Chapter 7 Linear Momentum

We add a new tool to the tool box: Momentum.

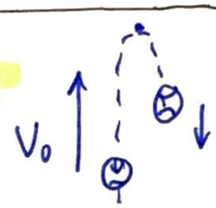
⊛ Each tool has it's strengths and weaknesses:

- Newtons Laws (kinematics)

$$\begin{cases} v = v_0 + gt \\ y = y_0 + v_0 t + \frac{1}{2} g t^2 \end{cases}$$

$$\left\{ \sum F = ma \right.$$

- Energy



$$P_1 + K_1 = P_2 + K_2 + W_{drag}$$

$$\frac{1}{2} m v_0^2 = mgh + F_d \cdot H$$

- Momentum we conserve the product of "mass · speed"
  "Best for collisions."

* Momentum

$$\boxed{\text{The product of an object's velocity and mass is called is momentum}}$$

$$\boxed{\vec{p} = m\vec{v}}$$

- Dimensions: $[p] = (kg)\left(\frac{m}{s}\right)$

- Note that we can mult. & div. by another "second" to get...

$$[p] = kg\,\frac{m}{s^2}\cdot s$$

this reveals another form of momentum:

$$\boxed{\Delta\vec{p} = \vec{F}\cdot\Delta t}$$ "impulse"

$\div\,\Delta t$ $$\boxed{\vec{F} = \frac{\Delta\vec{p}}{\Delta t}}$$ another form of Newton's Law

- momentum is best used in a "before vs. after" situation

Variations

- $\vec{F} = \frac{\Delta\vec{p}}{\Delta t} = m\frac{\Delta\vec{v}}{\Delta t} = m\vec{a}$ if mass is fixed

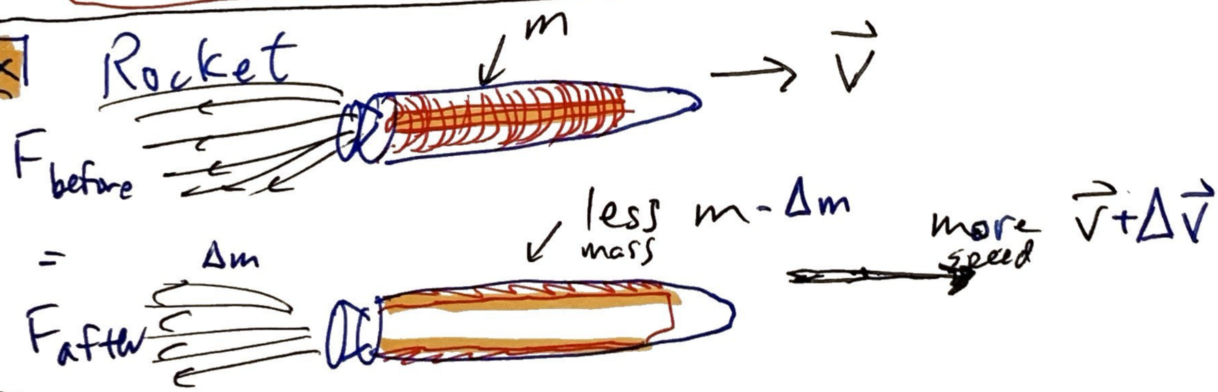
-or-

$$\vec{F} = \frac{\Delta\vec{p}}{\Delta t} = \vec{v}\,\frac{\Delta m}{\Delta t}$$ if velocity is fixed

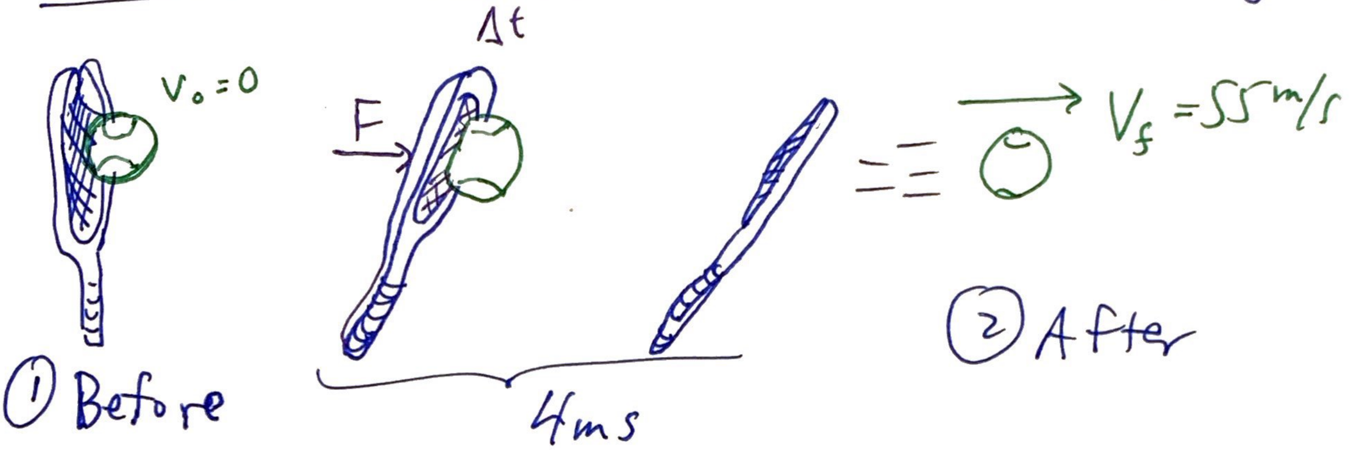
-or-

$$\vec{F} = \boxed{\frac{\Delta\vec{p}}{\Delta t} = \frac{\Delta(m\vec{v})}{\Delta t} = \frac{\vec{v}\,(\Delta m) + m\Delta\vec{v}}{\Delta t}}$$ neither velocity nor mass is fixed

