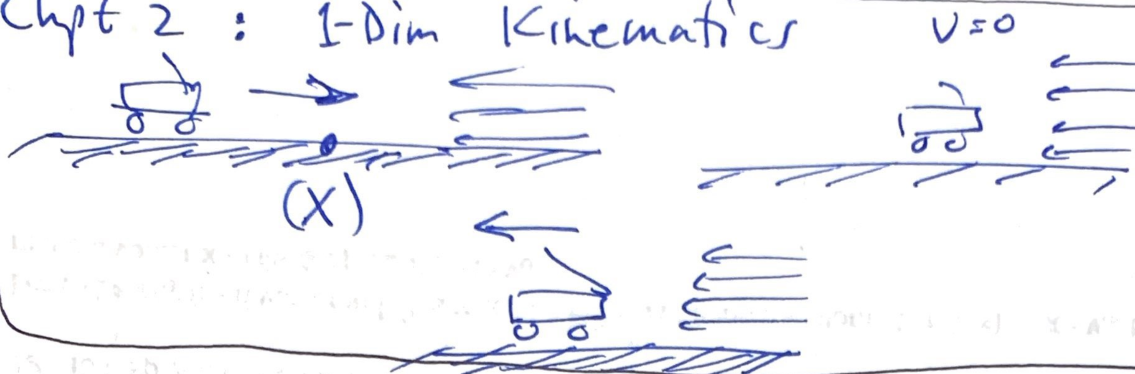
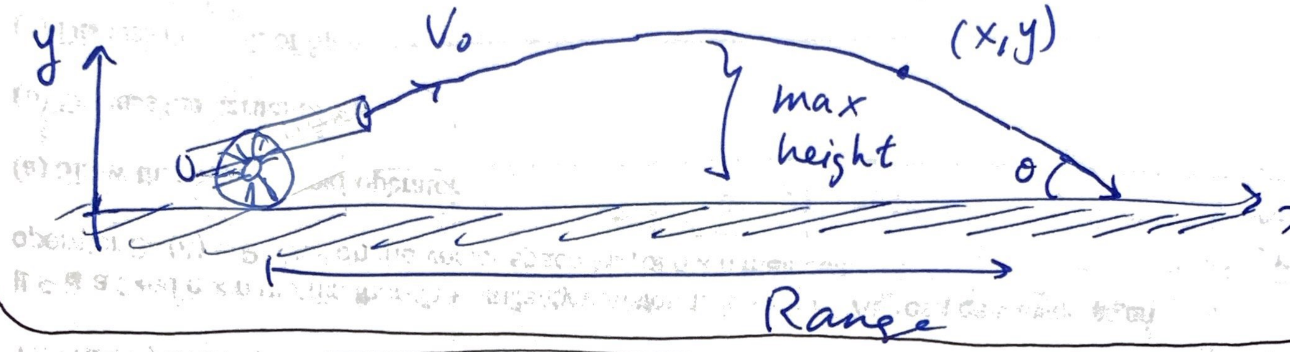
Chapter 3 A: vectors & B: 2-Dim Kinematics

3A

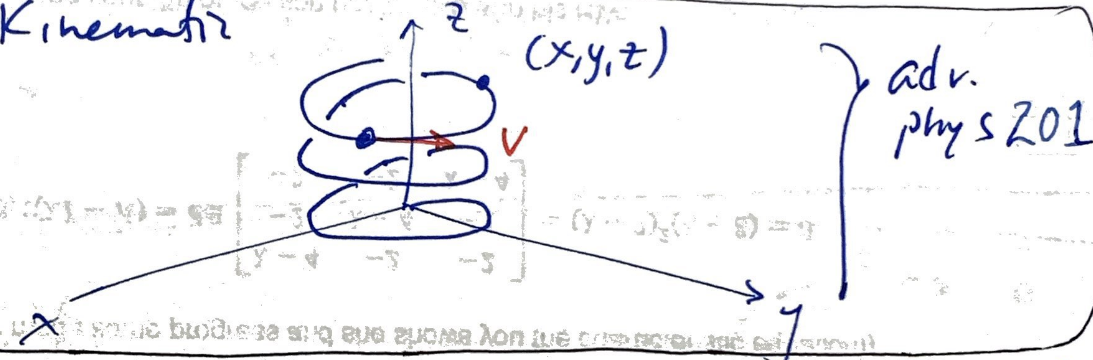
- Chpt 2 : 1-Dim Kinematics



- 2-Dim Kinematics



- 3-Dim Kinematics



EQNS 2-Dim Kinematics w/ a = const acc'ln

x-dir

$a = \text{constant}$

$V_x = V_{x_0} + a_x t$

$x = x_0 + V_0 t + \frac{1}{2} a_x t^2$

$V_f^2 = V_0^2 + 2a \|\vec{x} - \vec{x}_0\|$

y-dir

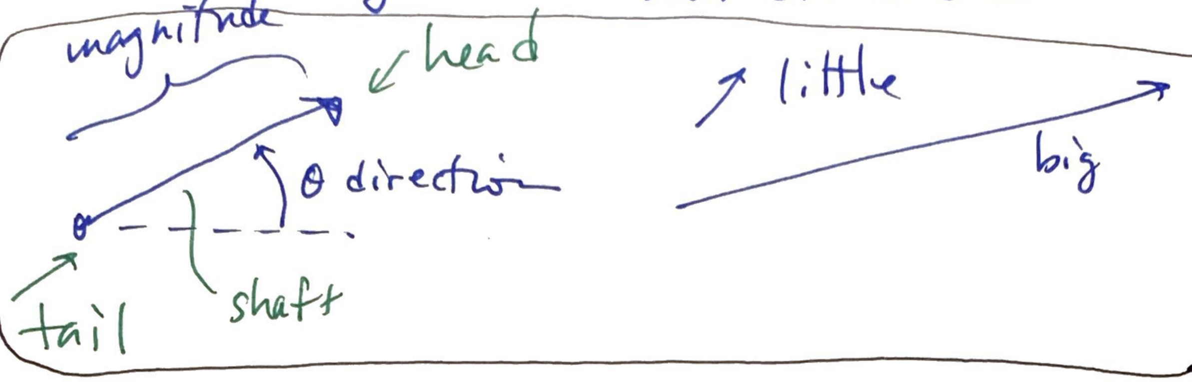
$a = -g$

$V_y = V_{y_0} + a_y t$

$Y = Y_0 + V_{y_0} t + \frac{1}{2} a_y t^2$

A. Vectors

A vector is a mathematical construct (object) that denotes magnitude and direction

