7.4 The CLThm for Proportions

Recall : mean values describe such items as averages of daily gallons of milk from Bessie the Cow or ave. number of eggs from Checkers the Chicken

Proportions describe items such as the percentage of white cars in the parking structure or the fraction of employees with an IRA account.

Much of the remaining material covered in Stats deals with mean values OR proportions

THINK : How many cigarettes do you smoke

vs.

What fraction of students smoke
Binomial Data
yes or No

7.3 covered CLThm for means

This section, 7.4 covers C.L. Thm for proportions

Def: • A <u>sample's proportion</u> is called ②

$$\boxed{\hat{p} \equiv x/n}$$

x = the number of qualifying outcomes in a sample of size "n" {# ppl smoke}

• A <u>population's proportion</u> (census) is given by $\boxed{p \equiv x/N}$

As we pull samples of size "n" from the population we will size different proportions.

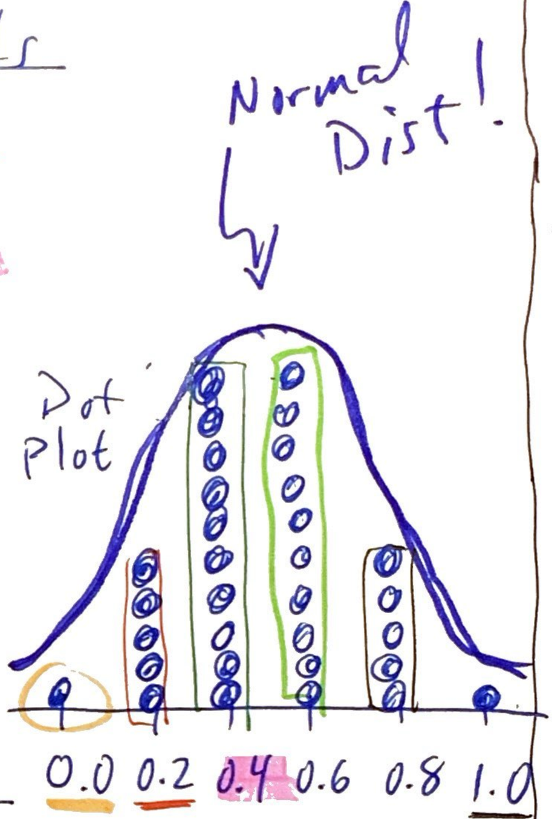
EX Visiting Freshmen classes in H. S. and ask how many use <u>Laptops</u> (vs phones/tables)

ex 35 of 100 randomly selected students had laptops so $\boxed{\hat{p} = 35/100 = 0.35}$

The distribution of these different proportions follows a normal distribution.

EX Toss a coin 5 times. Write out the sample space. There are $2^5 = 32$

outcome	$\hat{p}$ = proportion of heads
T T T T T	0/5 = 0.0
T T T T H	1/5 = 0.2
T T T H T	1/5 = 0.2
T T T H H	2/5 = 0.4
T T H T T	1/5 = 0.2
T T H T H	2/5 = 0.4
T T H H T	2/5 = 0.4
T T H H H	3/5 = 0.6
T H T T T	1/5 = 0.2
T H T T H	2/5 = 0.4
T H T H T	2/5 = 0.4
T H T H H	3/5 = 0.6
T H H T T	2/5 = 0.4
T H H T H	3/5 = 0.6
T H H H T	3/5 = 0.6
T H H H H	4/5 = 0.8
H T T T T	1/5 = 0.2
H T T T H	2/5 = 0.4
H T T H T	2/5 = 0.4
H T T H H	3/5 = 0.6
H T H T T	2/5 = 0.4
H T H T H	3/5 = 0.6
H T H H T	3/5 = 0.6
H T H H H	4/5 = 0.8
H H T T T	2/5 = 0.4
H H T T H	3/5 = 0.6
H H T H T	3/5 = 0.6
H H T H H	4/5 = 0.8
H H H T T	3/5 = 0.6
H H H T H	4/5 = 0.8
H H H H T	4/5 = 0.8
H H H H H	5/5 = 1.0



The C.L. Thm for proportions follow the normal model of

$$N\left(\mu_{pop} = p,\ \sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}\right)$$

