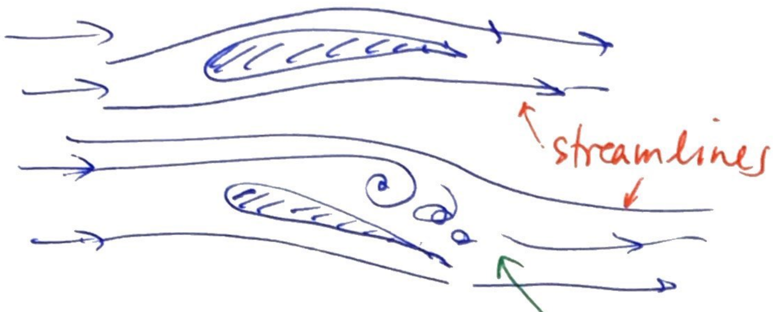
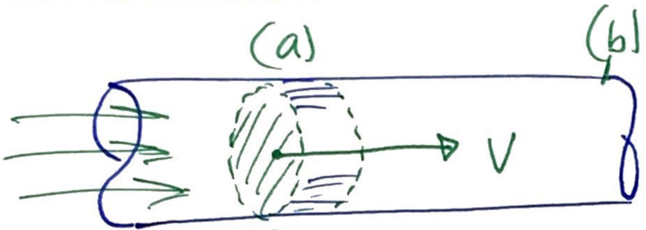
10b Fluid Dynamics

we study laminar fluid flow. In this type of flow the particles of liquid stay near by eachother.



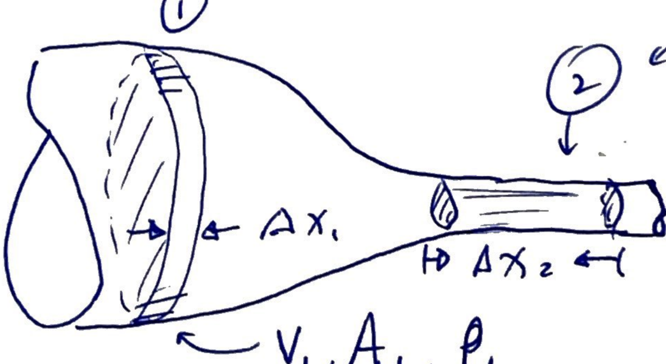
* mass flow rate



at station (a) $\boxed{\text{mass flow rate}\Big|_{\text{"a"}} = \dfrac{\text{mass of the "puck" of water}}{\text{change in time}}}$

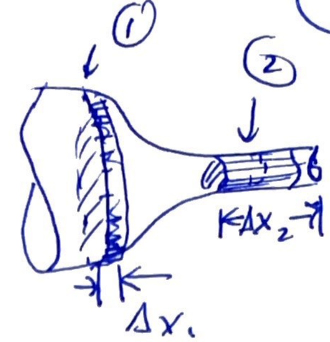
$$\boxed{\text{flow rate} = \frac{\Delta\,\text{mass}}{\Delta t}}$$

* If the pipe narrows then the "puck" changes shape



Conservation of mass tells us that fluid flow rates are the same...

flow rate at 1 = rate at 2



$$\frac{\Delta m_1}{\Delta t} = \frac{\Delta m_2}{\Delta t}$$

$$\frac{\rho_1 V_1}{\Delta t} = \frac{\rho_2 V_2}{\Delta t} \qquad V = A \cdot \Delta x$$

$$\rho_1 A_1 \underbrace{\frac{\Delta x_1}{\Delta t}}_{v_1} = \rho_2 A_2 \underbrace{\frac{\Delta x_2}{\Delta t}}_{v_2}$$

$$\boxed{\rho_1 A_1 v_1 = \rho_2 A_2 v_2}$$

mass flow rate eqn

- compressible flow

- Incompressible $\boxed{\rho_1 = \rho_2}$

Then the flow rate becomes $\boxed{A_1 v_1 = A_2 v_2}$

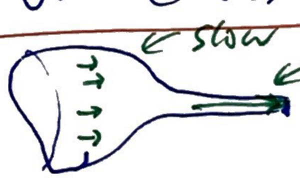
⊛ Application

- IF $A_1 > A_2$ then $\frac{A_1}{A_2} > 1$ so

$$v_2 = \left(\frac{A_1}{A_2}\right) v_1$$

shows us that $v_2 > v_1$

Narrow cross-sectional area has faster flow



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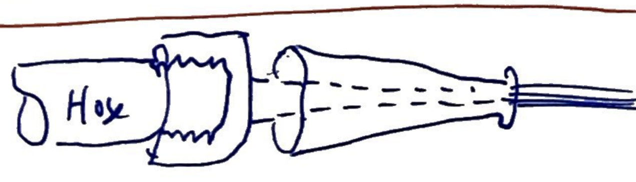