Ex Rocket



EX A top tennis player has the tennis ball leaving her racket at 55 m/s {120 mphr}! If the ball is in contact with the racket for 4 ms, estimate the average force that was delivered to the ball. $m_{ball} = 0.06\,kg$ {60 gm}



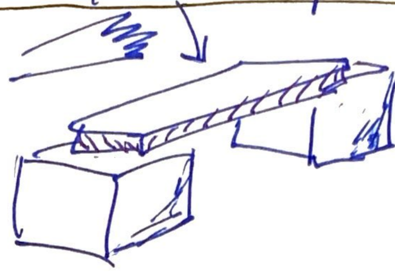
$$F_{ave} = \frac{\Delta p}{\Delta t} = \frac{m(\Delta v)}{\Delta t} = m\,\frac{V_2 - V_1}{\Delta t}$$

$$= (0.060\,kg)\frac{[55\,m/s - 0\,m/s]}{0.004\,s} = \underline{\underline{800\,N}}$$

- The weight of a 60 kg person $= (60)(9.8) \approx \underline{600\,N}$
- So $Force_{ave}$ about $\underline{\underline{1.3}}$ that of the $\underline{weight}$ of a 60 kg person ...

* **Impulse** {the rapid application of force}

Ex



Let the hand ~~Be~~ in contact with the board for Δt

If the speed of the hand @ contact is 10 m/s and the the board moves 1 cm before breaking, Find the impulse. {assume the hand is 1 kg in mass}

- Impulse $= I = F\cdot\Delta t$ but $F\cdot\Delta t = \Delta p$

$$I = m\Delta v = (1\,\text{kg})\left(\underset{\text{Before}}{10\,\text{m/s}} - \underset{\text{after}}{0\,\text{m/s}}\right) = 10\,\text{kg}\cdot\frac{\text{m}}{\text{s}}\ \{= N\cdot s\}$$

- Contact time

$$V = \frac{d}{t} \Rightarrow \Delta t = \frac{\Delta x}{V} = \frac{0.01\,\text{m}}{\frac{(10\,\text{m/s} + 0\,\text{m/s})}{2}}$$

10 m/s: instant before contact; 0 m/s: After contact

$$\Delta t = \frac{0.01\,\text{m}}{5\,\text{m/s}} = \boxed{2\,\text{ms}}$$

Contact.

- Average Force during contact: $(\vec{P}_{after} - \vec{P}_{before}) = \boxed{mv_{after} - mv_{before}}$

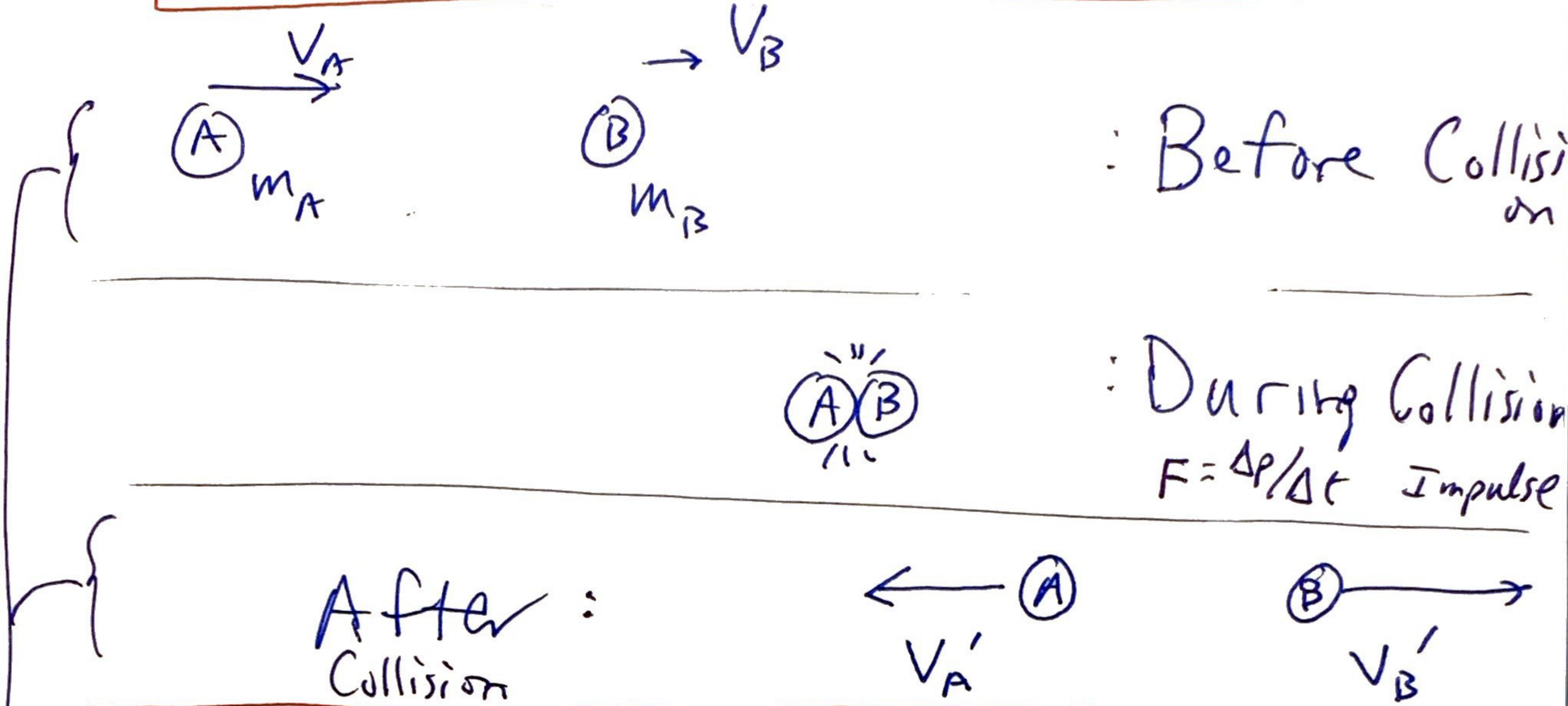
$$F_{ave} = \frac{\Delta p}{\Delta t} = \frac{m\Delta v}{\Delta t} = \frac{-10\,\text{kg}\cdot\text{m/s}}{2\times10^{-3}\,\text{s}} = -5000\,N$$