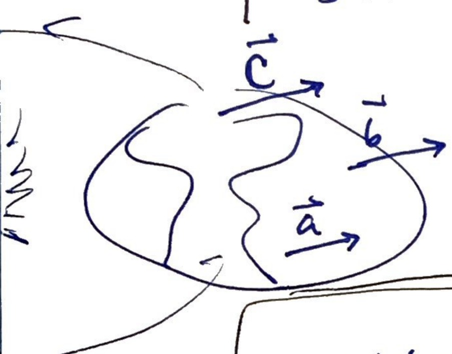


* properties

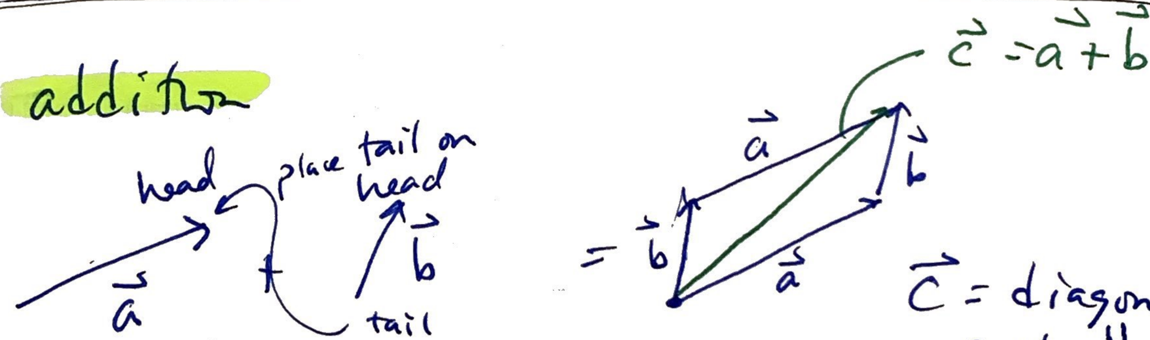
- Vectors are equal if they have the same magnitude and direction (points to the same far away star)

$$\vec{a} = \vec{b} = \vec{c}$$

* star



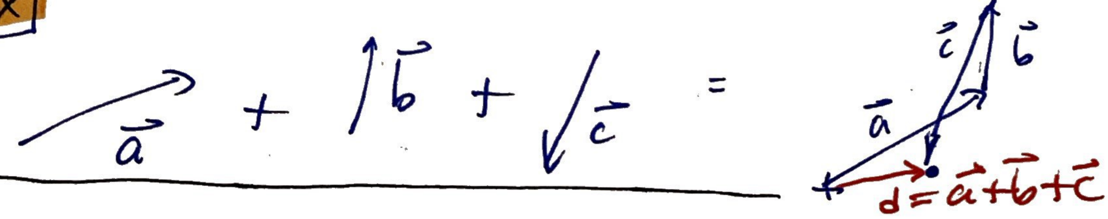
- addition



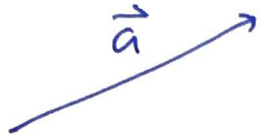
$\vec{c}$ = diagonal of the ||-ogram

- commute $\vec{a} + \vec{b} = \vec{b} + \vec{a}$

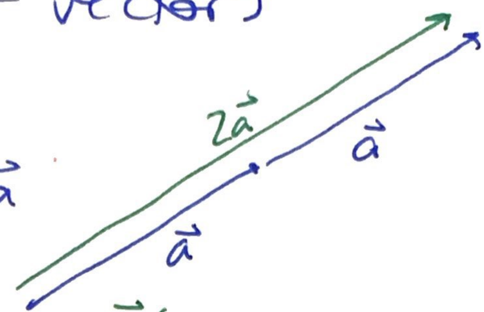
EX

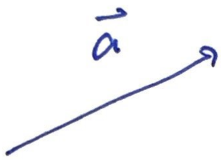


- Scalar multiples of vectors

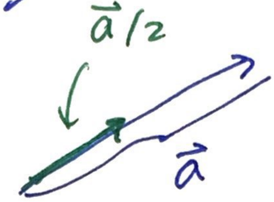


then $2\vec{a}$

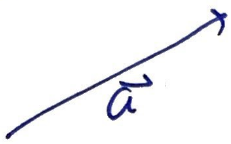




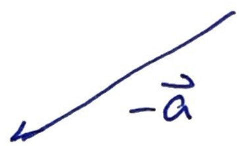
then $\frac{1}{2}\vec{a}$



- negate



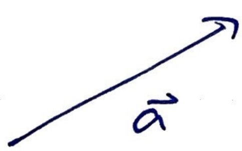
then $-\vec{a}$



Reverse direction

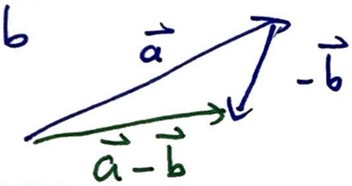
- Subtract vectors

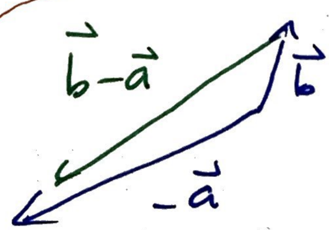
$\vec{a} - \vec{b}$ is really shorthand for $\vec{a} + (-\vec{b})$