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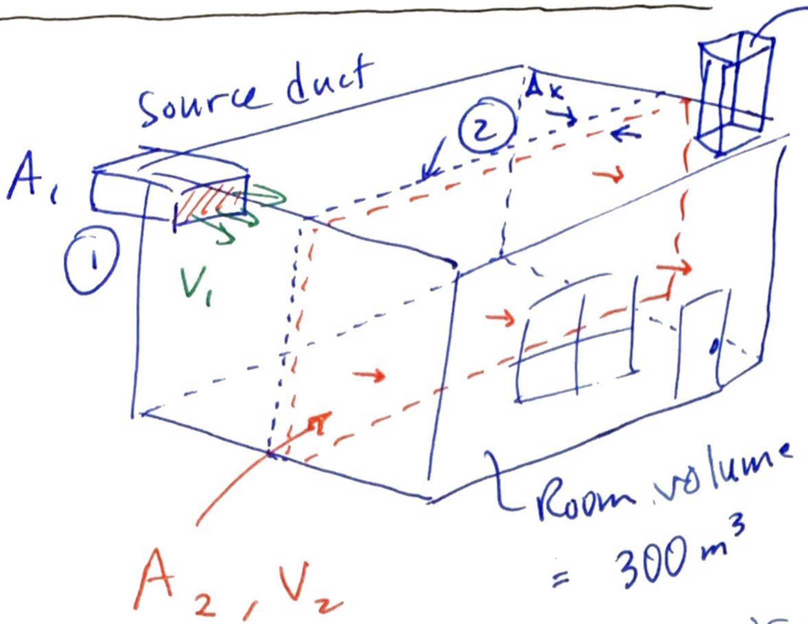

Q: what area of duct is need to replenish the air in the room every 15 minutes if $V_1 = 3 m/s$ is duct flow?

