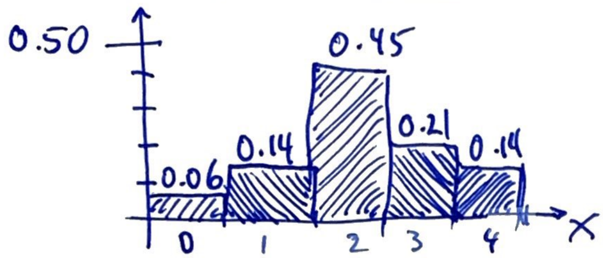
Math 140 Stats, Fa24, LACC, R Erickson $\frac{70}{70}$ TEST 3 (6.1, 7.1-4, 8.1) Name: KEY

Try to keep your work on these sheets. Show or explain ALL work for full credit. BOX in your answers please. DATA TABLES are on the last pages.

1. (10 pts) Following is the probability distribution of a random variable that represents the number extracurricular activities a college freshman participates in.

x	0	1	2	3	4
$P(x)$	0.06	0.14	0.45	0.21	0.14

a. Build a Probability Histogram from this data.



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b. Find the expected value, E_x, of the number of extracurricular activities the freshmen engage in.

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$$E_x = \sum x \cdot P(x)$$

$$= 0\cdot(0.06) + 1\cdot(0.14) + 2\cdot(0.45) + 3\cdot(0.21) + 4\cdot(0.14)$$

$$= \boxed{2.23} \text{ activities per student.}$$

c. Does the expected mean of extracurricular actives meet the 'healthy' region of between 2 to 3 ?

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Yes. $2 < 2.23 < 3$

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2. (10 pts) In a recent Pew poll, 50% of adults said that they play video games. Assume that 9 adults are randomly sampled. Use the binomial probability distribution to find the following probabilities, i.e. use

$$P(x) = {}_nC_x p^x (1-p)^{n-x}$$

a. Find the probability that exactly two of the 9 sampled adults play video games.

$x = 2$ successes

3

$$P(2) = \boxed{{}_9C_2 (0.5)^2 (1-0.5)^{9-2}}$$
$$= \frac{9!}{(9-2)!\,2!}(0.5)^2(0.5)^7$$
$$= \frac{9 \cdot \cancel{8}^{4} \cdot \cancel{7!}}{\cancel{7!}\,\cancel{2}\cdot 1}(0.5)^9$$
$$= 36 \cdot (0.00195)$$
$$= 0.0703$$

or $\boxed{\text{7\% chance exactly 2 play}}$

b. Find the probability that fewer than two of the sampled adults play video games.

4

$$P(x<2) = P(0) + P(1)$$
$$= \boxed{{}_9C_0 (0.5)^0 (0.5)^9 + {}_9C_1 (0.5)^1 (0.5)^8}$$
$$= \frac{9!}{(9-0)!\,0!} \cdot 1 \cdot (0.00195) + \frac{9!}{(9-1)!\,1!} \cdot (0.5)^9$$
$$= \underbrace{1 \cdot 1 \cdot (0.00195) + 9 \cdot (0.00195)}_{= 10 \cdot (0.00195)}$$
$$= \text{Total } \boxed{0.0195}$$
$$\approx 0.02$$

or $\boxed{\text{2\% chance that there is only 1 or none that play}}$

c. Find the probability that two or more sampled adults play video games. Hint: Compliment Rule

3

$$\boxed{P(x \geq 2) = 1 - P(x<2)}$$
$$= 1 - 0.0195$$
$$= 0.9805$$

or $\boxed{\text{98 \% chance}}$ a group of 9 will have 2 or more gamers.

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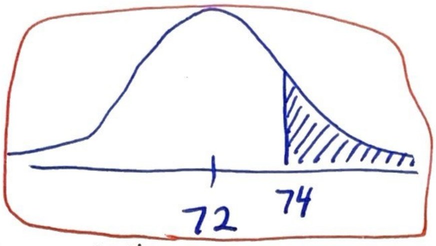
$\boxed{3.}$ **(10 pts)** *Individual* CoC men have heights that are normally distributed with mean μ = 72 inches and a standard deviation σ = 5 inches.

a. What proportion of CoC men are *more* than 74 inches tall?

(i) Draw and shade a distribution curve



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(ii) compute the z-score associated with a height of 74 in.