+

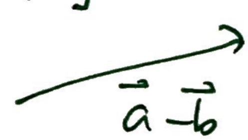
then $\vec{a} - \vec{b}$





this is $\vec{b} - \vec{a}$

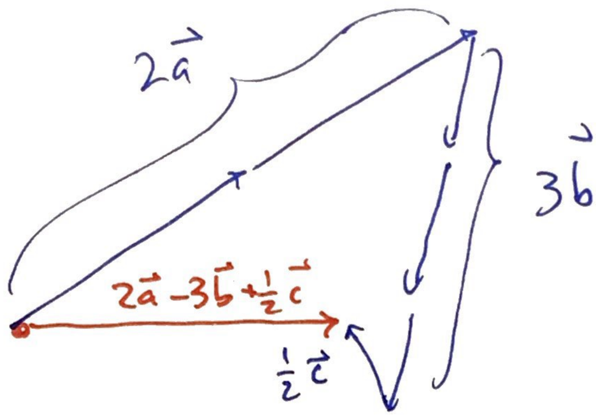
Same length



opposite direction

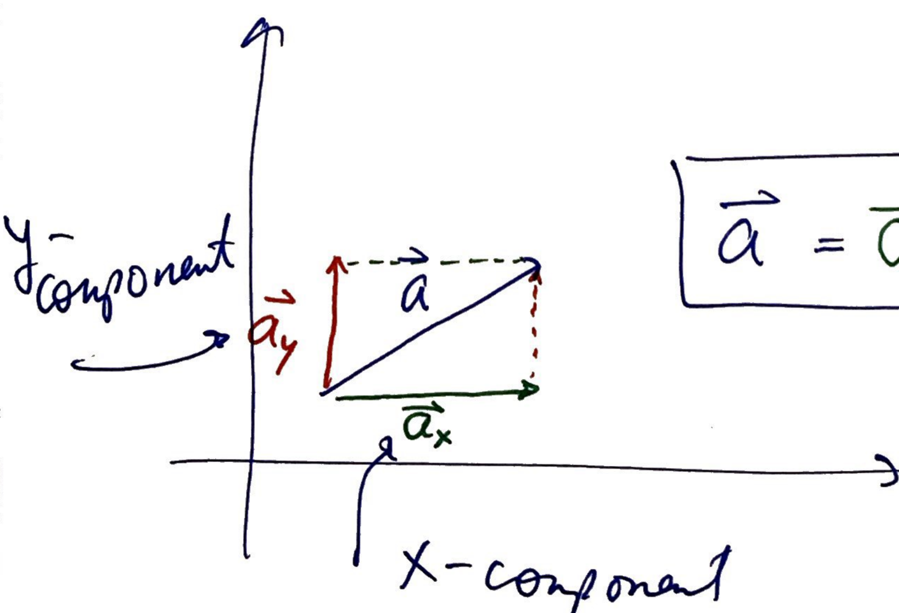
Ex graphically perform $2\vec{a}-3\vec{b}+\frac{1}{2}\vec{c}$ if

4



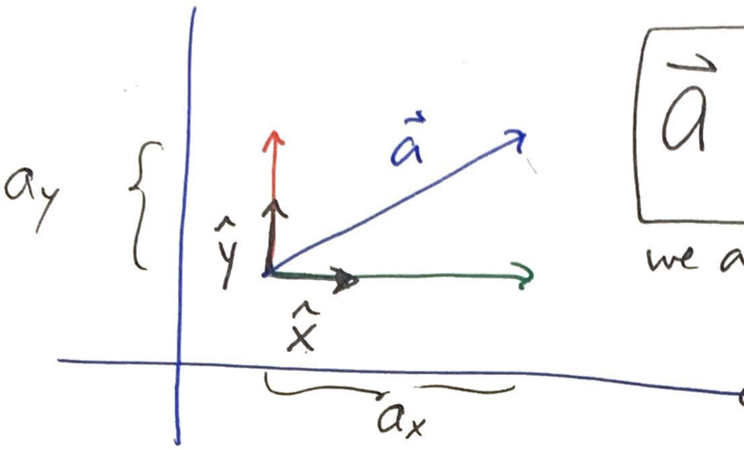
"Resultant vector"

* components of a vector: we can decompose a vector into two parts: one $\parallel$ to x-axis and one $\perp$ to x-axis (i.e., $\parallel$ to y axis)



$$\boxed{\vec{a}=\vec{a}_x+\vec{a}_y}$$

• $\hat{\imath}$ & $\hat{\jmath}$



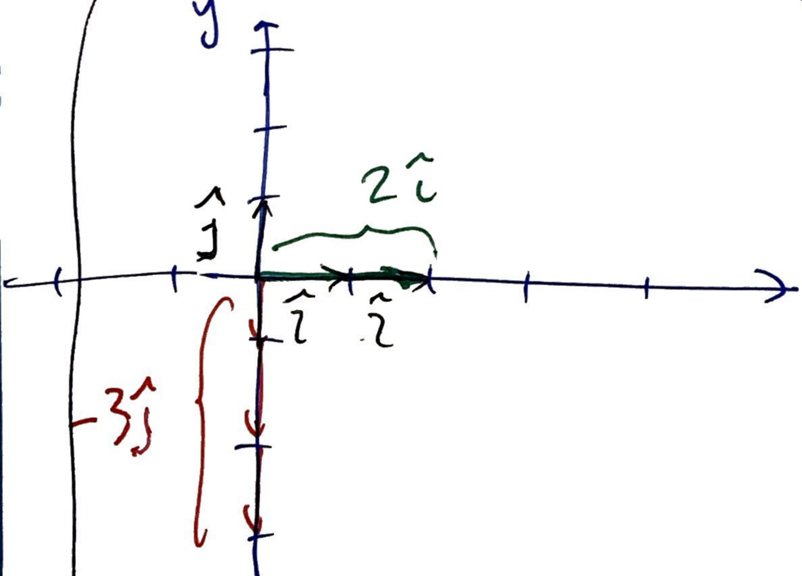
$$\boxed{\vec{a} = a_x\hat{x} + a_y\hat{y}}$$

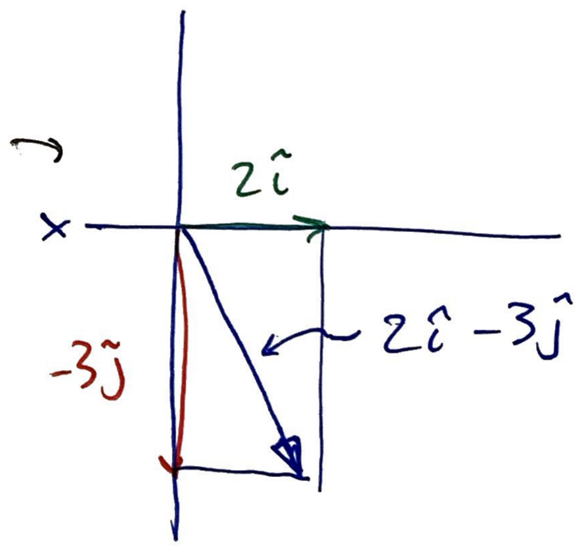
we also use
$\hat{\imath}$ for $\hat{x}$
and $\hat{\jmath}$ for $\hat{y}$

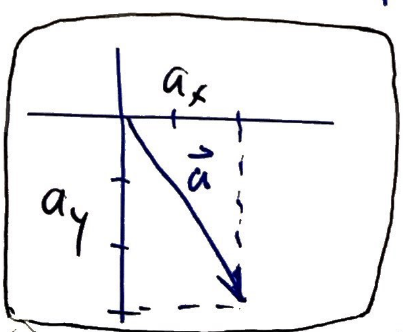
$\hat{x}$ has a length of 1
$\hat{y}$ has a length of 1

Then $\vec{a}_x = \overbrace{\|\vec{a}_x\|}^{a_x}\hat{x}$, $\vec{a}_y = \overbrace{\|\vec{a}_y\|}^{a_y}\hat{y}$

