) What is the velocity half-way between an extrema and the equilibrium?



We need v as a function of x, here $x = \frac{A}{2}$

So $V(x) = A\sqrt{\frac{k}{m}}\sqrt{1-\left(\frac{A/2}{A}\right)^2}$

$$V(x) = (0.10\,m)\sqrt{\frac{19.6\,N/m}{0.3\,kg}}\sqrt{1-\frac{1}{4}}$$

$$\boxed{V(x) = 0.70\,m/s}$$

(e) Total energy? $E_{TOT} = \frac{1}{2}kA^2 = \frac{1}{2}(19.6\,\frac{N}{m})(0.10\,m)^2$

$$\boxed{E_{TOT} = 0.098\,J}$$

(f) What is the PE & KE at a ½ A point?

$x = \frac{A}{2}$ • $PE_{1/2} = \frac{1}{2}kx^2 = \frac{1}{2}k\left(\frac{A}{2}\right)^2 = \frac{kA^2}{8} = \boxed{0.0245\,J}$

• $KE_{\frac{1}{2}} = \frac{1}{2}mv_{1/2}^2 = \frac{1}{2}(0.3\,kg)(0.70\,m/s)^2 = \boxed{0.0735\,J}$ +

0.0980 J TOTAL

BTW as a Test

$$KE_{\frac{1}{2}} = E_{TOT} - PE_{1/2}$$

(12)

✱ Recall
For general $\boxed{\text{SHO}}$ we have for any "ω":

$x(t) = A\cos(\omega t)$
$v(t) = -V_{max}\sin(\omega t)$
$a(t) = -a_{max}\cos(\omega t)$

$\omega = 2\pi f, \quad f = \frac{1}{T}$

#

$X_{max} = A, \quad V_{max} = 2\pi A f, \quad a_{max} = (2\pi f)^2 A$

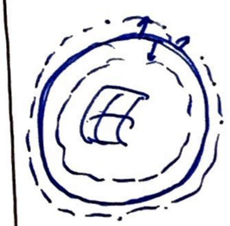
ω is given by the physical structure that oscillates

EX

An observer models an inflating/deflating balloon using an SHO model. The change of radius about an equilibrium radius maxes out at $x = 0.3\,m$ and the angular frequency is $8\ ^r/sec = \omega$

Find

(a) max displacement? $A = 0.3\,m$



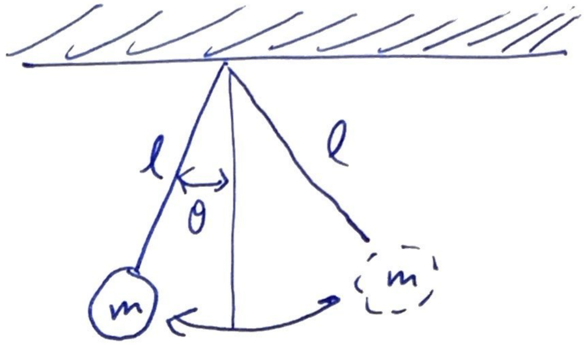
(b) Frequency? $f = \frac{\omega}{2\pi} = \frac{8.0}{2\pi} = 1.27\,Hz$

(c) period? $T = \frac{1}{f} = \frac{1}{1.27\ cy/sec} = 0.79\ s/cycle$

(d) max speed? $V_{max} = 2\pi(0.3\,m)(1.27\,Hz) = 2.4\,m/s$

(e) max acc'ln? $a_{max} = (2\pi f)^2 A = \omega^2 A$
$= (8.0\ ^{rad}/s)^2 (0.3\,m) = \boxed{19\,m/s^2}$

(*) Pendulum



It is shown in an advanced physics treatment that $\boxed{\omega = \sqrt{g/l}}$

$\theta(t) = \theta_{max} \cos\left(\sqrt{\frac{g}{l}}\, t\right)$

$\omega(t) = \omega_{max} \sin\left(\sqrt{\frac{g}{l}}\, t\right)$

$\alpha(t) = -\alpha_{max} \cos\left(\sqrt{\frac{g}{l}}\, t\right)$

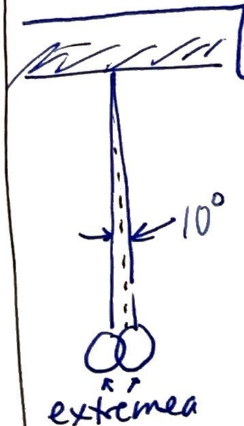
((Don't mix osc. freq with ang. velocity

Note also that arc length $S = l\theta$

so $\boxed{s(t) = \theta_{max}\, l \cos\left(\sqrt{g/l}\, t\right)}$

$$T = \frac{1}{f} = \frac{1}{\omega/2\pi} = \frac{2\pi}{\omega} = 2\pi\sqrt{l/g}$$

EX A pendulum at the planetarium has a length of 10 m ((Note: For these SHO formulas to work the angular displacement, θ_{max}, must be less than 5°))



(a) Period? $T = 2\pi\sqrt{\frac{l}{g}} = 2\pi\sqrt{\frac{10m}{9.8 m/s^2}} = \boxed{6 \text{ sec}}$

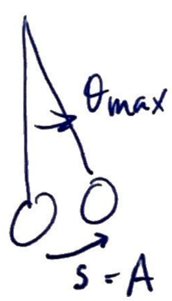
(b) If $\theta_{max} = 5°$ what is S_{max}?

