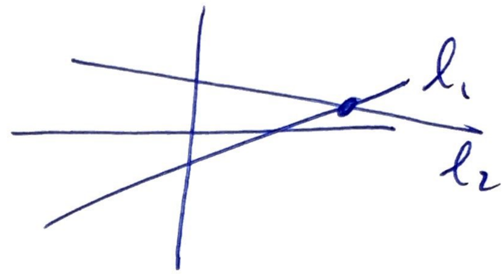
9.3 Non-Linear Systems

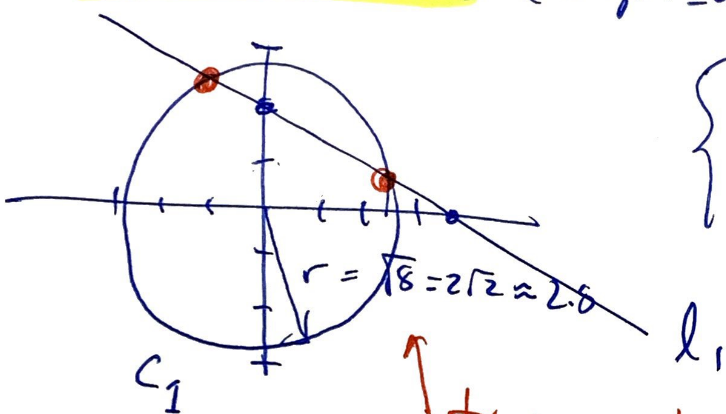
Notation

- **Linear**: x & y are power 1.
- **System**: $\begin{cases} 3x - 4y = 5 \\ -x + 2y = 11 \end{cases}$



- **2-Dim**: (x, y)

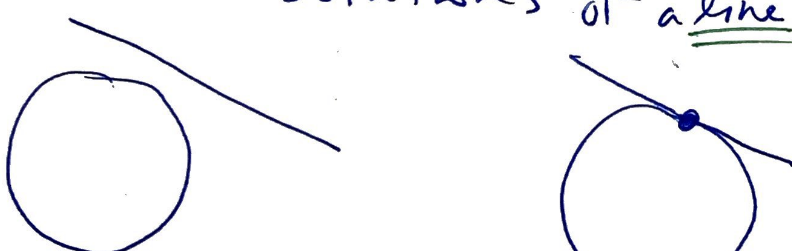
- **Non-Linear**: x^2 or x^3 or y^4, or $\sqrt{x}$ or $\sin(x)$
- Basic Conic Sections (chpt 10): circles, ellipses, hyperbola, parabolas,



$$\begin{cases} x^2 + y^2 = 8 & : C_1 \\ x + 2y = 4 & : l_1 \end{cases}$$

two solutions

other potential solutions of a line and circle



No-solutions

one-soln

Substitute: solve one eqn for one var and inset into the other eqn:

Ex

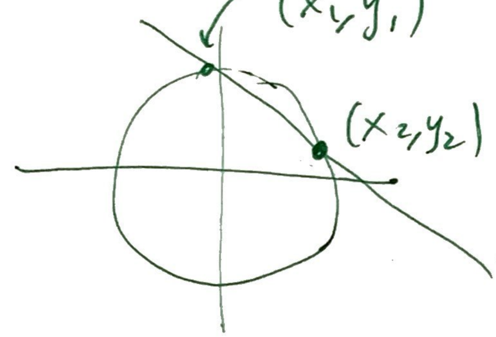
$$c_1: \quad x^2 + y^2 = 8$$
$$l_1: \quad x + 2y = 4$$

- **Solve** l_1 for x: $\quad x = 4 - 2y$ (chosen Eqn)
- **Sub** into c_1: $\quad (4-2y)^2 + y^2 = 8$
- Solve: $\quad 16 - 16y + 4y^2 + y^2 = 8$

$$\boxed{5y^2 - 16y + 8 = 0}$$

- factorable? $\quad 5, 1 \qquad \cancel{4, 2} \quad \cancel{8, 1} \qquad$ No!

$\Rightarrow$ use **Q. Formula**: $\quad y = \dfrac{-(-16) \pm \sqrt{(-16)^2 - 4(5)(8)}}{2 \cdot 5}$



$$= \frac{16 \pm \sqrt{256 - 160}}{10}$$

$$= \frac{16 \pm \sqrt{96}}{10} = \frac{16 \pm 4\sqrt{6}}{10} = \boxed{\begin{matrix} \frac{16}{10} - \frac{2\sqrt{6}}{5} \\ \frac{16}{10} + \frac{2\sqrt{6}}{5} \end{matrix}}$$

$$\Rightarrow \boxed{y_{1,2} = \frac{8 - 2\sqrt{6}}{5}, \ \frac{8 + 2\sqrt{6}}{5}}$$