Ex graph $\vec{a} = 2\hat{\imath} - 3\hat{\jmath}$







- bracket notation

$$\vec{a} = 2\hat{i} - 3\hat{j} = \langle 2, -3 \rangle$$

x-component

y-component

- vector addition with bracket notation

$$\vec{a} = \langle 2, -3 \rangle$$

$$\vec{b} = \langle 1, 2 \rangle$$

$$\vec{c} = \langle -1, 3 \rangle$$

then

$$\boxed{2\vec{a} - 3\vec{b} + \tfrac{1}{2}\vec{c}}$$

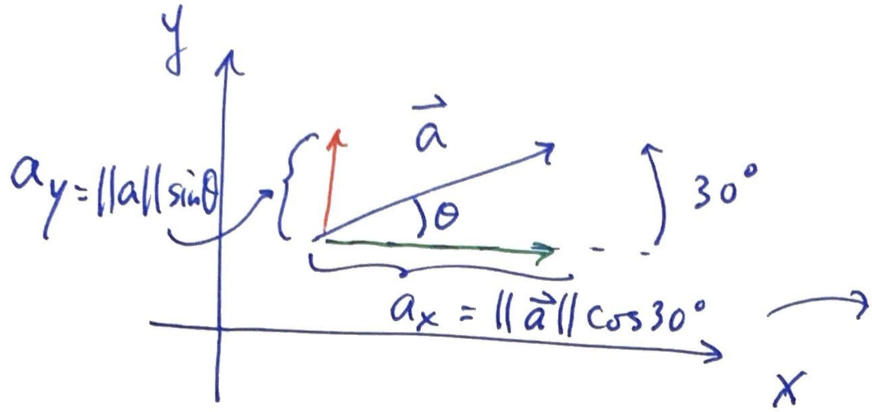
$$= 2\langle 2, -3 \rangle - 3\langle 1, 2 \rangle + \tfrac{1}{2}\langle -1, 3 \rangle$$

$$= \langle 4, -6 \rangle + \langle -3, -6 \rangle + \langle -\tfrac{1}{2}, \tfrac{3}{2} \rangle$$

$$= \langle 4 - 3 - \tfrac{1}{2}, -6 - 6 + \tfrac{3}{2} \rangle$$

$$= \boxed{\langle \tfrac{1}{2}, -\tfrac{21}{2} \rangle}$$

• vector decomposition via trig



$a = \|\vec{a}\|$ → length of

$$\boxed{a_x = a\cos\theta}$$

$$\boxed{a_y = a\sin\theta}$$

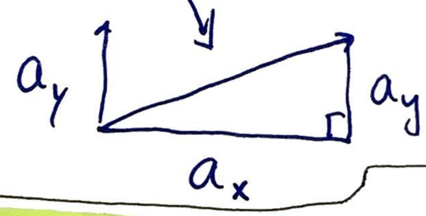
$$\vec{a} = \langle a\cos\theta, a\sin\theta \rangle$$

or

$$\vec{a} = a\cos\theta\,\hat{i} + a\sin\theta\,\hat{j}$$

• Calculating length of a vector: given $\vec{a} = \langle a_x, a_y \rangle$

$\|\vec{a}\|$

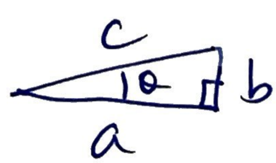


$$\boxed{\|\vec{a}\| = \sqrt{a_x^2 + a_y^2}}$$

• angle of a vector

$$\boxed{\theta = \tan^{-1}\left(\frac{a_y}{a_x}\right)}$$

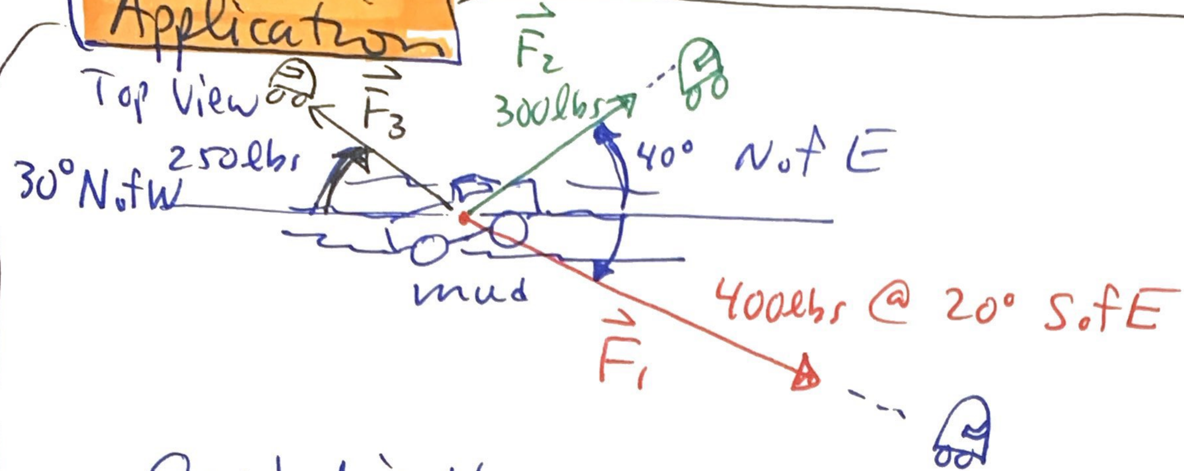
since



$$c = \sqrt{a^2 + b^2}$$

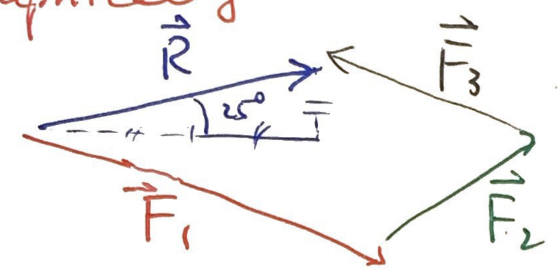
$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{b}{a}$$

Application



Q: what is the resultant Force on the truck?

graphically



$$\vec{R} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$$

Component addition (Tables)

	vector	x-component	y-component
F_1	400 20° S of E	$400 \cos 20° = 375.8770$	$-400 \sin 20° = -136.8080$
F_2	300 40° N of E	$300 \cos 40° = 229.8133$	$300 \sin 40° = 192.8363$
F_3	250 30° N of W	$-250 \cos 30° = -216.5064$	$250 \sin 30° = 125.0000$
		389.1839	181.0280