$\boxed{S_{max} = l\,\theta_{max}} = (10m)(5°)\left(\frac{2\pi^r}{360°}\right) = 87 cm$

(c) max acc'ln?

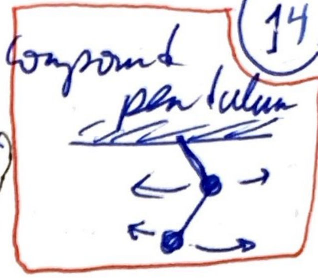
$a_{max} = (\overbrace{2\pi f}^{\omega})^2 A = \left(\sqrt{\frac{g}{l}}\right)^2 A = \frac{gA}{l} = \frac{(9.8)}{10m} \cdot 0.87m$

$\boxed{a_{max} = 0.85 \, m/s^2}$

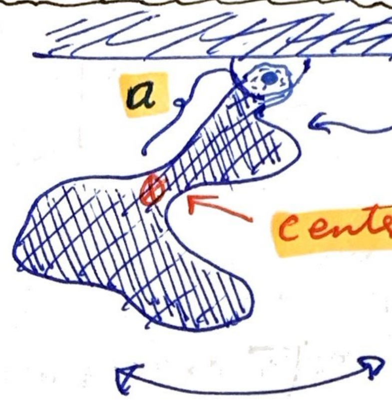


(d) $s(t) = ?$ $s(t) = A\cos(\omega t) = 10m \cos\left(\sqrt{\frac{9.8 m/s^2}{10m}}\, t\right)$

Physical pendulum



- A physical Pendulum is any object hung from a point not its Center of Mass. oscillation will occur.



center of mass

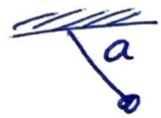
I = moment of inertia is about the C.M.

$$\boxed{\omega = \sqrt{\frac{mga}{I}}}$$

Then $f = \frac{1}{2\pi}\sqrt{\frac{mga}{I}}$

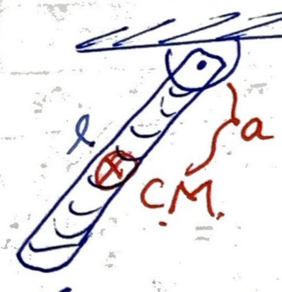
and $$\boxed{T = 2\pi\sqrt{\frac{I}{mga}}}$$

Note: $I = ma^2$ for traditional pendulum



so $T = 2\pi\sqrt{\frac{ma^2}{mga}} = \underbrace{2\pi\sqrt{\frac{a}{g}}}_{\text{simple pend. formula}}$

EX Hang a rod by it's end.



$$T = 2\pi\sqrt{\frac{\frac{1}{3}ml^2}{mga}} \quad a = l/2$$

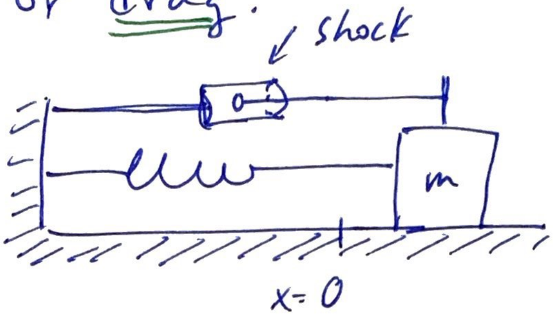
$I = \frac{1}{3}ml^2$

$$= 2\pi\sqrt{\frac{ml^2}{3mg\left(\frac{l}{2}\right)}}$$

$$\boxed{T = 2\pi\sqrt{\frac{2}{3}\frac{l}{g}}}$$

⊛ Damped SHO

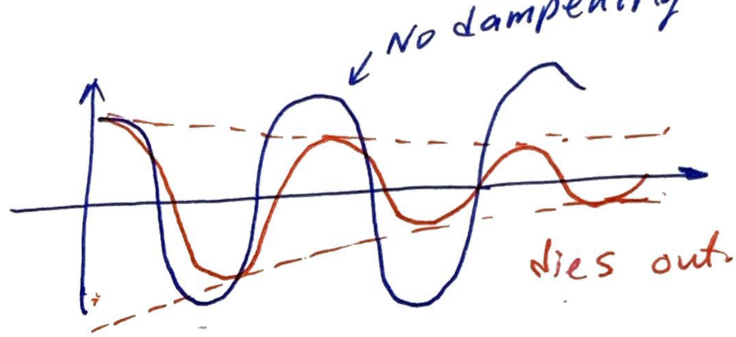
Here we add some dampening, or friction or drag.



