# 5.4 Counting



## multiplication rule for counting

If an activity can be performed in "m-ways" and a 2nd activity can be performed in n-ways then the total number of ways to perform the two activities, one after the other, is $\boxed{m \cdot n}$

**EX** A cafe has you pick one Protien and one condiment for a kids sandwich

$$\underbrace{\begin{array}{l}\text{Turkey}\\ \text{Tuna}\\ \text{Chicken}\\ \text{Beef}\\ \text{Veggies}\end{array}}_{5} \qquad \underbrace{\begin{array}{l}\text{mustard}\\ \text{Ketchup}\\ \text{Mayonaise}\\ \text{BBQ sauce}\\ \text{Relish}\\ \text{Pesto Sauce}\end{array}}_{6}$$

| Protein | Condiment List |
| --- | --- |
| Turkey | mustard |
| Tuna | Ketchup |
| Chicken | Mayonaise |
| Beef | BBQ sauce |
| Veggies | Relish |
| | Pesto Sauce |

Q: How many pairings can be selected?

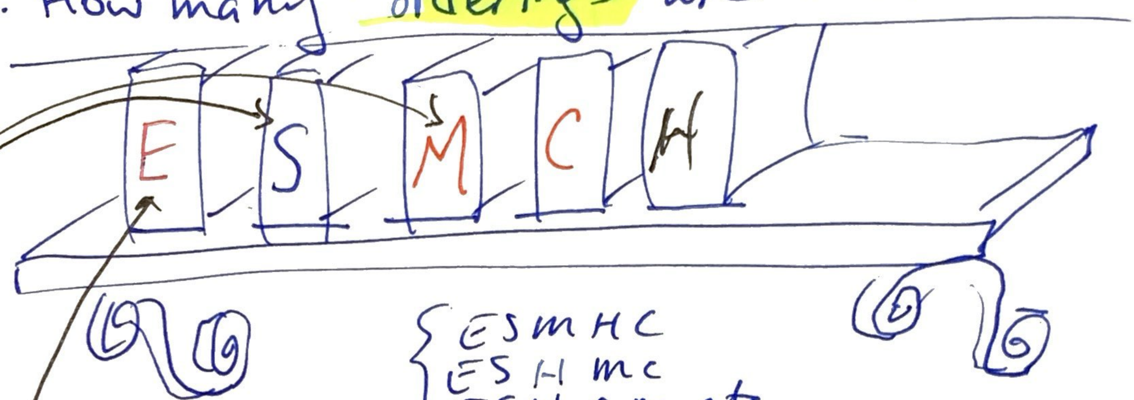
(5 protiens) $\cdot$ (6 condimants) $= 30$ combinations

The Keywork is $\boxed{\text{AND}}$, so both activities occur. We multiply

$\boxed{\text{EX}}$ we have 5 textbooks on a table.


we need to place these onto a bookshelf, in order,

Q: How many orderings are there?




{ ESMHC
ESHMC
ESHCM etc...

- For the 1st position how many choices of textbooks are there? 5

Lets assume we put the english text up 1st.

- For the 2nd position how many choices? 4

assume stats is in 2nd position

- For the 3rd? 3

assume its math.

- 2 choices & then Finally one text is left so only 1 choice.

$\boxed{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = \boxed{120 \text{ ways! orders!}}$

# Permutations

**A permutation of n-objects, none alike, is the number of orderings in which these objects can be ordered.** {order is important}

**Ex** List the number of ways we can select 3 texts from a table


ABC
ACB
BCA
BAC
CAB
CBA

$\boxed{\text{OR}}$ $\underline{3} \cdot \underline{2} \cdot \underline{1} = \boxed{6}$ ways.