- **Back Substitute** into the chosen eqn above:

$$(x_1, y_1) = \left(4 - 2\left(\frac{8 - 2\sqrt{6}}{5}\right), \ \frac{8 - 2\sqrt{6}}{5}\right) = \left(\frac{20 - 16 - 4\sqrt{6}}{5}, \ \frac{8 - 2\sqrt{6}}{5}\right)$$

$$\boxed{(x_1, y_1) = \left(\frac{4 - 4\sqrt{6}}{5}, \ \frac{8 - 2\sqrt{6}}{5}\right)} \ \& \ \boxed{(x_2, y_2) = \left(\frac{4 + 4\sqrt{6}}{5}, \ \frac{8 + 2\sqrt{6}}{5}\right)}$$

Elimination

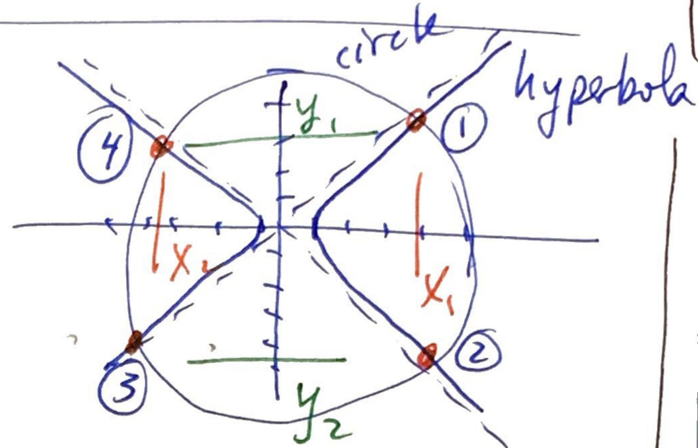
Ex

$$x^2+y^2=25 \leftarrow \text{circle}$$
$$x^2-y^2=1 \leftarrow \text{hyperbola}$$

- Just add eqns...

$$\begin{aligned} & x^2+y^2=25 \\ + \; & x^2-y^2=1 \\ \hline & 2x^2 = 26 \\ & x^2 = 13 \end{aligned}$$

$$\boxed{x=\pm\sqrt{13}}$$



- to get y, substitute into either eqn.

OR

- eliminate x by multiplying 2nd eqn by "-1" and add

$$\begin{aligned} & x^2+y^2=25 \\ + \; & -x^2+y^2=-1 \\ \hline & 2y^2=24 \\ & y^2=12 \end{aligned}$$

$$\boxed{y=\pm\sqrt{12}}$$

$\Downarrow$

due to symmetry

$(x_1,y_1)=(\sqrt{13},\sqrt{12})$

$(x_2,y_2)=(\sqrt{13},-\sqrt{12})$

$(x_3,y_3)=(-\sqrt{13},-\sqrt{12})$

$(x_4,y_4)=(-\sqrt{13},\sqrt{12})$

- substitution.

pick bottom eqn

$(+\sqrt{13})^2-y^2=1$

$13-y^2=1$

$13-1=y^2$

$12=y^2$

$\pm\sqrt{12}=y \Rightarrow (\sqrt{13},\sqrt{12}),(\sqrt{13},-\sqrt{12})$

//

$(-\sqrt{13})^2-y^2=1$

$13-y^2=1$

$y^2=12$

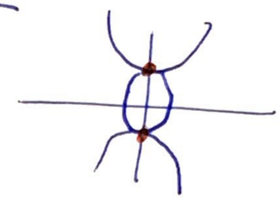
$y=\pm\sqrt{12} \Rightarrow (-\sqrt{13},\sqrt{12})$ & $(-\sqrt{13},-\sqrt{12})$

(*) variation on substitution

Ex $16x^2 - 9y^2 + 144 = 0$

$x^2 + y^2 = 16$

desmos



- Bottom : solve for x^2 and sub into top eqn
  $\hookrightarrow x^2 = 16 - y^2$
- $16(16 - y^2) - 9y^2 + 144 = 0$

  $256 - 16y^2 - 9y^2 + 144 = 0$

  $400 - 25y^2 = 0 \quad \div 25$

  $16 - y^2 = 0 \Rightarrow \boxed{y = \pm 4}$
- plug ± 4 into either eqn:
  ((Bottom looks easier))

$+4$: $\left\{ \begin{aligned} x^2 + (+4)^2 &= 16 \\ x^2 &= 16 - 16 \\ x^2 &= 0 \end{aligned} \right. \rightarrow \boxed{x = 0}$ $\hookrightarrow (0, 4)$