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rate @ 1 = rate @ 2

$$\boxed{\rho_1 A_1 v_1 = \rho_2 A_2 V_2}$$

incompressible flow $\rho_1 = \rho_2$

$$A_1 V_1 = A_2 V_2$$

$v = \frac{\Delta x}{\Delta t}$

$$A_1 V_1 = A_2 \frac{\Delta x}{\Delta t}$$

$A_2 \Delta x = \Delta Vol.$

$$A_1 V_1 = \frac{V_2}{\Delta t}$$

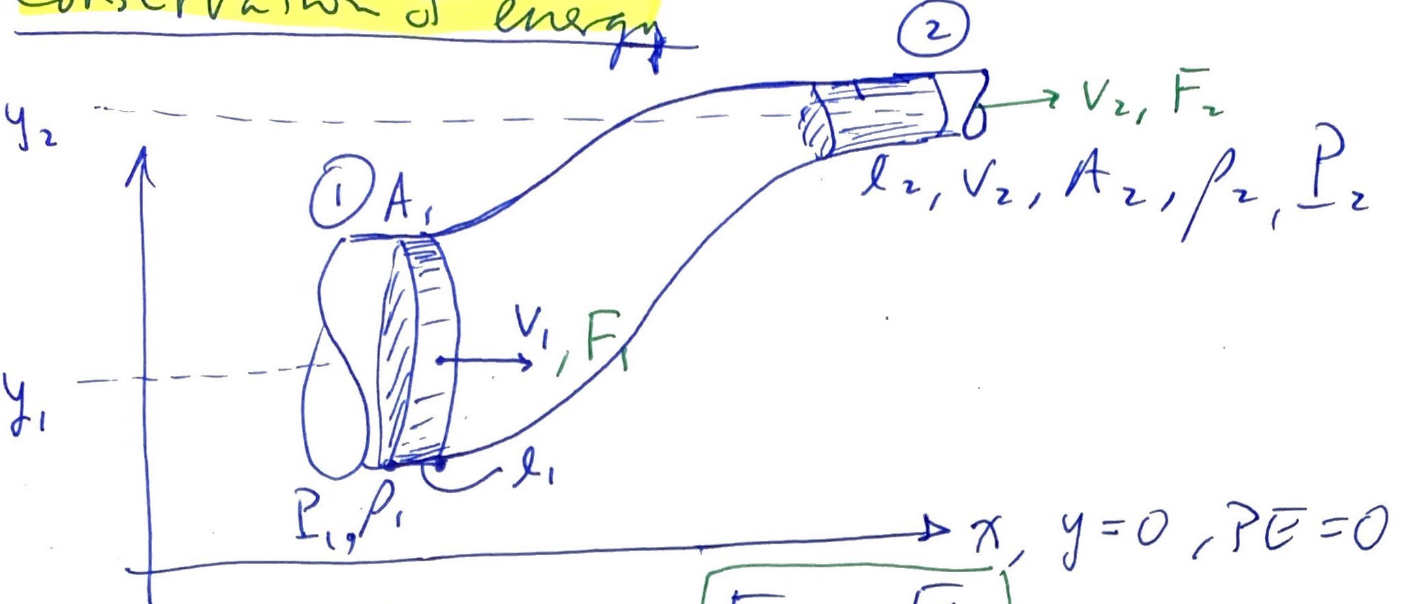
← volume of Room

$$\Rightarrow A_1 = \frac{V_2}{V_1 \Delta t} = \frac{300 m^2}{(3 m/s)(15 min)\left(\frac{60 s}{min}\right)}$$

$$\boxed{A_1 = 0.11 m^2}$$

1/10 of the size of the dishwasher box.

Conservation of energy



- Conservation of energy $\boxed{E_1 = E_2}$

OR $\Delta W@1 + KE@1 + PE@1 = \Delta W@2 + KE@2 + PE@2$

$$F_1 \cdot l_1 + \frac{1}{2} m_1 v_1^2 + m_1 g y_1 = F_2 \cdot l_2 + \frac{1}{2} m_2 V_2^2 + m_2 g y_2$$

$$(P_1 A_1) l_1 + \frac{1}{2}\rho_1 V_1 v_1^2 + \rho_1 V_1 g y_1 = P_2 A_2 l_2 + \frac{1}{2}\rho_2 V_2 v_2^2 + \rho_2 V_2 g y_2$$

let $A_1 \cdot l_1$ and $A_2 l_2$, factor out

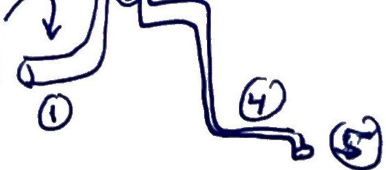
$$\boxed{\cancel{A_1 l_1}\left[P_1 + \frac{1}{2}\rho_1 v_1^2 + \rho_1 g y_1\right] = \cancel{A_2 l_2}\left[P_2 + \frac{1}{2}\rho_2 v_2^2 + \rho_2 g y_2\right]}$$

- use conservation of mass flow $A_1 l_1 = A_2 l_2$

$$\boxed{P_1 + \frac{1}{2}\rho_1 V_1^2 + \rho_1 y_1 g = P_2 + \frac{1}{2}\rho_2 V_2^2 + \rho_2 y_2 g}$$

Bernoulli's Equation : Cons. of energy.

$$\Rightarrow \boxed{P + \frac{1}{2}\rho v^2 + \rho y g = \text{constant}}$$



Applications

- level pipe flow $y_1 = y_2$

$$\Rightarrow \boxed{P_1 + \frac{1}{2}\rho_1 V_1^2 = \frac{1}{2}\rho_2 V_2^2 + P_2}$$

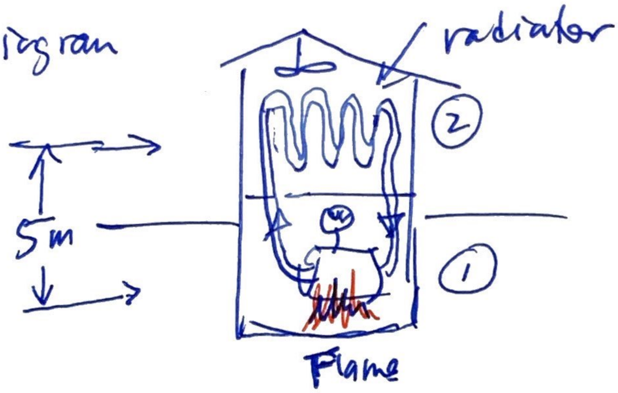
- multi height pipe but no (or "slow") flow $V_1 = 0, V_2 = 0$
  - incompressible

$$\Rightarrow \boxed{\Delta P = \rho g \Delta y}$$

This is just hydrostatic pressures seen in 10a $P = \rho g h$

Ex Consider a hot water radiative heating in an older house

(i) Diagram



- Water is pumped through the hot water pipe at 0.5 m/s and the pipe is 4.0 cm in diameter. The boiler runs at 3.0 atm.

