7.3 The Central Limit Thm AND Statistics of "Grouped Means"

The Class work we just finished contained the following activity

1. Studied "Bear" weight: shape
   we noticed the data is skewed left.

2. Used random.org to tell us how to collect a sample of size $N = 15$.

3. We selected from the Bear Database those 15 individual bear's weights. and placed them into Statdisk.

4. We analyzed our samples and created a histogram and Box plot. We generally noticed that our sample data was likewise skewed left like the population it came from.

5. Finally, we collected the mean from each student sample ~~and~~ placed those into Statdisk. We saw a Normal Distribution of the sample means from the class.

Recap

I

II

sample #1	sample #2	sample #3	...	sample #N

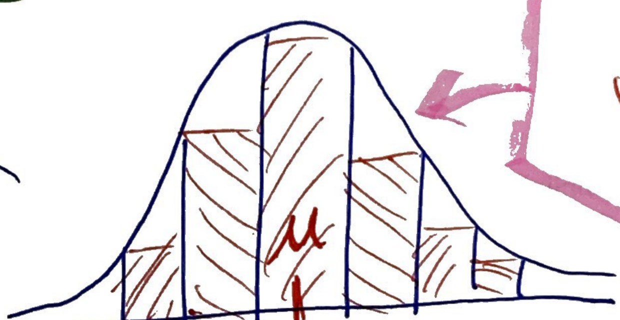
$\bar{x}_1 =$	$\bar{x}_2 =$	$\bar{x}_3 =$	...	$\bar{x}_{15} =$
$s_1 =$	$s_2 =$	$s_3 =$	...	$s_{15} =$
$N=15$	$N=15$	$N=15$		$N=15$

- Frequency Table

$\bar{x}$	Freq
150-169	ll
170-189	llll
190-209	~~llll~~ ~~llll~~ ll
210-229	llll
230-249	lll

III

- Histogram



model $\boxed{N\left(\mu_{\bar{x}} = \mu_{pop},\ \sigma_{\bar{x}} = \frac{\sigma_{pop}}{\sqrt{N}}\right)}$

distribution shape of the mean values of the sample is unimodal and symmetric (N. Dist!!)

The Central Limit Thm has $\boxed{\text{conditions}}$ that must be met: (3)

Independant Samples:

- **Independence** Subjects need to be independant from each other.
- **Random** Data must be randomly gathered.
- **10% Condition** We collect the sample w/o replacement. So the sample must not exceed 10% of the population
- **Sample size based on skewness** If the population's distribution of the parameter of interes is skewed we need a sample size of $N \geq 30$ ((The less skewed the data a smaller the sample size is allowed.))

⊛ Application: "Grouped Means"

- This application allows us to describe the attributes of a group of data's mean values

- For a group of sampled data of size N, the mean value follows, per the CL Thm, the model: $N(\mu, \sigma/\sqrt{N})$

  std. dev.
  mean
  normal dist.

  {think z-tables}

- We are given the $\boxed{\text{population's}}$ mean value AND standard deviation for some parameter (Individual)

- We then calculate probabilities for whole groups of data pulled from the population

{We are only discussing the group's mean value of a parameter, not the individual's parameter}

$\boxed{EX}$ The national mean SAT score is 500 with a standard deviation of 100.

Q: what can you say for an average class of 20 students SAT scores?

Hint: Use the emperical Rule-of-thumb

- Individuals follow $N(\mu = 500, \sigma = 100)$
- Groups of 20 follow $N\left(\bar{x} = 500, s = \frac{100}{\sqrt{20}}\right)$

★ Assuming we meet the conditions

1. Independence: Assume the 20 students come from different H.S.

2. Randomness: Assumed the sample was choosen randomly

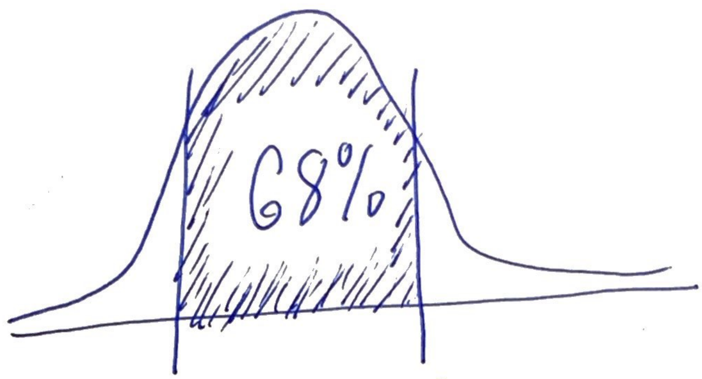
3. 10% condition: 20 students form less than 10% of national SAT takers. {we have more than 200 individual in the population}

4. Sample size: $N = 20$ is less than 30 but we assume the Population distribution shape is unimodal & symmetric.

EX Continued

- 68% of groups of sample size 20 will have a mean SAT score between

$$500 \pm \frac{100}{\sqrt{20}}$$



$-S$ $\qquad\qquad$ $+S$

$500 - \frac{100}{\sqrt{20}}$ $\qquad$ $500 + \frac{100}{\sqrt{20}}$

$= 500 - 22.36$ $\qquad$ $= 500 + 22.36$

$= \boxed{477.64}$ $\qquad$ $= \boxed{522.36}$

i.e. 68% of groups of 20 have a group mean value between 478 – 522

Likewise 95% of groups of 20 have a group mean value between 456 to 545

NOTE: Individual SAT scores will find 68% of SAT test takers fall between

$500 - 100$ to $500 + 100$

i.e. $\boxed{400 \text{ to } 600}$

EX Use the z-tables

Engineers design a large elevator to hold 40 people.
The maximum payload is 8120 lbs.
The National Health Database reports that the weights of men follows a mean of 194 lbs with a std. dev. of 68 lbs.
{for Women $\mu = 164$, $\sigma = 77$ lbs}

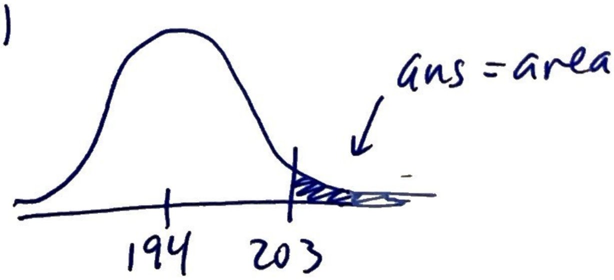
(a) If 40 men are on the elevator what is their average weight if indeed their total weight is the 8120 lb max weight?

$$8120 / 40 = \underline{\underline{203 \text{ lbs}}}$$

(b) NOW, If a random sample of 40 men ride the elevator, what is the probability the group's mean weight exceeds 203 lbs [8120 lb limit]?

$$\mu_{\bar{x}} = \mu_{pop} = 194 \text{ lbs}, \quad \sigma_{\bar{x}} = \frac{\sigma_{pop}}{\sqrt{40}} = \frac{68 \text{ lb}}{\sqrt{40}} = \underline{\underline{10.75 \text{ lb}}}$$

(i)



(ii)

$$z_{\bar{x}} = \frac{203 - 194}{10.75} = \underline{\underline{0.84}}$$

(iii)

$$P(\bar{x} > 203) = 1 - P(\bar{x} < 203)$$
$$= 1 - P(z < 0.84)$$
$$= 1 - 0.7995 \quad \{\text{row } 0.8, \text{ col } 0.04$$
$$= 0.2005$$

or

20% chance