$z_{74} = \frac{74-72}{5} = 0.4$

(iii) Use the z-table to obtain the area in the shaded region of your curve:

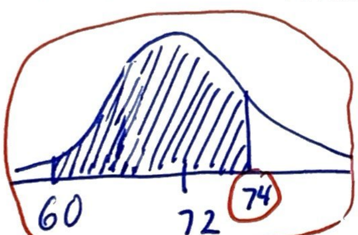
$P(z < 0.4) = 0.6554$

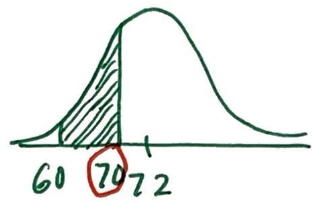
so $P(z > 0.4) = 1 - 0.6554$

$= \boxed{0.3446}$

b. What is the probability that a randomly chosen CoC man is *between* 60 and ~~70~~ 74 inches tall?

(i) Draw and shade the distribution curve involved.





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(ii) compute the z-scores for the two limits:

- $z_{74} = 0.4$
- $z_{60} = \frac{60-72}{5}$

  $= -12/5 = -2.40$

$Z_{70} = \frac{70-72}{5} = -\frac{2}{5} = -0.40$

$Z_{60} = \frac{60-72}{5} = -\frac{12}{5} = -2.40$

(iii) Use the z-table to obtain the area in the shaded region of your curve:

- $P(x<74) = P(z<0.4) = 0.6554$
- $P(x<60) = P(z<-2.40) = 0.0082$

$\underline{\underline{0.6472}}$

- $P(x<70) = P(z<-0.40) = 0.3446$
- $P(x<60) = P(z<-2.40) = 0.0082$

$\underline{\underline{0.3364}}$

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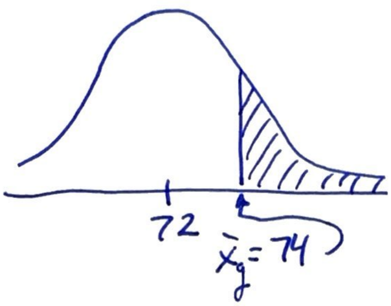
$\boxed{4.}$ **(10 pts)** *Samples (groups) of 6* CoC men are taken where the *individual* heights are normally distributed with mean $\mu = 72$ inches and standard deviation $\sigma = 5$ inches.

What is the probability that a randomly chosen *group of 6* CoC men have a mean height of *more* than 74 inches tall?

(i) Draw and shade a distribution curve



$N\left(72, \sigma_g = \frac{5}{\sqrt{6}}\right)$

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(ii) Compute the group's z-score associated with a mean height of 74 in.

$$z_g = \frac{74 - 72}{5/\sqrt{6}} = \underline{\underline{0.980}} \quad 0.979796$$

3

group

(iii) Use the z-table to obtain the area in the shaded region of your curve:

$$P(\bar{x}_g < 74") = P(z_g < 0.980) = 0.8365$$

$$\text{so } P(\bar{x}_g > 74") = 1 - P(\bar{x}_g < 74")$$

$$= 1 - 0.8365$$

$$= \underline{\underline{0.1635}}$$

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Or $\underline{16.4\%}$ chance a sample of size 6 will have a mean height of 74" or more

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5. (10 pts) According to a Harris poll, chocolate is the favorite ice cream flavor for 27% of Americans. If a sample of 10 Americans is taken, what is the probability that the sample proportion of those who prefer chocolate is greater than 0.30? SHOW ENOUGH WORK.

a) What are the conditions to be met so we can use the Central Limit Theorem to calculate the group's proportion of chocolate lovers?

- SRS → assumed
- $n < 10\%$ → 10 pple is less than 10% of pop.
- $np > 10$ & $n(1-p) > 10$ → $10(0.27) \approx 3 \overset{?}{>} 10$ NO! $10(0.73) \approx 7 \overset{?}{>} 10$ NO!

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b) Is it appropriate to use a normal model in finding the probability that the sample proportion of those who prefer chocolate is greater than 0.30? Explain.

2 We need at least 10 failures (don't like chocolate Ice Cr.) and we need at least 10 success (do like choc.). we have neither.

c) A new sample of 100 people are chosen. What is the probability that the sample proportion of those who prefer chocolate is greater than 0.30. now # succ $= 100(0.27) = 27$, #fails $= 73$ both > 10.