Sometimes "$1-p$" is called "q" so $\sqrt{\frac{pq}{n}}$

- Steps to solve (evaluate) group's proportions

1. Identify and state $\mu_{pop} = p = x/n$
2. Calculate the std. dev. of proportions
   $$\sigma = \sqrt{\frac{p(1-p)}{n}}$$
3. Calculate the z-score for a given sample value requested
4. Draw and lable the curve.
5. Calculate probabilites from the z-tables

- Conditions to be met:

(i) Trials must be independant
(ii) we need 10 success and 10 failures
i.e. $n \cdot p \geq 10$ and $n \cdot (1-p) \geq 10$

EX A Harris poll finds that 27% of Americans choose chocolate Ice Cream as their favorite flavor.

If a sample of size 100 people is taken then what is the probability that the groups love of chocolate ice cream exceeds 30%

Conditions : 1. Indpendence: assume each person has equal chance of being selected and they are not connected to any other in the group.

2. successes :

$100 \cdot 27\% = 27 \geq 10$ ✓

failures

$100 \cdot 73\% = 73 \geq 10$ ✓

Model $\mu_p = 0.27$, $\sigma = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.27(1-0.27)}{100}}$

$$\boxed{N(\mu, \sigma) = N(0.27, 0.044)}$$

Solve

$P(\hat{p}_g > 0.3)$
$= 1 - P(\hat{p}_g < 0.3)$

$= \frac{0.30 - 0.27}{0.04} = 0.68$

$P(\hat{p}_g > 0.3)$
$= P(z > 0.68)$
$= 1 - P(z < 0.68) = 1 - 0.7517$ z-table
$= 83$

EX At 7-11 stores soft-drink cups have tickets attached to them. The proportion of winning tks is $p_{pop} = 0.25$

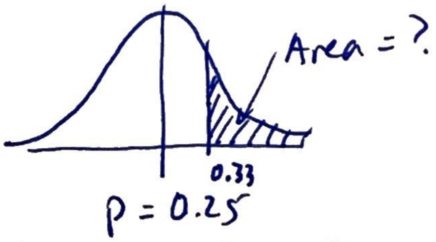
A total of 70 people purchase the soft-drink during lunch. $\boxed{Q_1}$ Find the mean and standard deviation of winners during lunch. (The Model)

- C.L.Thm says $\begin{cases} \mu_{\hat{p}} = \mu_{pop} = p = 0.25 \\ \sigma_{\hat{p}} = \sqrt{\dfrac{p(1-p)}{n}} \\ \quad = \sqrt{\dfrac{0.25(0.75)}{70}} = \underline{\underline{0.052}} \end{cases}$

the Model to use

So $\boxed{N(0.25, 0.052)}$

$\boxed{Q_2}$ In any group of 70 purchases what is the probability the proportion of winners is larger than 0.33?



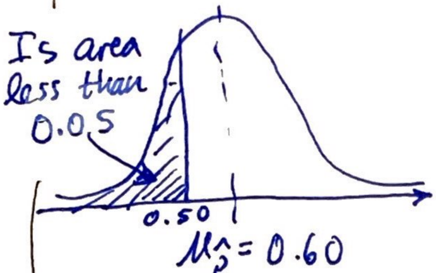
So the model used is:

$N(0.25, 0.052)$

- $Z_{0.33} = \dfrac{0.33 - 0.25}{0.052} = \underline{\underline{1.54}}$
- $P(\hat{p} \geq 0.33) = P(z \geq 1.54)$
  $= 1 - P(z \leq 1.54)$
  $= 1 - 0.9382 = \boxed{0.0618}$ 6.2%

EX Approx 60% of eligible voters voted in 2016. If a sample of 88 eligible voters were polled would it be unusual to find less than ½ voted?

Model $\left(0.60,\ \sigma_{\hat{p}} = \sqrt{\frac{0.60(1-0.60)}{88}} = 0.0522\right) = \boxed{N(0.6, 0.05)}$



$$Z = \frac{0.50 - 0.60}{0.0522} = -1.91$$

z-table

$$P(\hat{p} \leq 0.50) = P(z \leq -1.91) = \boxed{0.0281}$$

So since the probability of having a group of 88 voter-eligible people possessing less than 50% having voted is at 2.8%, we state that this is unusual

$\leq 5\%$