Q: what will be the speed and pressure on the second floor if the pipe diameter narrows to 2.6 cm. Assume 5 meters between boiler and radiator.

Variables @ 1

$$\begin{cases} V_1 = 0.5\,m/s \\ r_1 = 2\,cm \\ P_1 = 3\,atm \\ y_1 = 0 \end{cases}$$

Variables @ 2

$V_2 = ?$

$r_2 = 1.3\,cm$

$P_2 = ?$

$y_2 = 5\,m$

Cont. ⇓

cont.

Ex

- Assume incompressible $\rho_1 = \rho_2$
- we have one eqn and two unknowns

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Bernie's Eqn $\left\{ P_1 + \frac{1}{2}\rho_1 v_1^2 + \rho_1 y_1 g = P_2 + \frac{1}{2}\rho_2 V_2^2 + \rho_2 y_2 g \right.$

? ?

- mass flow $\left\{ \cancel{\rho_1} V_1 A_1 = \cancel{\rho_2} V_2 A_2 \right.$ $\rho_1 = \rho_2 \Rightarrow \rho_{H_2O}$

$$\Rightarrow V_2 = \left(\frac{A_1}{A_2}\right) V_1$$

$$= \left(\frac{\cancel{\pi} r_1^2}{\cancel{\pi} r_2^2}\right) V_1 \;\rightarrow\; = \left(\frac{2cm}{1.3cm}\right)^2 (0.5 m/s)$$

$$\boxed{V_2 = 1.2 m/s}$$

Insert this into Bernoulli's eqn:

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho V_2^2 + \rho g y_2$$

$$P_1 + \frac{1}{2}\rho(v_1^2 - V_2^2) + \rho g\,(\underbrace{y_1 - y_2}_{\Delta y}) = P_2$$

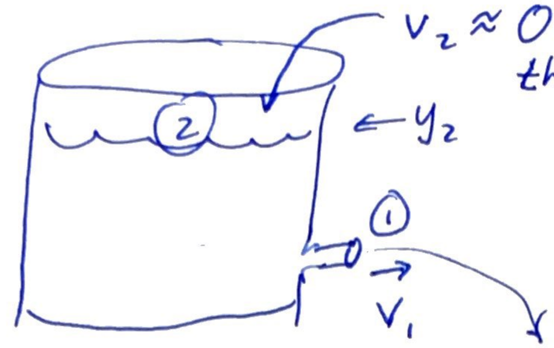
$$\underbrace{\left(3 * 1.01 * 10^5 \frac{N}{m^2}\right) + \frac{1}{2}(10^3 kg/m^3)\left[(0.5\tfrac{m}{s})^2 - (1.2 m/s)^2\right] + \left(10^3 \frac{kg}{m^3}\right)[0 - 5m]}_{= P_2}$$

- Higher altitude
- narrow pipe
- Faster Flow
- Lower pressure

$$P_2 = 2.5 \times 10^5 \frac{N}{m^2} \text{ or } \boxed{2.5\, atm}$$

Applications

- Torricelli Flow



since both top & pipe are exposed to the air we will let $P_1 = P_2$

Bernolli's

$$\underbrace{\cancel{P_1} + \frac{1}{2}\rho v_1^2 + \rho g y_1}_{\text{Bottom}} = \underbrace{\cancel{P_2} + \frac{1}{2}\rho \cancel{v_2^2} + \rho g y_2}_{\text{Top}}$$

$\cancel{v_2^2} \to 0$ m/s

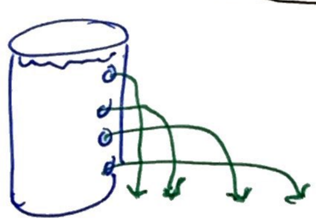
$$\frac{1}{2}\cancel{\rho} v_1^2 = \cancel{\rho} g y_2 - \cancel{\rho} g y_1$$

$$\boxed{v_1 = \sqrt{2g\,(y_2 - y_1)}}$$

Torricelli's eqn.