opposite motion

about 8 X the weight of a 60 kg person.
but only for 2 milliseconds

* Conservation of Momentum

The momentum total before a collision is equal to the momentum total after



$$\boxed{m_A \vec{V}_A + m_B \vec{V}_B = m_A \vec{V}_A' + m_B \vec{V}_B'}$$

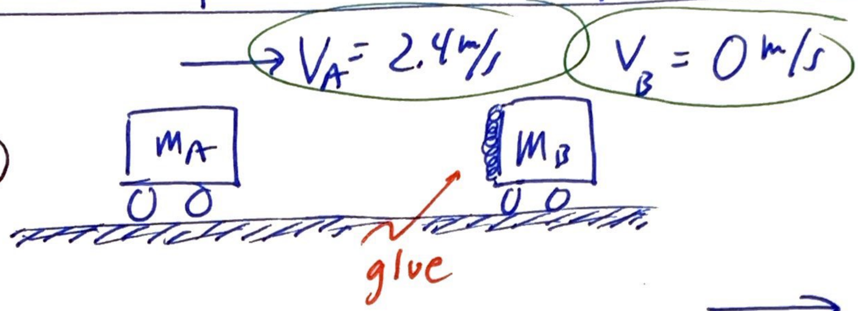
6a

EX Two carts collide and stick together. (Totally Inellastic)

Find the post-collision speed

(i)

Before ①



$\left\{ \begin{array}{l} m_A = 0.5\,kg \\ m_B = 0.7\,kg \end{array} \right.$

After ②

(iii) Eqns

$$\underline{\text{Before}} = \underline{\text{After}}$$

$$m_A V_A + m_B V_B = m_A V_A' + m_B V_B'$$

$$\boxed{m_A V_A + 0 = (m_A + m_B) V_f'}$$

(iv) do the math:

$$\boxed{V_f' = \left(\frac{m_A}{m_A + m_B}\right) V_A}$$

dim'less

populate:

$$V_f' = \left(\frac{0.5\,kg}{0.5 + 0.7}\right)(2.4\,m/s)$$

$$= (0.417)(2.4\,m/s)$$

$$V_f' = \boxed{1.0\,m/s}$$

combined speed to the right

6b

Ex FB Training Exercise

Before

$m_{FBall} = 0.5\,kg$

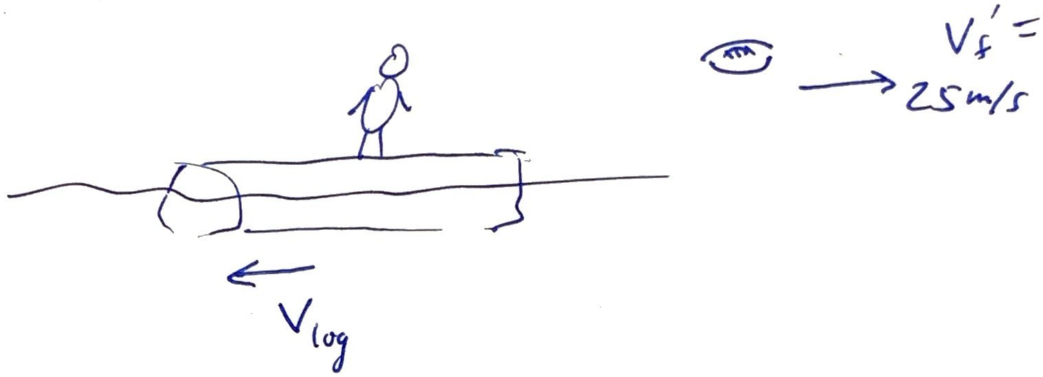
$m_{person} = 80\,kg$

$m_{log} = 100\,kg$

(i)

$$V_{log} = 0, \quad V_{football} = 0$$

After



Find the person/log speed after the pass

(iii) $m_A V_A + m_B V_B = m_A V_A' + m_B V_B'$

person + Log	football
m_A = mass of log + person,	m_B = mass of football
$V_A = 0$	$V_B = 0$
$V_A' = ?$	$V_B' = 25\,m/s$

(iv)

$$0 + 0 = (m_l + m_p) V_{log}' + m_{FB} V_{FB}'$$

$$\boxed{V_{log}' = -\frac{m_{FB}}{(m_{log} + m_{person})} V_{F.Ball}'}$$

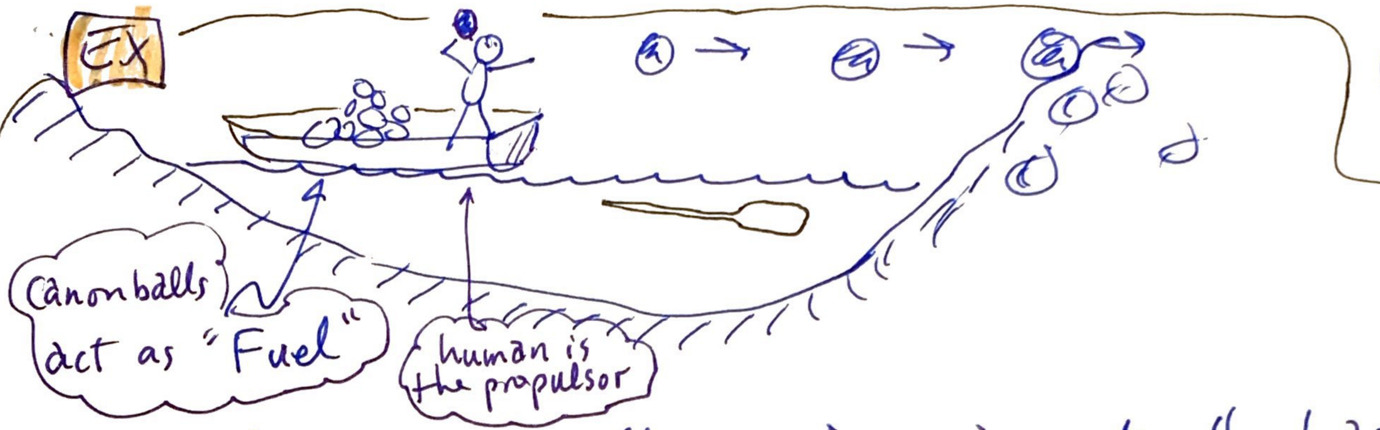
populate ...