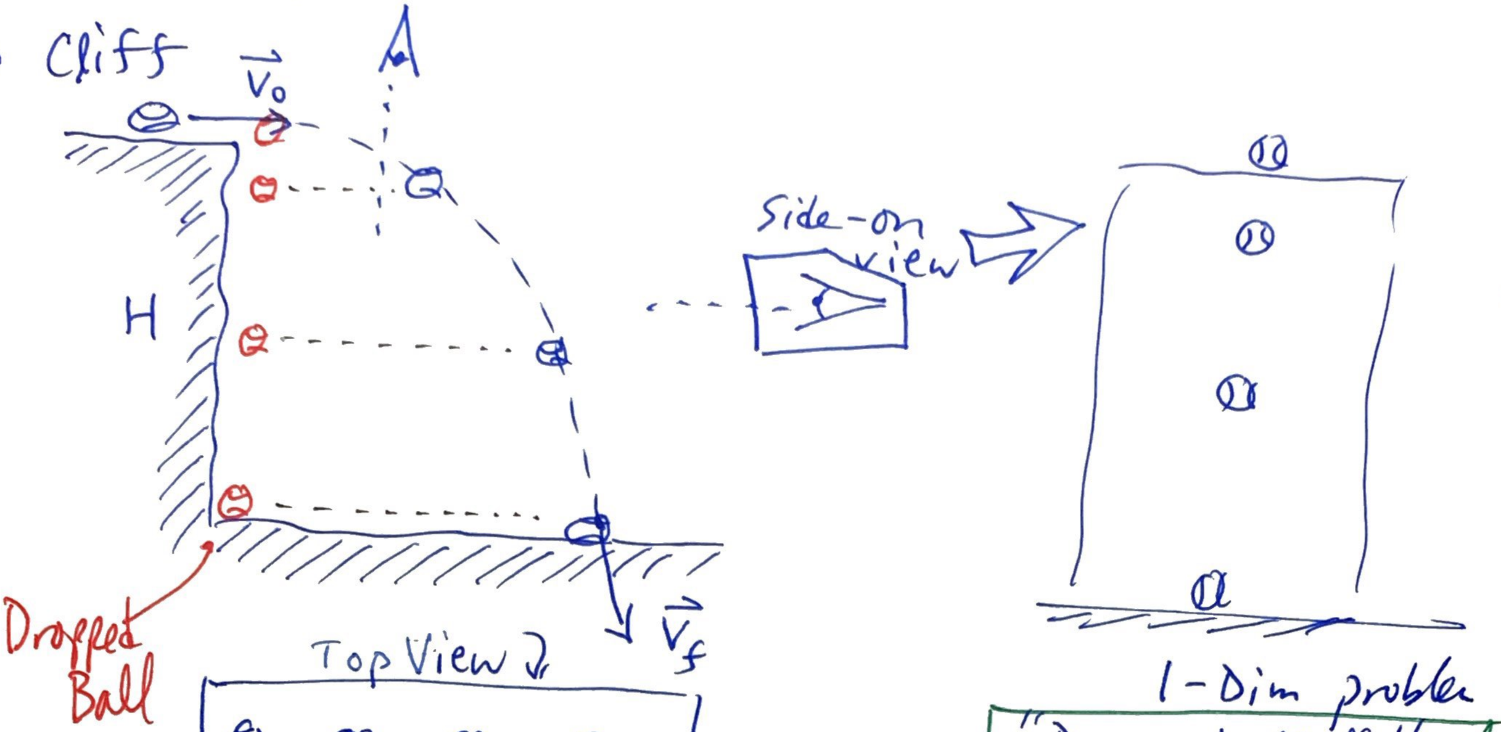
points west

Ans: $\boxed{\text{Resultant vector} = \langle 389.184, 181.028 \rangle}$

- magnitude of net force $= \sqrt{389.184^2 + 181.028^2} = \underline{\underline{429.23}}$ lbs
- Direction $\theta = \tan^{-1}\left(\dfrac{181.028}{389.184}\right) = \tan^{-1}(0.465) = \boxed{24.9°}$ N of E.

A Common problems in 2-D is the projectile prob.

• Cliff

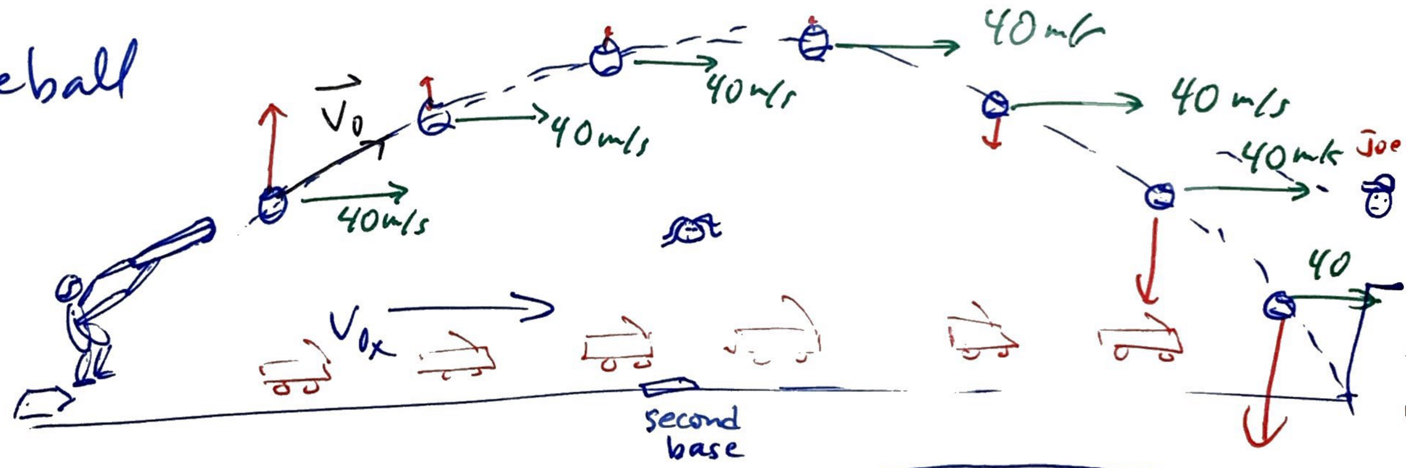


uniform spacing
No Accl'n

1-Dim problem

"Drop a ball off the cliff."