-4: $\left\{ \begin{aligned} x^2 + (-4)^2 &= 16 \\ x^2 &= 16 - 16 \\ x^2 &= 0 \end{aligned} \right. \rightarrow \boxed{x = 0}$ $\hookrightarrow (0, -4)$

⑤

(*) a no solution case:

Ex

$$x^2+y^2=1$$
$$x+y=10$$

- Bot eqn: $x=10-y$ sub into top eqn

$$\Rightarrow (10-y)^2+y^2=1$$

$$100-20y+y^2+y^2=1$$

$$\boxed{2y^2-20y+99=0}$$

- Q. Formula:

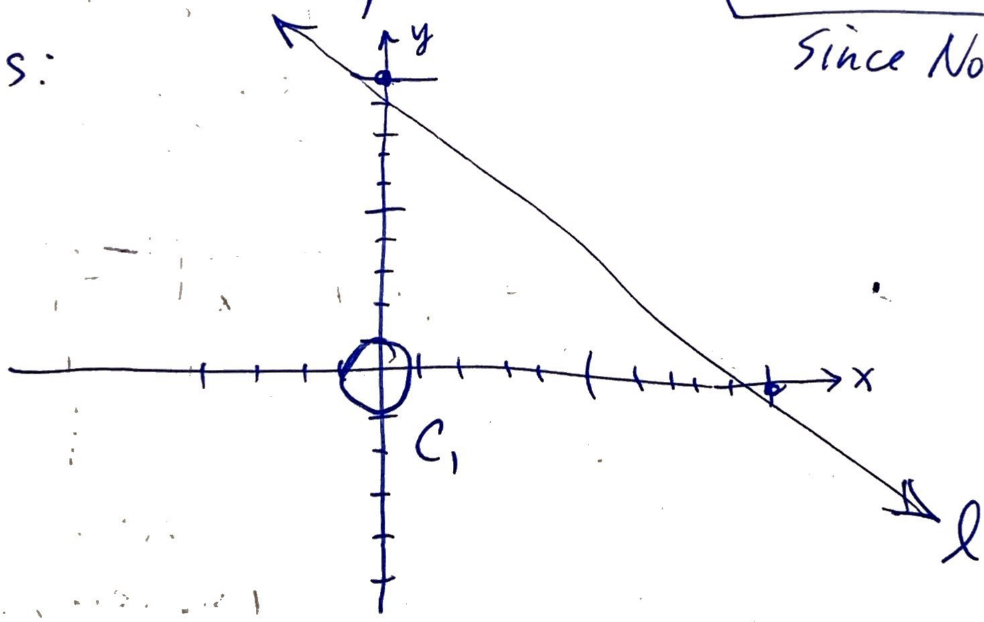
$$y=\frac{-(-20)\pm\sqrt{(-20)^2-4(2)(99)}}{2\cdot 2}$$

$$y=\frac{20\pm\sqrt{400-792}}{4}$$

$$y=5\pm\frac{1}{4}\sqrt{-392}$$

No Real Solution since No intersection

- graphs:

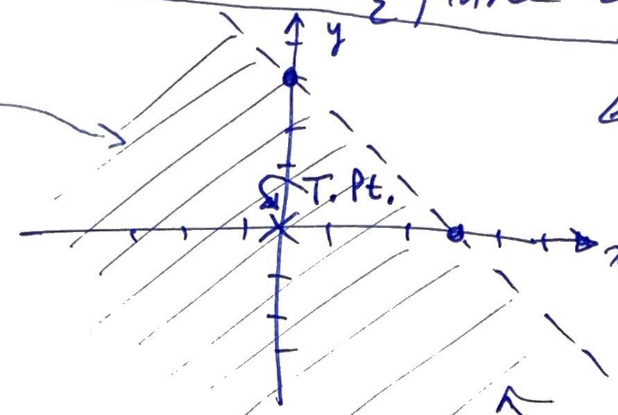


(A) Inequalities

- **Lines:** $\boxed{\text{Ex}}$ $x + y < 3$ ← (i) graph
  - graph $x + y = 3$
  - (ii) test point: $(0,0)$

    $\Rightarrow 0 + 0 \overset{?}{<} 3$ ans: T
  - (iii) shade the $\frac{1}{2}$ plane below the line

"Solution space" to $x+y<3$



$\frac{1}{2}$-plane

- any (x,y) pair does NOT satisfy $x+y<3$
- any (x,y) pair does satisfy $x+y<3$

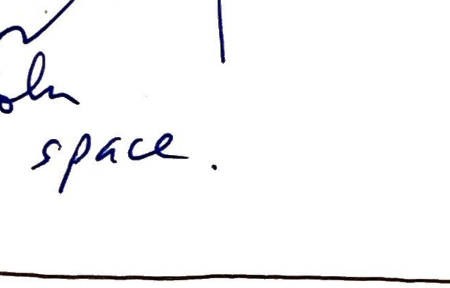
- **Conic sections**

$\boxed{\text{Ex}}$

$$x^2 + y^2 < 1$$

(i) - graph boundary

$x^2+y^2=1$



soln space.