{ Recall In Chpt 6 : $\frac{1}{2}mv^2 = mgh \to v = \sqrt{2gh}$ }

EX



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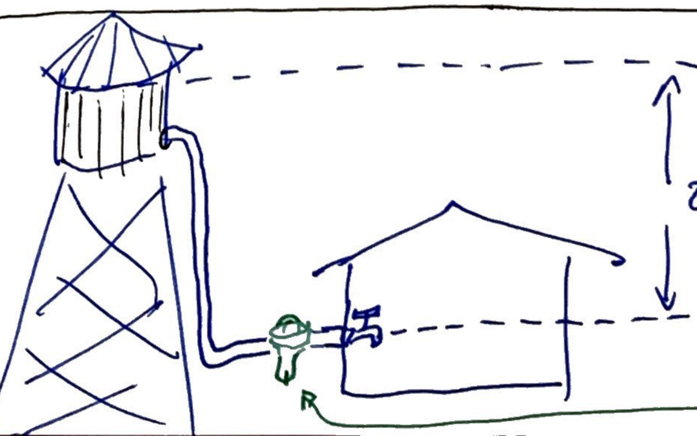

$$v_1 = \sqrt{2(9.8)(20\text{m})}$$

$$\underline{v_1 = 19.8\ \text{m/s}}$$ ouch!

we need a pressure regulator

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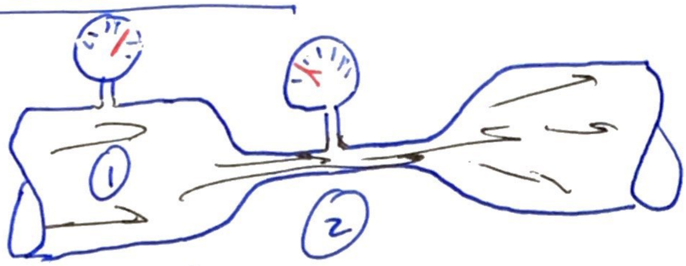
- Venturi Flow Meter : Find flow speed in a pipe.



r_1 = known, r_2 = known

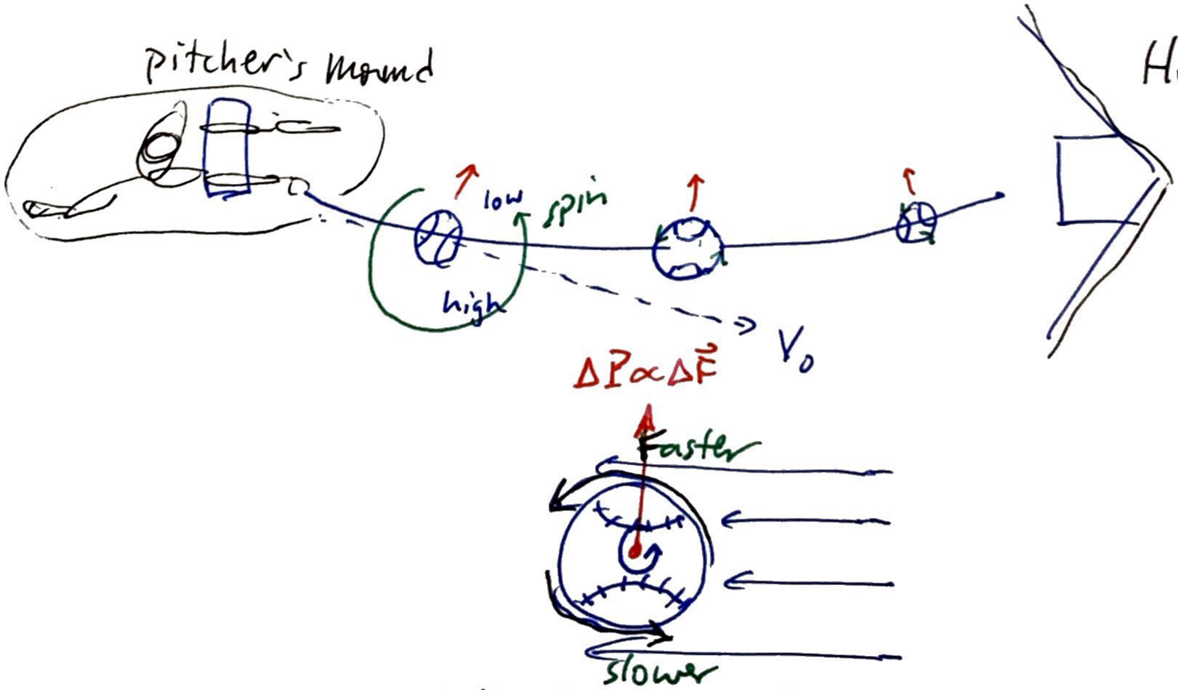
By measuring P_2, P_1 we can calculate V_1

Faster flow lower pressures.