} strut

drag $\propto$ velocity.

The amplitude decays

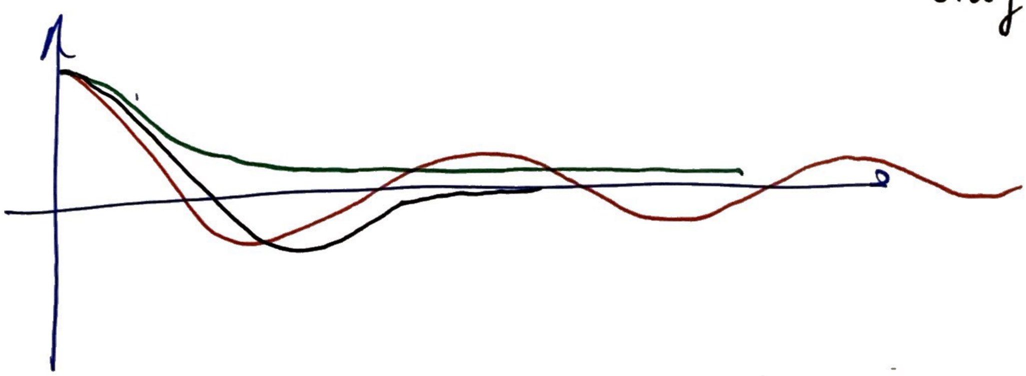


The piston (drag) converts energy into heat.

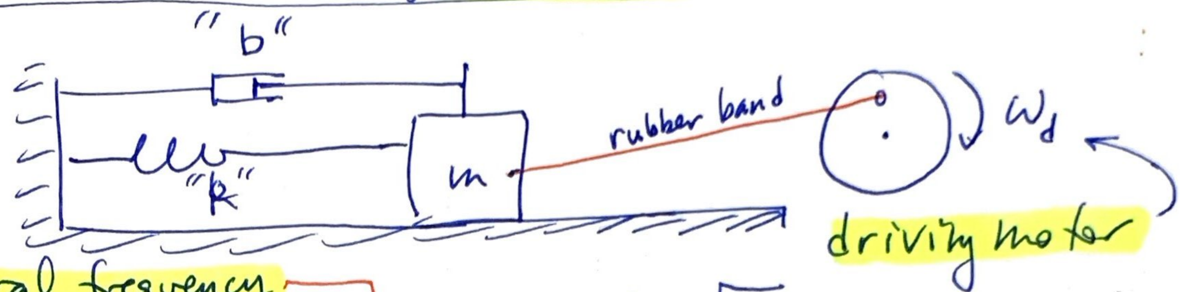
*There are three forms of damped motion:

- $\boxed{\text{underdamped}}$ (worn out shock absorber)
- $\boxed{\text{critically damped}}$ (optimal shock)
- $\boxed{\text{overdamped}}$ (shock is too stiff)

"drag to mass ratio."



⊛ Forced (Driven) Harmonic Oscillator



- natural frequency

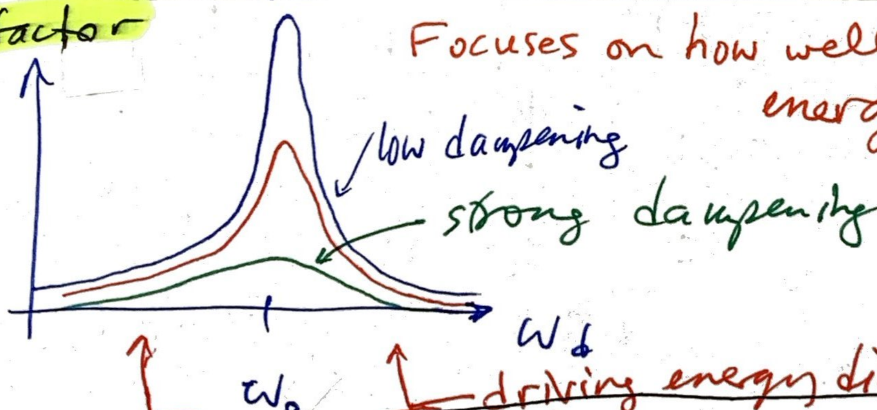
$$\boxed{\omega_0 = \sqrt{k/m}}\,, \quad f_0 = \frac{1}{2\pi}\sqrt{k/m}$$

- driving frequency is ω_d

When we drive the system at ω_0, i.e. $\omega_d = \omega_0$
then we are pumping energy into the system
and if the damping is not sufficient to "bleed"
away the extra energy we will have a
resonance situation "Tacoma Narrows
bridge Collapse" { "practical Engineering" Bridge Collapse

- Q factor

Focuses on how well a system retains energy driven into it



driving energy dissapated as heat

- END part I : SHO
- Next part II : waves on a string (Guitar)