(i) Draw and shade a distribution curve

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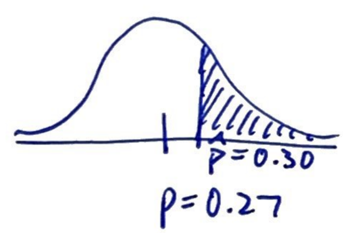

$$N\left(p, \sqrt{\frac{p(1-p)}{n}}\right) = N\left(0.27, \sqrt{\frac{0.27(0.73)}{100}}\right) = \underline{N(0.27, 0.0444)} \leftarrow \text{our model}$$

(ii) compute the group (sample mean) z-score

2 $$\boxed{z_g = \frac{0.30 - 0.27}{\sqrt{\frac{p(1-p)}{n}}}} = \frac{0.03}{0.0444} = 0.675 \approx \underline{0.68}$$

(iii) Use the z-table to obtain the area in the shaded region of your curve:

$$P(\hat{p} > 0.30) = P(z_g > 0.68) = 1 - P(z_g < 0.68)$$

2 $$= 1 - 0.7517$$

$$= \boxed{0.2483}$$

So, 25% chance that a sample of size 100 will have 30% of the ppl liking Chocolate as their fav. ice creame.

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6. (20 pts) A random sample of 11 smartphones sold over the internet had the following prices, in dollars: standard deviation is known to be $\sigma = 85$.

199	169	385	329	269	149	135	249	349	299	249

(a) Briefly state why it is necessary to check whether the population is approximately normal before constructing a confidence interval.

2 We have only a sample size of 11 which is is well below the 30 needed by the C. L. Thm.

(b) Make a quick a dotplot of these data on top of the number line below. Is it reasonable to assume that the population is approximately normal?

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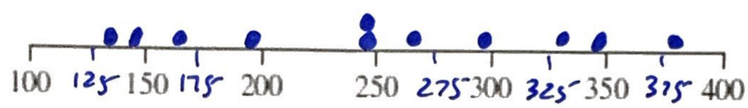

yes, it is reasonably unimodal and symmetric

(c) Assuming it is appropriate to proceed, construct a 95% confidence interval for the mean price for all phones of this type being sold on the internet. Fill out the follow form, as was done in class.

STEP 0: **Assumptions (state the general and justify your application's)**

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- SRS, $n < 10\%$ — assumed & there are more than 110 smart phones
- unimodal & sym or $n \geq 30$ — dot plot looks acceptable

3 **STEP 1:** **Compute the point estimate (use ~~Statdisk~~ calc.):** $\bar{x} = 252.818$, $\sigma_{n-1} = 83.386$

2 **STEP 2:** **State the Confidence Level:** 0.95 **critical value (circle one):** z t = 1.96

STEP 3: **Compute standard error.**

proportion problems ⟵|⟶ mean value problems

(a) Formula SE (circle): $\sqrt{\frac{\hat{p}\hat{q}}{n}}$ $\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1}+\frac{\hat{p}_2\hat{q}_2}{n_2}}$ $\frac{s}{\sqrt{n}}$ $\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}$ $\frac{s_d}{\sqrt{n}}$

3 $SE = \frac{85}{\sqrt{11}} = 25.628$

SE Value = 25.628

STEP 4: **Compute the Margin of Error = critical value * SE**

3 ME = 1.96 * 25.628 = 50.232

17

STEP 5: **Construct the Confidence Interval:** **point estimate ± ME**

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$252.82 - 50.23 < \mu < 252.82 + 50.23$

Place the clean Confidence Interval in the box below

$$\boxed{202.59 < \mu < 303.05}$$

7. (10 pts) A dean at a certain college looked up the GPA for a random sample of 85 students. The sample mean GPA was 2.82, and a 95% confidence interval for the mean GPA of all students in the college was 2.76 < μ < 2.88. True or false, and explain:

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a. We are 95% confident that the mean GPA of all students in the college is between 2.76 and 2.88.

2 T : 19 of 20 such samples will produce a C. Intvl that captures the true population mean GPA

b. We are 95% confident that the mean GPA of all students in the sample is between 2.76 and 2.88.

2 F the mean of the sample is 2.82, fixed, not an Intvl

c. The probability is 0.95 that the mean GPA of all students in the college is between 2.76 and 2.88.

2 T 19 of 20 such samples is equiv. to 0.95 prob.

d. 95% of the students in the sample had a GPA between 2.76 and 2.88.

2 F, cannot detail the actual GPA's. We only have the mean of a sample and can infer only a range of GPAs

13