Vertically

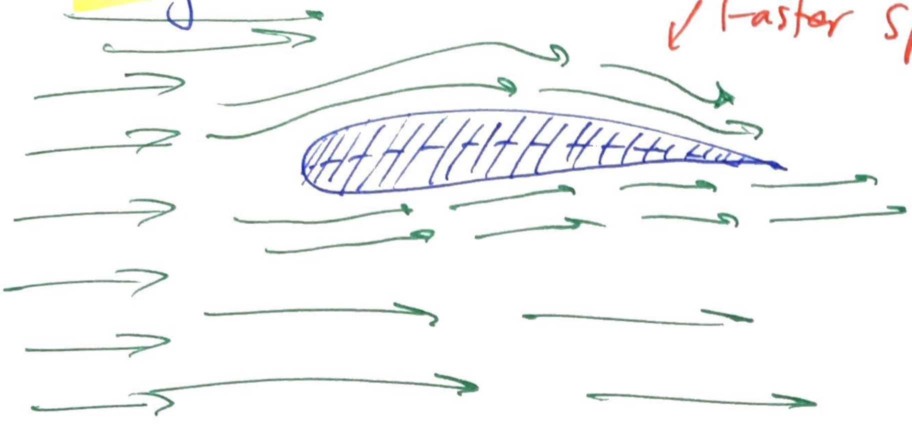
ping pong ball

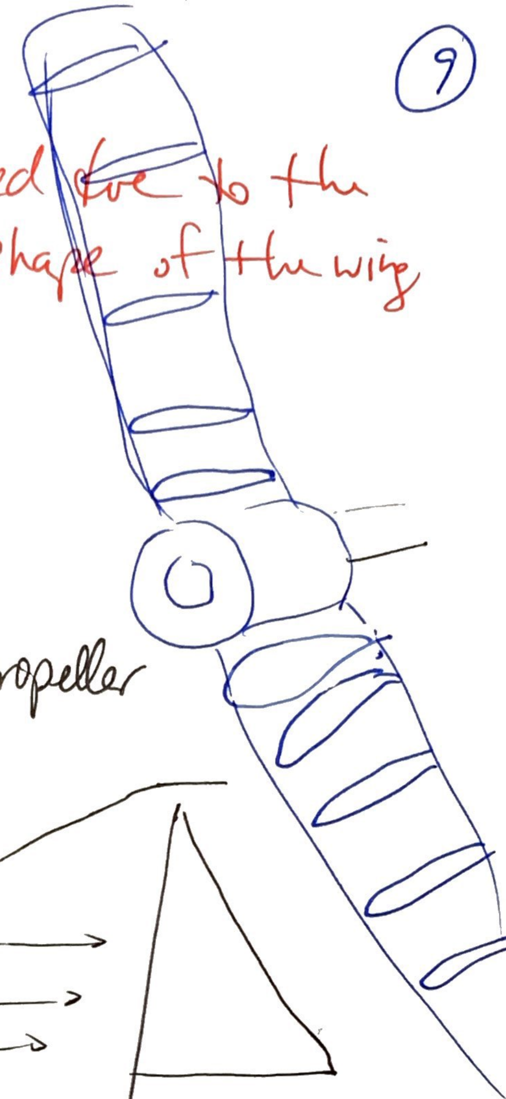
- curve pitches in baseball

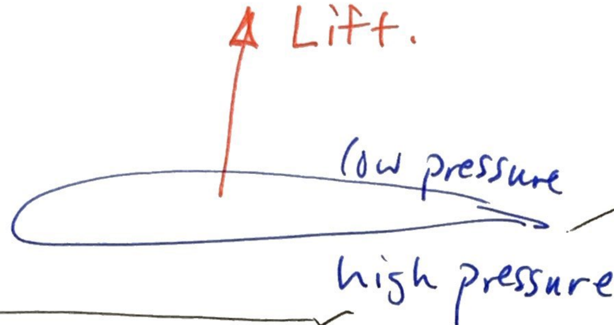


Wind tunnel view

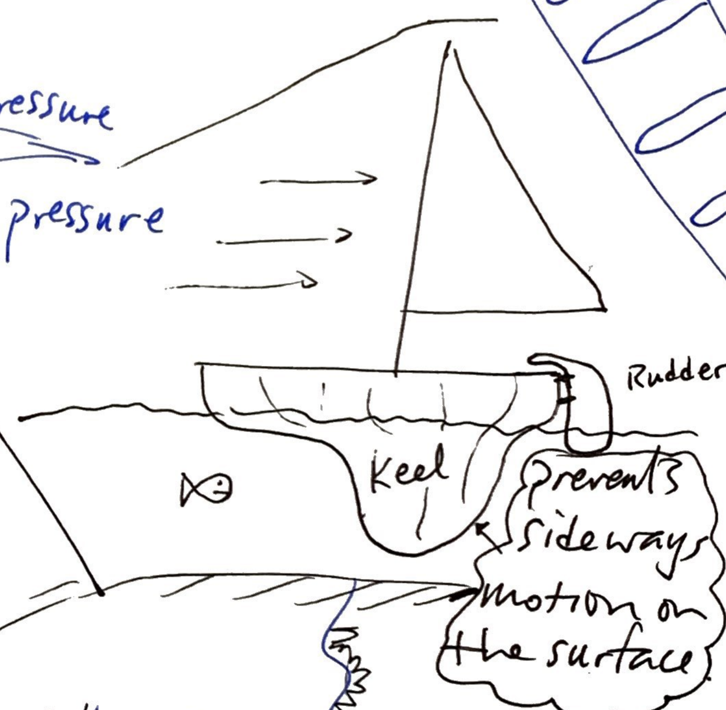
- Wings and lift

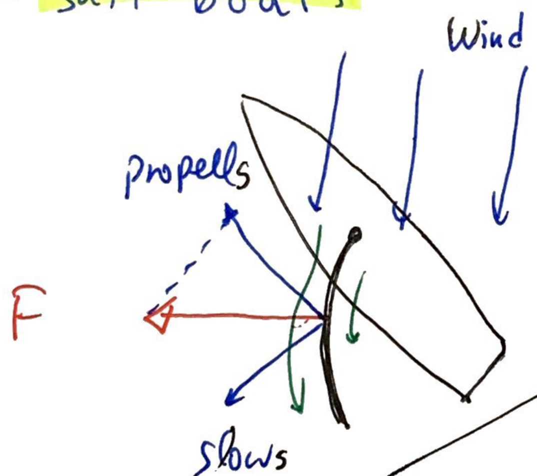


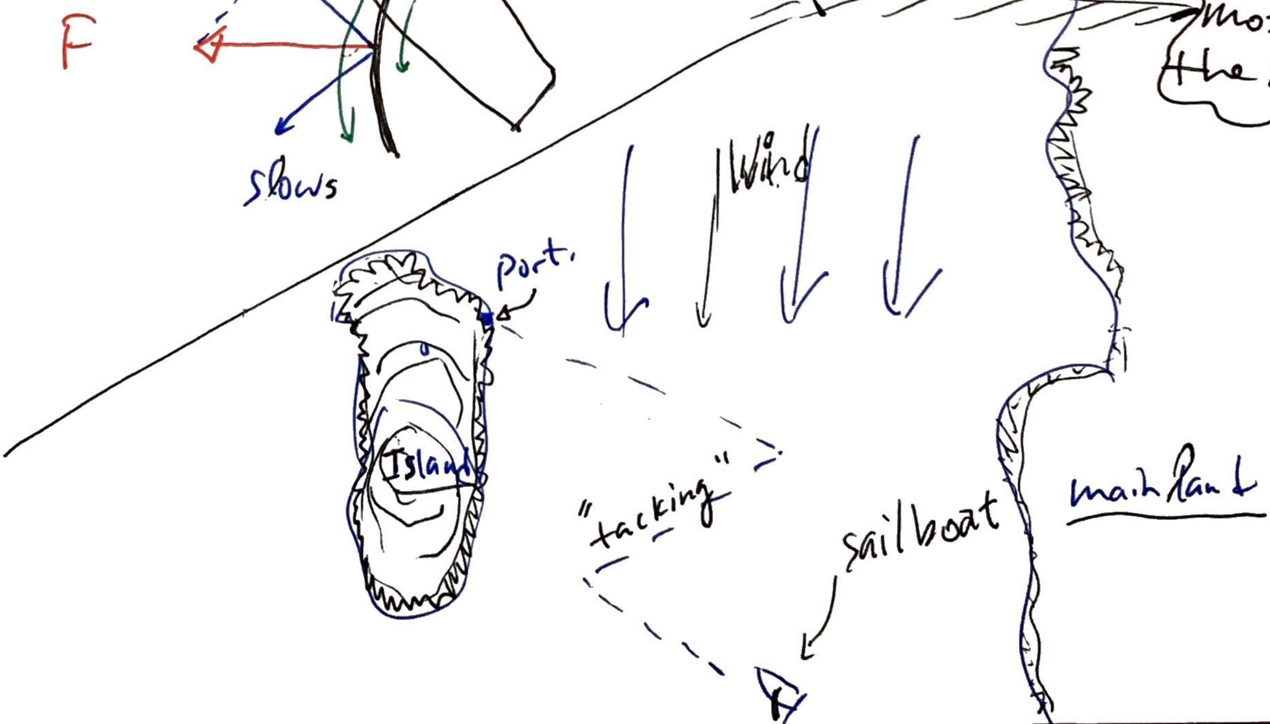