• Baseball



horizontally $V_x = 40$ m/s $\boxed{x = V_x t + x_0}$

vertically

$$\boxed{\begin{aligned} V_y &= V_{0y} - gt \\ y &= y_0 + V_{0y}t - \tfrac{1}{2}gt^2 \end{aligned}}$$

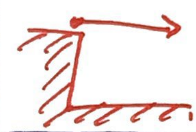
sees

General 2-D Kinematic Eqns

Horizontal (x)	vertical (y)
$V_x = V_{x_0} + a_x t$	$V_y = V_{y_0} + a_y t$
$x = x_0 + V_{x_0} t + \frac{1}{2} a_x t^2$	$y = y_0 + V_{y_0} t + \frac{1}{2} a_y t^2$
$V_x^2 = V_{x_0}^2 + 2a_x(x - x_0)$	$V_y^2 = V_{y_0}^2 + 2a_y(y - y_0)$

constant acc'ln only

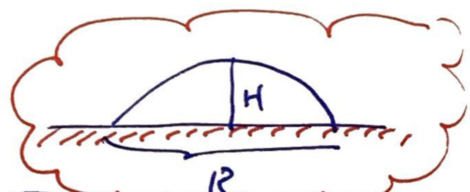
- Range Formula (Horizontal departure) $R = V_{x_0}\sqrt{\frac{2H}{g}}$



Projectile Motion

- Range Formula in general, starting at the ground and ending at the ground

$$R = \frac{V_0^2 \sin(2\theta)}{g}$$

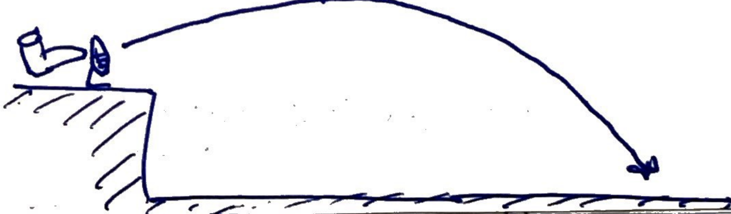


- max height: $H = \frac{V_{0y}^2}{2g}$

- Time of flight $t = 2\left(\frac{V_0 \sin\theta}{g}\right)$

Warning: does not apply to non-level applications

Ex:

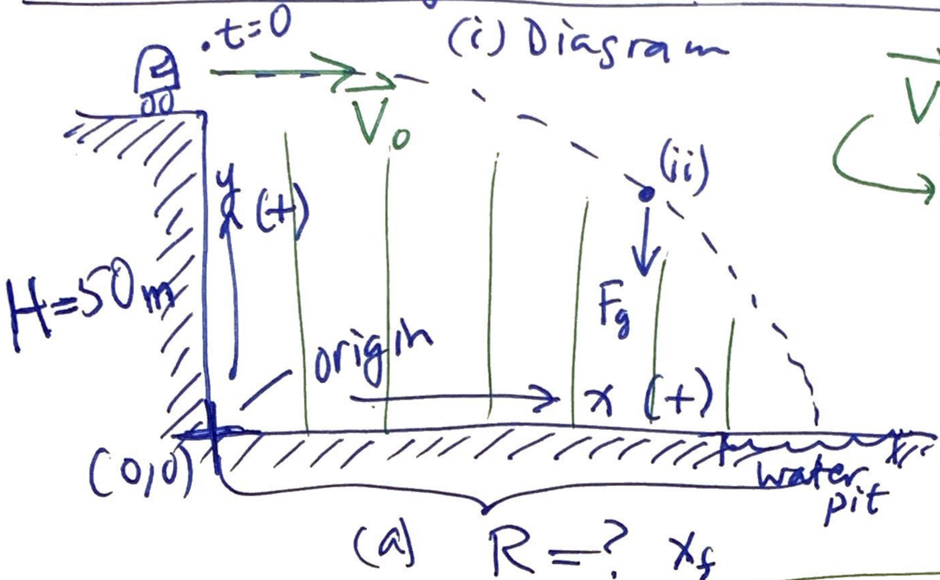


must use top eqns in the Boxes

Ex A stunt driver drives a car off a cliff. Her speed is 30 m/s after crossing the threshhold. The cliff is 50 m High ③

(a) Where does she land

(b) How long is she in the air?



$\vec{V}_0 = 30\,m/s\,\hat{i} + 0\,m/s\,\hat{j}$

$V_{x_0} = 30\frac{m}{s}$, $V_{y_0} = 0\,m/s$

(b) $t = ?$

Data

$a_x = 0$	$a_y = -g$	$g = 9.8\,m/s/s = 9.8\,m/s^2$
$V_{x_0} = 30\frac{m}{s}$	$V_{y_0} = 0\,m/s$	$t_0 = 0$
$x_0 = 0\,m$	$y_0 = 50\,m$	

Seek x_f @ time of impact...

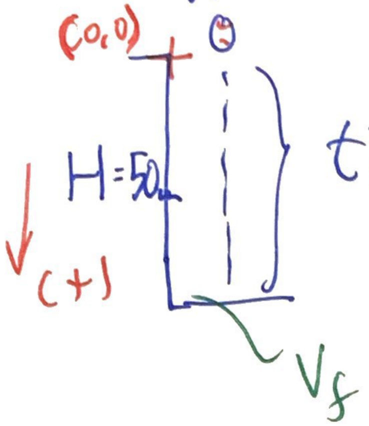
(iii) Formula choice:

$$x = \cancel{x_0}^{\,0} + \cancel{V_{x_0}}^{\,30\,m/s} t + \frac{1}{2}\cancel{a_x}^{\,0} t^2 \rightarrow x = vt$$

$$\boxed{x = (30\,m/s)\,t}$$

Q: How do we get "t"? { part b }

Ans: to get "t" consider an object dropped straight down from $H = 50m$



1-D Kinematics

$$V_y = \cancel{V_{y_0}}^{\,0} + \cancel{a_y}^{\,g} t$$

$$V_f = g t_f \rightarrow \boxed{t_f = \frac{V_f}{g}}$$

wait ... we do not know the final speed V_f

- Back to the drawing board ...
- Next eqn ...

Try $\cancel{y_f}^{\,H} = \cancel{y_0}^{\,0} + \cancel{V_{y_0}}^{\,0} t + \frac{1}{2}\cancel{a_y}^{\,g} t^2$