$$= -\left(\frac{0.5\,kg}{100\,kg + 80\,kg}\right)(25\,m/s) = \boxed{-0.07\,m/s}$$

opposite direction

or 7 cm/s to the left

EX



- propulsion system is "throwing mass out the back" in order to propel you forward.
- this is what a rocket or jet engine does

From calculus we get this eqn

$$\boxed{\Delta u = V_{exhaust} \ln\left(\frac{\Delta m_f/\Delta t}{\Delta m_e/\Delta t}\right)}$$

where

$\frac{\Delta m_f}{\Delta t}$ = mass rate of burned propellant ejected

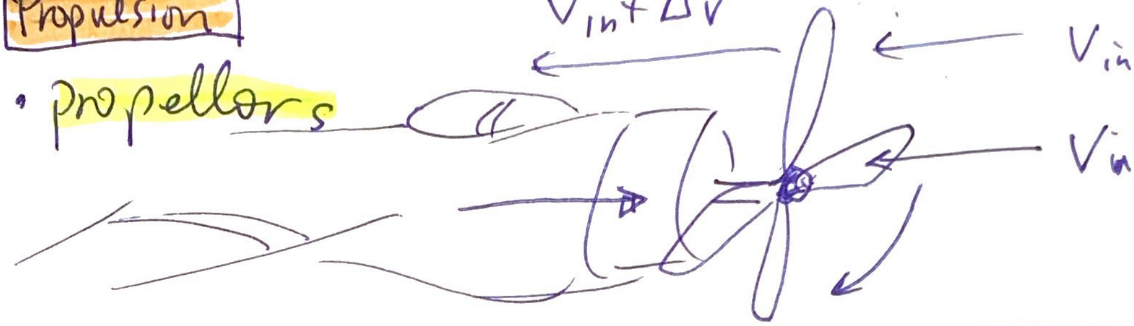
$\frac{\Delta m_e}{\Delta t}$ = mass rate decrease of existing unburned propellant.

Δu = change in rocket speed.

youtube: "Best of the Best Shuttle Launches"

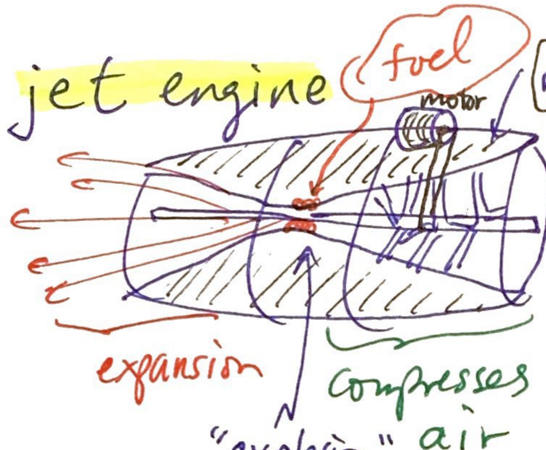
Propulsion

- propellors



- jet engine

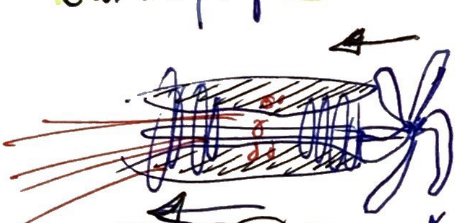
row after row of "propellors blades" insite a duct.



- loose ~~the motor~~
- add the turbine

use some of the energy of the expanding gas to spin the shaft.

- "turboprop"

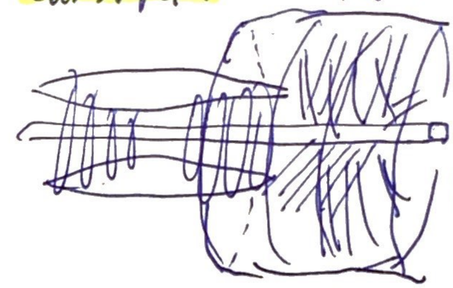


add an advanced high tech propellor

Thrust from jet

Thrust from the prop.