- sail boats





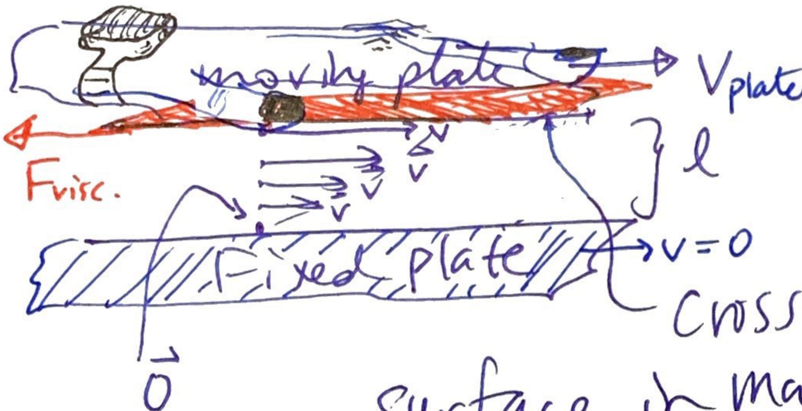


* **viscosity** due to the friction of air mole with themselves (their neighbors – "bonds")
the more dense a liquid the more viscous it tends to be.

$$F_{drag} = \boxed{F_{viscosity} = \eta A \left(\frac{v}{\ell}\right)}$$

contact area — velocity of plate — channel width

coefficent of viscosity