$$\boxed{H = \frac{1}{2} g t^2} \rightarrow t = \sqrt{\frac{2H}{g}}$$

{ Range formula: $\boxed{R = V_{0_x}\sqrt{\frac{2H}{g}}}$ } Horiz

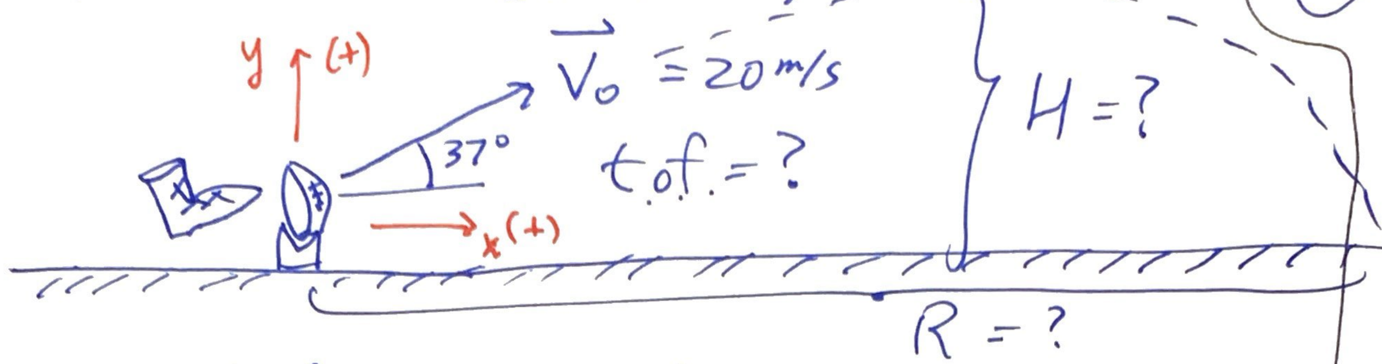
(b) Time of flight

$$t = \sqrt{\frac{2(50m)}{9.8m/s^2}} = \sqrt{\frac{100}{9.8}}$$

$\boxed{t.o.f. = 3.19\ sec}$

(a) $x_f = (30\frac{m}{s})(3.19s)$ → $\boxed{R = 95.7m}$

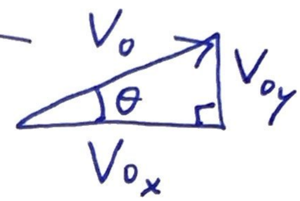
Ex Place Kicker (or Canon)



(a) Time to Reach max height?

Know:

$$\boxed{\begin{aligned} V_{x_0} &= V_0 \cos\theta \\ V_{y_0} &= V_0 \sin\theta \end{aligned}}$$



- $V_{x_0} = 20\,m/s \cos 37° = \underline{\underline{16.0\,m/s}}$ horiz.
- $V_{y_0} = 20\,m/s \sin 37° = \underline{\underline{12.0\,m/s}}$ vert.

we want H? ((vertical problem for now))

Formulas:

$$y = y_0 + V_{y_0} t + \frac{1}{2} a_y t^2$$

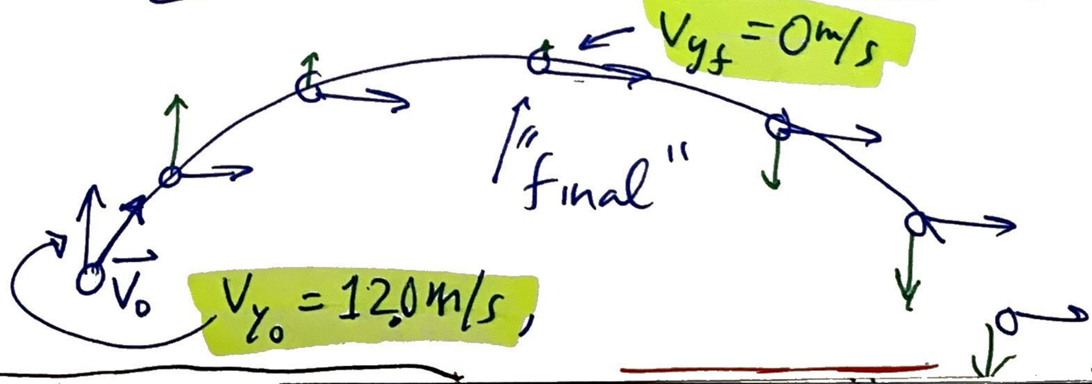
we need "t" ← Quadratic Formula

So use

$$\boxed{V_{y_f}^2 = V_{y_0}^2 + 2a_y \overbrace{(y_f - y_0)}^{H}}$$

f = final

Top of Arc...



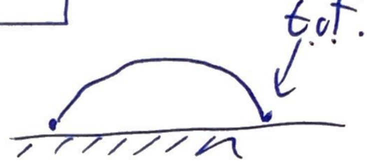
$$V_{y_f}^2 = V_{y_0}^2 + 2a_y(y_f - y_0)$$

$$(0\,\text{m/s})^2 = (12.0\,\text{m/s})^2 + 2(-9.8\,\text{m/s}^2)(H - 0)$$

Solve for $\Longrightarrow$ $H = \dfrac{-(12.0\,\text{m/s})^2}{-2(9.8\,\text{m/s}^2)} = \underline{\underline{7.347\,\text{m}}}$

$$\boxed{\text{max height is } 7.35\,\text{m}}$$

(b) <u>Find time of flight</u>



so now we can use

$$y = y_0 + V_{y_0}t + \tfrac{1}{2}a_y t^2$$

-OR-

$$V_y = V_{y_0} + a_y t$$ ← easier

$$V_{y_f} = V_{y_0} - g\,t$$

$$0 = 12.0\,\text{m/s} - 9.8\,\text{m/s}^2 \cdot t$$

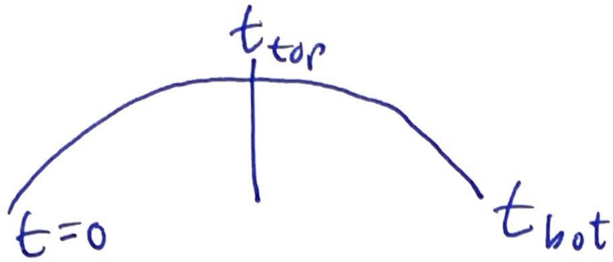
$$t = \frac{12.0\,\text{m/s}}{9.8\,\text{m/s}^2}$$

$$\frac{1}{\frac{1}{s}} = s$$

$$\boxed{t = 1.22\,\text{sec}}$$ time to top of arc

(b) cont.

Total Time of flight?



$$\boxed{t_{bot} = 2 \cdot t_{top}}$$

$$t \text{ of flight} = 2(1.22\,s)$$

$$\boxed{t = 2.44\,s}$$

$$x_s = \cancelto{0}{x_0} + \cancelto{16.0\,m/s}{v_{0x}}\,t + \tfrac{1}{2}\cancelto{0}{a_x}\,t^2$$

$$\boxed{D = vt}$$

(c) <u>Range</u>

$$R = V_{0x} \cdot t$$

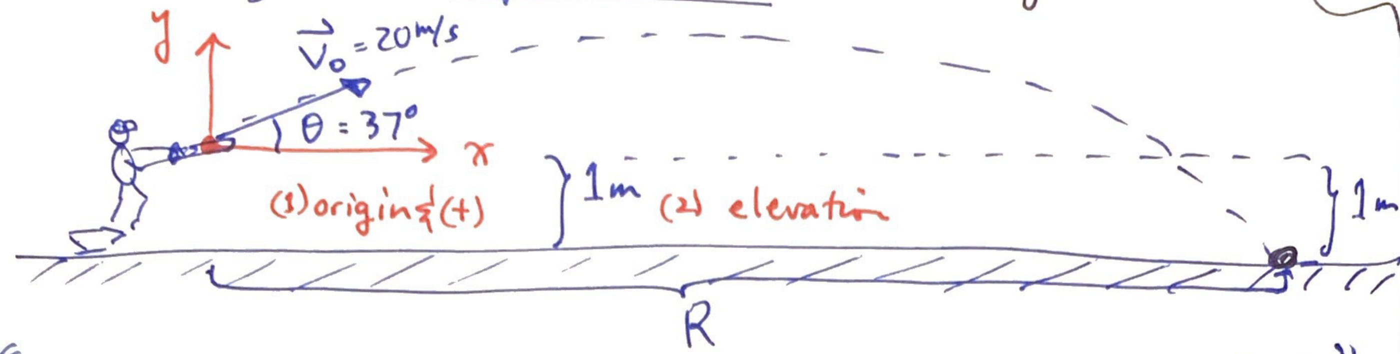
$$= (16.0\,m/s)(2.44\,s)$$

$$\boxed{R = 39.0\,m} \approx \tfrac{1}{3} \text{ soccer field.}$$

$$\approx 100 \text{ feet}$$

EX Starting above the ground : Find the range R

(i) diag.