(ii) - use test point pick $(0,0)$

$0^2 + 0^2 \overset{?}{<} 1$

(iii) ans: T

- So shade inside

System of two inequalities

Ex

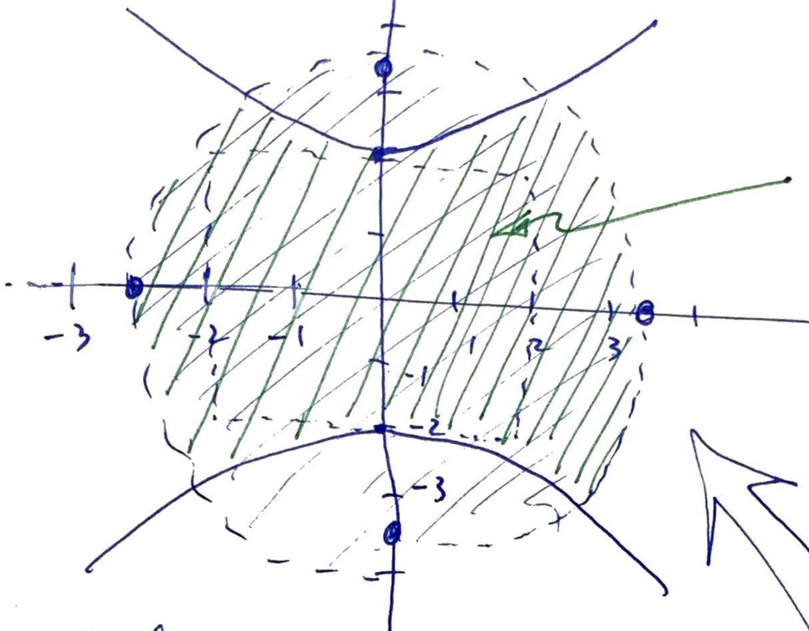
$$x^2 - y^2 > -4 \quad *(-1) \qquad \boxed{\begin{array}{l} y^2 - x^2 < 4 \\ x^2 + y^2 < 12 \end{array}}$$

$$x^2 + y^2 < 12$$

Top: hyperbola opens in the $\pm y$ axis

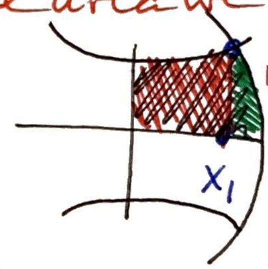
BOT: circle of radius $\sqrt{12} \approx 2\sqrt{3} = 3.4$

- circle



solution space of the system.

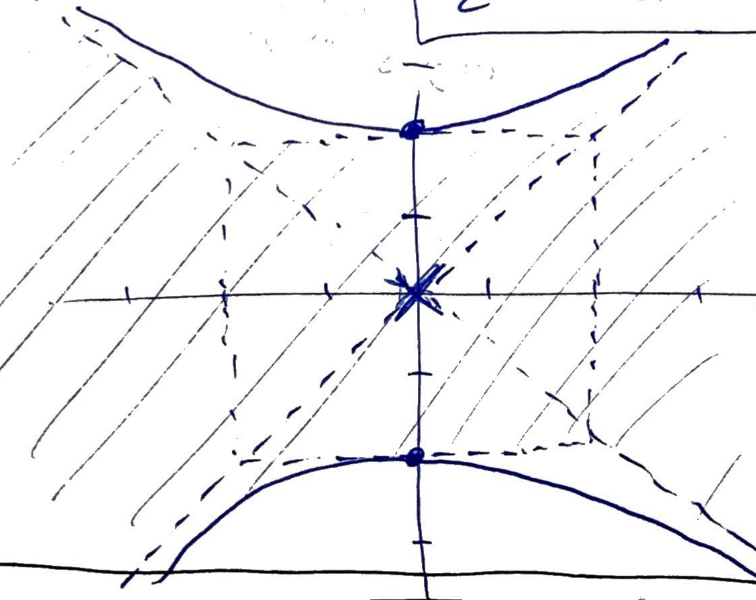
In calculus to find the area we will need x_1



- hyperbola

$$\frac{y^2}{4} - \frac{x^2}{4} < \frac{1}{}$$

$$\Rightarrow \boxed{\frac{y^2}{2^2} - \frac{x^2}{2^2} < 1}$$



Inequality:

Test: $(x,y)=(0,0)$

$$\frac{0^2}{2^2} - \frac{0^2}{2^2} \overset{?}{<} 1 \quad \text{ans:}$$

yes T