- turboFan



GeNX

most efficient comercial Jet Liners

- Ramjet (no moving parts)

"Testing GEJ79 with after-burne

- - Attached to military aircraft
  → Afterburners

- supersonic Scramjet

Thrust Forward

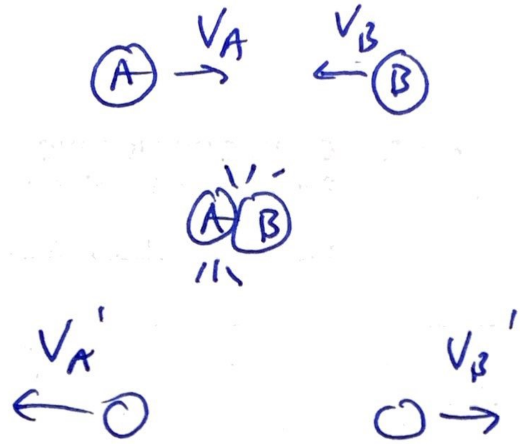
- ted x Kevin Bowcutt

- starship test launc

A Types of Collisions

Elastic Collision

- Before



- After:

- If elastic then

$$KE_{Before} = KE_{after}$$

{No absorbtion of heat)

- momentum before = momentum after

$$\text{I} \quad \boxed{m_A V_A + m_B V_B = m_A' V_A' + m_B' V_B'}$$

- KE

$$\text{II} \quad \boxed{\begin{aligned} &\tfrac{1}{2} m_A V_A^2 + \tfrac{1}{2} m_B V_B^2 \\ W=0 \quad &= \tfrac{1}{2} m_A' V_A'^2 + \tfrac{1}{2} m_B' V_B'^2 \end{aligned}}$$

In-elastic Collision

SAME

- If in-elastic then

$$KE_B = KE_{after} + \text{Work}$$

Heat loss

I $\boxed{\text{Same eqn}}$

- KE

$$\text{II} \quad \boxed{\begin{aligned} &\tfrac{1}{2} m_A V_A^2 + \tfrac{1}{2} m_B V_B^2 \\ &= \tfrac{1}{2} m_A' V_A'^2 + \tfrac{1}{2} m_B' V_B'^2 \\ &\quad + W_{friction\ loss} \end{aligned}}$$

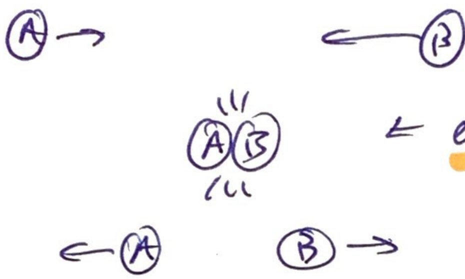
$\boxed{\text{Two equations I \& II and two unknowns } V_A', V_B'}$

- **100% Inelastic Collision** : objects stick together

← No energy lost

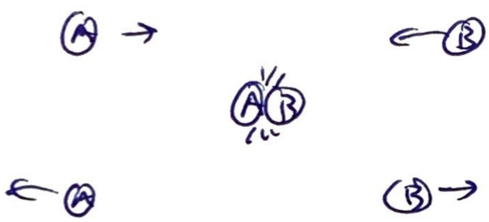
$V_A' = V_B' = V'$

- **Partial Inelastic Collision**



← energy lost due to internal means.

- Elastic (100%)



$$\begin{cases} m_A V_A + m_B V_B = m_A V_A' + m_B V_B' \\ \{ m_A = m_A' \ \& \ m_B = m_B' \} \\ \frac{1}{2} m_A V_A^2 + \frac{1}{2} m_B V_B^2 = \frac{1}{2} m_A V_A'^2 + \frac{1}{2} m_B V_B'^2 \end{cases}$$

we typically know V_A & V_B before and we seek V_A' & V_B' after collision

- Solve by substitution : rearrange mom : $m_A (V_A - V_A') = m_B (V_B - V_B')$

rearrange KE : $\cancel{\frac{1}{2}} m_A (V_A^2 - V_A'^2) = \cancel{\frac{1}{2}} m_B (V_B^2 - V_B'^2)$

factor $a^2 - b^2 = (a+b)(a-b)$

$$m_A (V_A - V_A')(V_A + V_A') = m_B (V_B - V_B')(V_B + V_B')$$

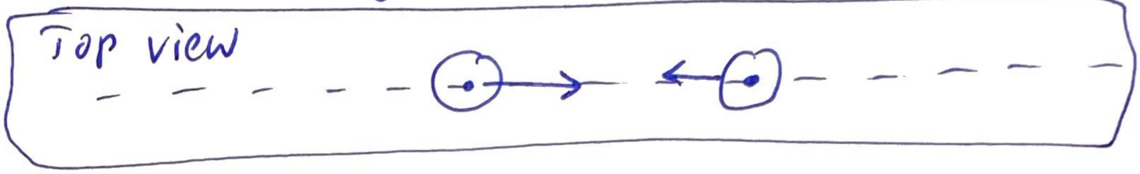
$\boxed{V_A + V_A' = V_B + V_B'}$ 100% elastic

-OR-

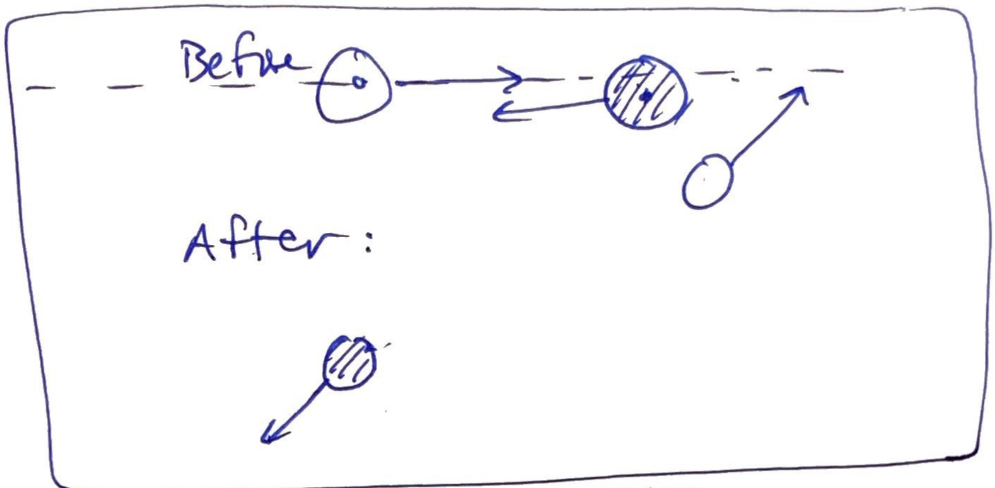
more usefull $\boxed{V_A - V_B = -(V_A' - V_B')}$ -OR- $\Rightarrow$ $\boxed{\Delta V_{Before} = -\Delta V_{After}}$ equal but opposite