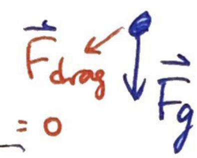
((place origin at point of where ball leaves the bat.))

let $x_0 = 0$ $\quad x_f = R$ want $\quad V_{0x} = 20\cos 37° = 12.0$ m/s

$y_0 = 0$ $\quad y_f = -1$ m $\quad V_{0y} = 20\sin 37° = 16.0$ m/s

want R

(ii) Free Body diagram



(iii) Formula :

$$\boxed{x_f = V_{0x} t}$$

but we need "t" 1st: $\quad$ opposite directions of grav. & y-axis

$$y_f = y_0 + V_{y_0} t - \frac{1}{2} g t^2$$

(iv) Populate and solve :

$$-1\text{m} = 0 + 12\text{ m/s} \cdot t - \frac{1}{2} 9.8 \cdot t^2$$

· put in Quadratic eqn form: (*2)

$$0 = 2 + 24t - 9.8t^2 \quad *(-1)$$

$$\rightarrow 9.8t^2 + 24t + 2 = 0 \quad \div 2$$

$$\boxed{4.9t^2 - 12t - 1 = 0}$$

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t = \frac{-(12) \pm \sqrt{12^2 - 4(4.9)(-1)}}{2(4.9)}$$

T.O.F. $\boxed{t = 2.53\text{s}}$ $\quad \boxed{-0.081\text{s}}$ ignore (−)

$$R = V_{0x} \cdot t$$

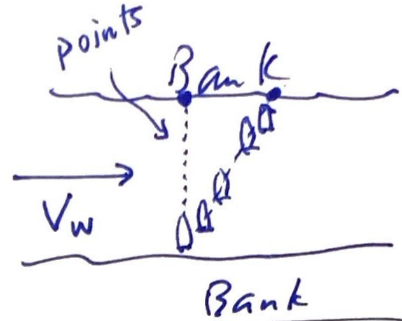
$$= (16\text{ m/s})(2.53\text{ s})$$

$$\boxed{R = 40.48\text{m}}$$

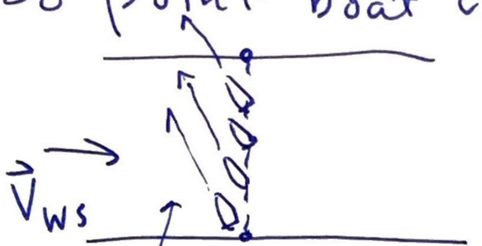
Relative Velocity

Scenario of a boat crossing a river

If you point boat @ shore then you are pushed down stream.



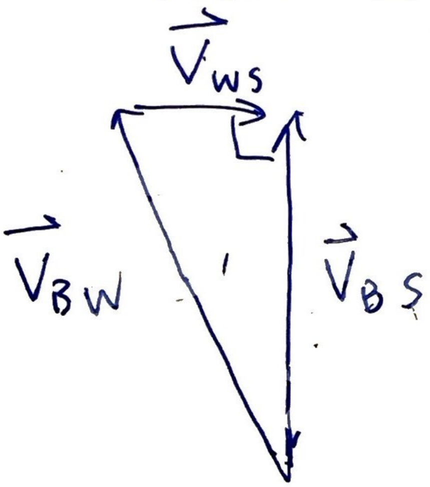
So point boat upstream:



a component of the vector counteracts the stream...

To correct for the water point the boat upstream.

- Vector Layout



$\vec{V}_{BW}$ = velocity of boat w.r.t. water

$\vec{V}_{BS}$ = Vel of Boat w.r.t. Shore

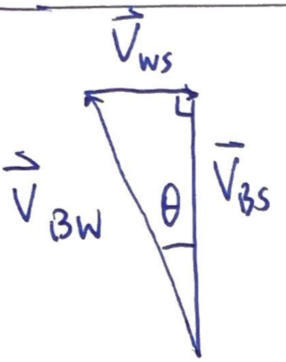
$\vec{V}_{WS}$ = Vel of water w.r.t. shore

$$\boxed{\vec{V}_{BW} = \vec{V}_{BS} + \vec{V}_{WS}}$$

Example

The navigator of a large ferry needs to point up stream in order to cross a large river and end up at the destination

Q_1: If $V_{BW} = 1.85\,m/s$ and $V_{WS} = 1.20\,m/s$ what direction does the navigator point?

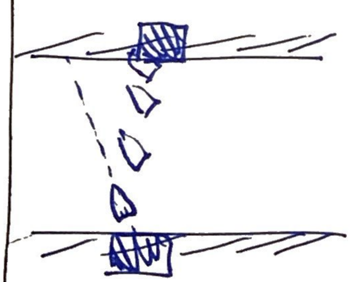


Trig $\boxed{\sin\theta = \frac{opp}{hyp}}$

$$\sin\theta = \frac{\|\vec{V}_{WS}\|}{\|\vec{V}_{BW}\|}$$

$$\theta = \sin^{-1}\left(\frac{1.20\,m/s}{1.85\,m/s}\right)$$

$$\boxed{\theta = 40.44°F}$$ pointing upstream



Q_2: what is the speed of the boat w/ shore?
{Radar gun speed from port of departure}

$$\|\vec{V}_{BW}\|^2 = \|\vec{V}_{BS}\|^2 + \|\vec{V}_{WS}\|^2$$

$$1.85^2 = V_{BS}^2 + 1.20^2$$

$$V_{BS} = \sqrt{1.85^2 - 1.20^2} = 1.41\,m/s$$

Q_3: How long to cross if dist from shore to shore is 1 km?

$$t = \frac{x}{v} = \frac{1000\,m}{1.41\,m/s}$$

$$\boxed{t = 710\ sec}$$ 12 min