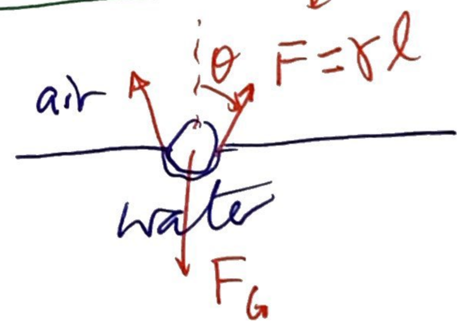


cross sectional area. A of surface in material contact to liquid.

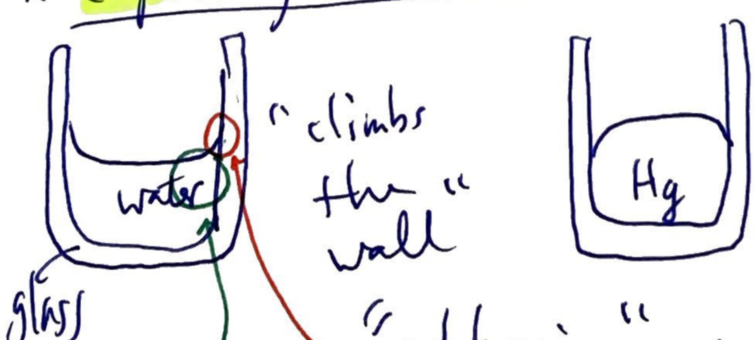
$[\eta]$ Pa·s , N/m²·s

* **surface Tension**

surface tension



* **capillary Action**



"Cohesion" Force between molecules of the same type

"adhesion" : Force between molecules of diff't types.

Cohesion