Ex Billiards • ($m_A = m_B$) • assume 100% ellastic

• assume "straight-on" collision



1 Dim



2 Dim

• Before $v_A = 10 m/s$ $v_B = -2 m/s$ A B → (+)

• After v_A' v_B' → (+)

• math KE + mom: $$\boxed{v_A - v_B = -(v_A' - v_B')}$$ if 100% ellastic — eqn #1

mom $m_A = m_B$ so $\cancel{m}v_A + \cancel{m}v_B = \cancel{m}v_A' + \cancel{m}v_B'$

$$\Rightarrow \boxed{v_A + v_B = v_A' + v_B'}$$ eqn #2

• Apply numbers

TOP: $10\,m/s - (-2\,m/s) = -v_A' + v_B'$

BOT: $10\,m/s + (-2\,m/s) = v_A' + v_B'$

• Solve the 2X2:

$12 = -v_A' + v_B'$

$\Rightarrow \oplus\ 8 = v_A' + v_B'$

$20 = 2v_B'$ → $\boxed{v_B' = 10\,m/s}$

$8 = v_A' + 10\,m/s$ → $\boxed{v_A' = -2\,m/s}$

Ex Particle Physics

p

$\rightarrow (+)$

$1\,u = 1.66E{-}27$ kg

$\vec{V}_A = 3.6\times10^4$ m/s

$m_A = 1.01\,u$

He @ $V_B = 0$

$m_B = 4.00\,u$

Q: What are the velocities after collision if the interaction is elastic.

• **Equations**

momentum: $\{\, m_A V_A + 0 = m_A V_A' + m_B V_B'$

mom.+KE: $\{\, V_A - 0 = -V_A' + V_B'$

$V_A' = V_B' - V_A$ substitute

$\Rightarrow m_A V_A = m_A (V_B' - V_A) + m_B V_B'$

$\Rightarrow m_A V_A + m_A V_A = m_A V_B' + m_B V_B'$?

$2 m_A V_A = (m_A + m_B) V_B'$

$$\boxed{V_B' = \left(\frac{2m_A}{m_A + m_B}\right) V_A}$$

• **Populate**

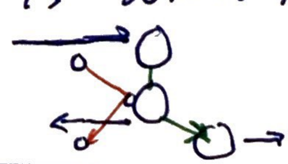
$V_B' = \left(\frac{2u}{u + 4u}\right) V_A = \frac{2}{5} V_A = \frac{2}{5}\left(36{,}000\frac{m}{s}\right)$

$\boxed{V_B' = 14{,}400 \text{ m/s to the right}}$

• **Back Substitute**:

$V_A' = V_B' - V_A = 14{,}400\,\text{m/s} - 36{,}000\,\text{m/s} = -21{,}600\,\text{m/s}$

$\boxed{V_A' = -21{,}600\,\text{m/s}}$

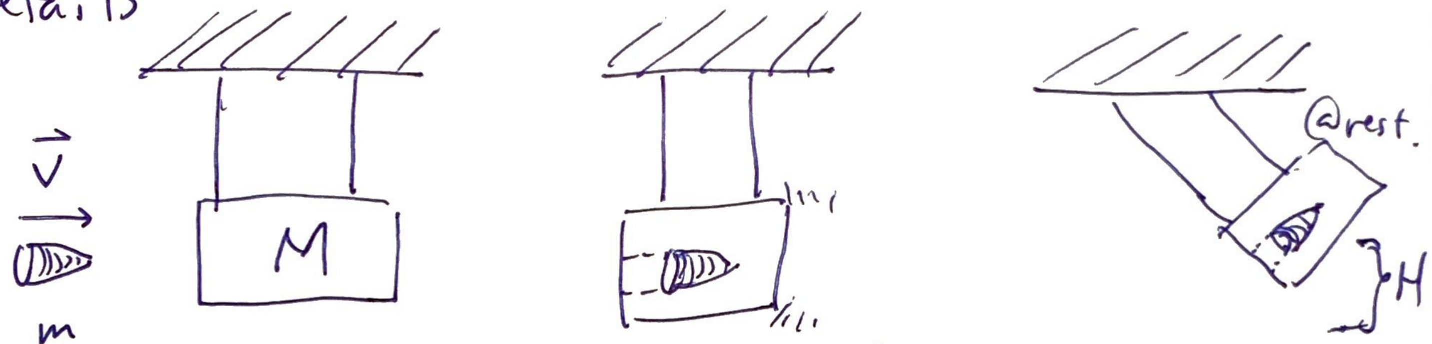


proton bounces off

Ex: 100% Inelastic collision – Ballistic Pendulum

Used to determine muzzle velocities of guns.

- Details



$V_M = 0$ | $V_{M+m} = V'$ |

- $KE = \frac{1}{2}(m+M)v'^2$
- $PE = 0$

- $KE = 0$
- $PE = (M+m)gH$

$\boxed{\text{I}}$: use momentum

$\boxed{\text{II}}$: use energy conservation

$$\text{I}: \quad mv + 0 = (m+M)v'$$

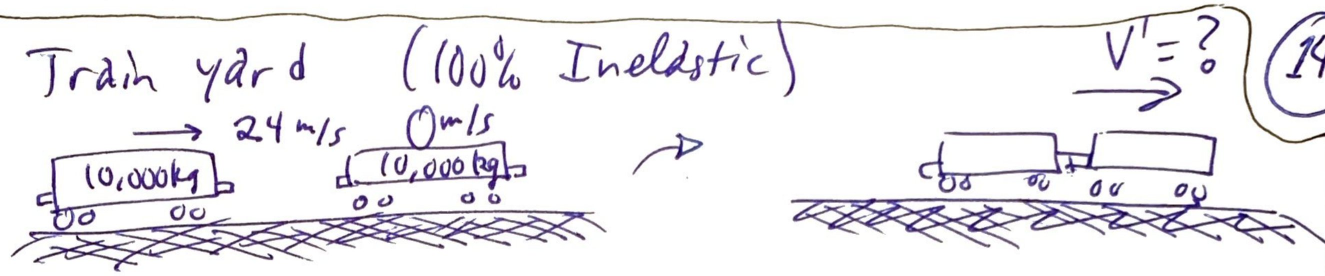
$$\text{II}: \quad \underbrace{\frac{1}{2}\cancel{(m+M)}v'^2}_{KE} = \underbrace{\cancel{(m+M)}gH}_{PE} \rightarrow v' = \sqrt{2gH}$$

$$\div (m+M)$$

Ballistic Equation

$$mv = (m+M)\sqrt{2gH} \Rightarrow \boxed{V_{Bullet} = \left(\frac{m+M}{m}\right)\sqrt{2gH}}$$

$\boxed{EX}$ Train yard (100% Inelastic)



Q: What is the KE lost to thermal Energy?

- **KE:** $\frac{1}{2}m_A V_A^2 = \frac{1}{2}(m_A + m_B)V'^2$
- **Mom:** $m_A V_A = m_A V' + m_B V' \longrightarrow V' = \frac{m_A}{(m_A+m_B)} V_A$
- let $m_A = m_B \rightarrow \underline{V' = \frac{1}{2} V_A} = \frac{24\,m/s}{2} = \underline{12\,m/s}$ combined final velocity
- **Heat loss:** $\Delta KE = Heat$

$$\frac{1}{2} m V_A^2 - \frac{1}{2}(2m)V'^2 = Heat$$

$$\frac{1}{2}[10{,}000\,kg]\left((24\,m/s)^2 - 2(\overset{\frac{1}{2}\cdot 24}{12\,m/s})^2\right) = Heat$$

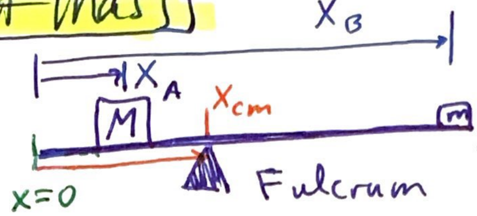
$$\underbrace{\frac{1}{2}\,10{,}000\,kg\,(24\,m/s)^2}_{KE_A}\underbrace{[1 - \tfrac{1}{2}]}_{\cdot\,\frac{1}{2}} = Heat$$

we loose $\frac{1}{2}$ of car A's KE to heat

OR → $\frac{2{,}880{,}000}{2}$ or $\boxed{1.44MJ}$ heat.

* Center of Mass

• 1-Dim

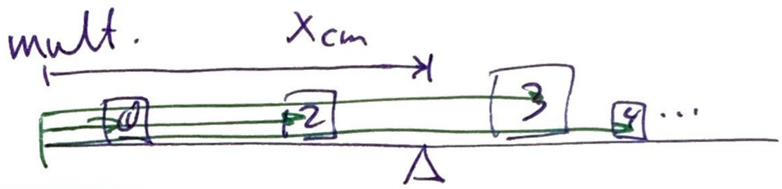


$$\sum md = 0$$

$$\boxed{X_{cm} = \frac{m_A X_A + m_B X_B}{m_A + m_B}}$$

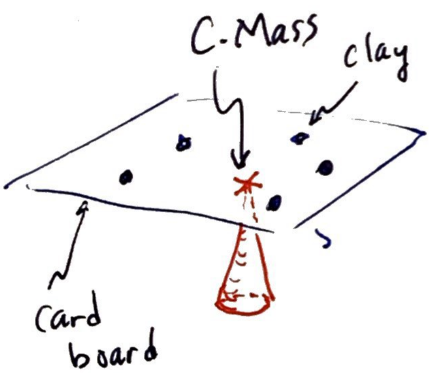
"leverage"

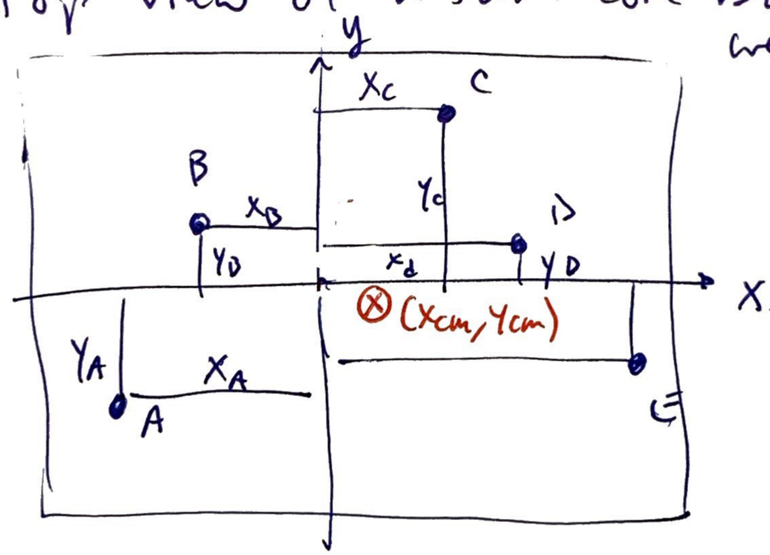
If many blocks...



$$\boxed{X_{cm} = \frac{m_A X_A + m_B X_B + \cdots m_N X_N}{m_A + m_B + \cdots + m_N}} = \frac{\int m \cdot x \cdot dx}{\int m\, dx}$$

• 2-Dim Top view of a foam-core board that has weights on it.

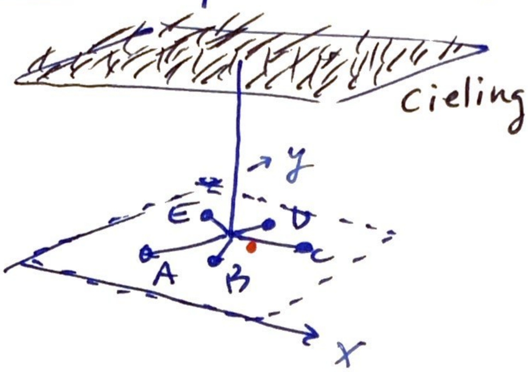




$$X_{cm} = \frac{m_A X_A + m_B X_B + \cdots + m_N X_N}{m_A + m_B + \cdots + m_N}$$

$$Y_{cm} = \frac{m_A Y_A + m_B Y_B + \cdots + m_N Y_N}{m_A + m_B + \cdots + m_N}$$

EX Find the (x_{CM}, y_{CM}) of a distribution of suspended lights in a modernistic chandalier



Light	mass	(x, y)
A :	1kg	(9cm, 3cm)
B :	0.75kg	(15cm, 7cm)
C :	1kg	(40cm, 30cm)
D :	1.3 kg	(10cm, 40cm)
E :	1kg	(4cm, 33cm)

centimeter

$$x_{CM} = \frac{(1\,\text{kg})(9\,\text{cm}) + (0.75)(15\,\text{cm}) + (1)(40\,\text{cm}) + (1.3)(10\,\text{cm}) + (1)(4\,\text{cm})}{1 + 0.75 + 1 + 1.3 + 1}$$

$$x_{CM} = \frac{9 + 11.25 + 40 + 13 + 4}{5.05\,\text{kg}} = \frac{77.25\,\text{kg}\cdot\text{cm}}{5.05\,\text{cm}} = \underline{\underline{15.3\,\text{cm}}}$$

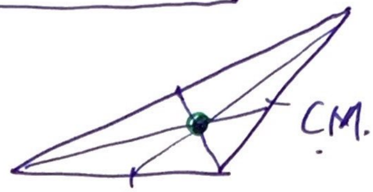
$$y_{CM} = \frac{(1)(3) + (0.75)(7) + (1)(30) + (1.3)(40) + (1)(33)}{5.05\,\text{kg}}$$

$$y_{CM} = \frac{3 + 5.25 + 30 + 52 + 33}{5.05} = \frac{123.25}{5.05} = \underline{\underline{24.4\,\text{cm}}}$$

CM is at $\boxed{(15.3\,\text{cm}, 24.4\,\text{cm})}$

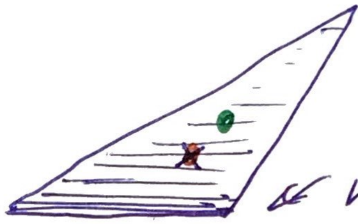
Ex of Centers : The Triangle

- geometrical center (center of geometry) centroid



uniform material in the shape of a triangle

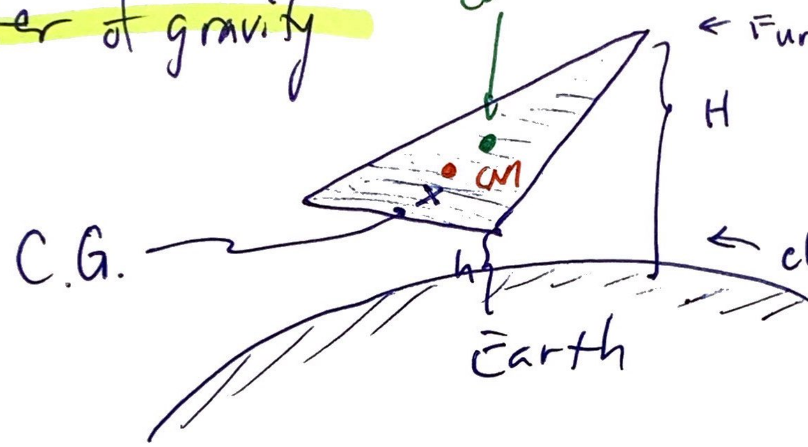
- center of mass



$$x = \frac{\iint x \cdot m \, dx\, dy}{\iint m}$$

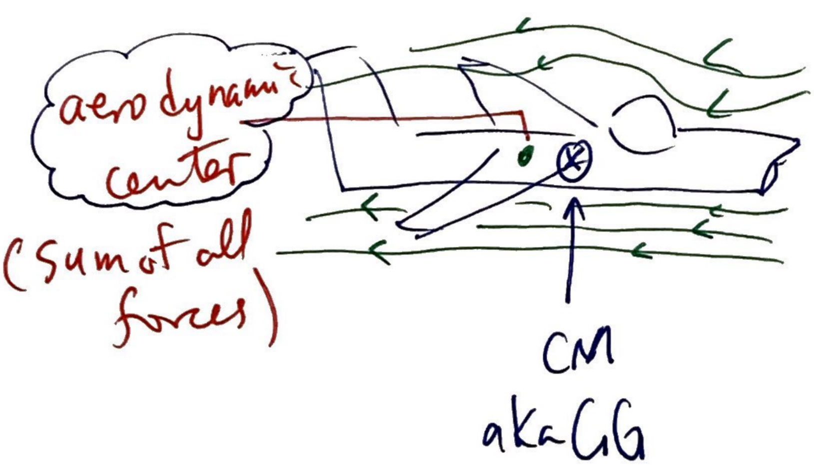
$$y = \frac{\iint y\, m \, dx\, dy}{\iint m}$$

- center of gravity



$h < H$

- center of aerodynamic forces



Google : Walter Lewin

"8.01x - Lect 15"

- go to 47 min

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