**Def:** n Factorial $= n \cdot (n-1) \cdot (n-2) \cdot \dots \cdot 3 \cdot 2 \cdot 1$

$\boxed{n!}$

**ex** $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

**ex** $11! = 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

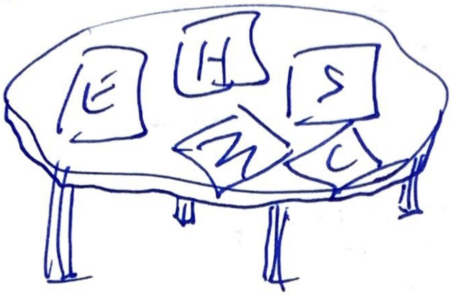
**Thm** The number of orderings of n objects, none alike, is $n!$

- Calculator

$11 \; \boxed{2^{nd}} \; \boxed{x!} = 39{,}916{,}800$

## * Permutation of 'r' objects taken from 'n' objects {order is important & non-alike objects}

**EX**




$\underline{5} \cdot \underline{4} \cdot \underline{3}$

• 3rd slot

• 2nd slot has $\underline{4}$ choices

• 1st slot has $\underline{5}$ choices

$5 \cdot 4 \cdot 3 = \boxed{60 \text{ ways}}$

**Def** the number of ways to order 'r' objects from 'n' objects, all non-alike, is

$$\underbrace{n \cdot (n-1) \cdot (n-2) \cdots (n-r+1)}$$

Notation is $\boxed{{}_nP_r}$ select "r" objects from "n" - order impt.

**EX** take 4 objects from 7

$$\underbrace{7 \cdot 6 \cdot 5 \cdot 4}_{840} = {}_7P_4$$ "permutations of 4 of 7 objects"

• Calculator button:

7 $\boxed{2^{nd}}$ $\boxed{nPr}$ 4 $\boxed{=}$ "840"

Alternative way to calculate $_nP_r$

$$\boxed{{}_nP_r = \frac{n!}{(n-r)!}}$$

$$= \frac{n\cdot(n-1)\cdots(n-r+1)\cancel{(n-r)\cdot(n-r-1)\cdots 3\cdot 2\cdot 1}}{\cancel{(n-r)(n-r-1)\cdots 3\cdot 2\cdot 1}}$$

Ex

$$_7P_4 = \frac{7!}{(7-4)!} = \frac{7!}{3!} = \frac{7\cdot 6\cdot 5\cdot 4\cdot\cancel{3\cdot 2\cdot 1}}{\cancel{3\cdot 2\cdot 1}}$$

also

$$= \frac{7!}{3!} = \frac{7\cdot 6\cdot 5\cdot 4\cdot\cancel{3!}}{\cancel{3!}}$$

Application

We need to send a presidency of President, V-Prez & Sectry to the ASG. From a class of 12 students, how many presidencies can we send out?

$$_{12}P_3 = \boxed{1320} \text{ ways.}$$

$= 12\cdot 11\cdot 10$

# * General multiplication Rule

EX Sarah graduates from Law School and will purchase 3 cars to populate her 3-car garage.

She will chose one from 3 cars she likes at Toyota, one from 2 cars she likes at Subaru, and 1 from 6 cars she likes @ Lamborgini.

Q: How many different ways can she populate those garage spaces.

$$\underset{G1}{\underline{3}} \cdot \underset{G2}{\underline{2}} \cdot \underset{G3}{\underline{6}} = \boxed{36}$$ possible arrangement of cars in these 3 garages.

Toyota | Subaru | Lamborgini

G1 and G2 and G3

all 3 activities will occur

## * Add Probability into Counting



**Ex** 5 lifegaurds : Abbey, Bruce, Chris, Danna and Esmeralda are part of the Magic Mountain' Hurricane Harbor's weekend group.

3 are selected for the North, South and west chair of the big Wave Pool.

Q: what is the probability that Bruce is assigned to the North Chair, and Danna is assigned to the West chair and Abbey is assigned to the South chair

- Generic: $\frac{5^{\text{choices}}}{N} \; \frac{4^{\text{choices}}}{W} \; \frac{3^{\text{choices}}}{S} = 60 = {}_5P_3$ order is important
- Specific: $\frac{B}{N} \; \frac{D}{W} \; \frac{A}{S} = 1$ way

$$\boxed{P(B, D, A) = \frac{\text{specific ways}}{\text{generic ways}}} = \boxed{\frac{1}{60}}$$

chance that we have the specified order of Lifegaurds to chairs.

$$= \frac{1}{{}_5P_3}$$

# 8 Addition Rule ("or")

If one activity has "m" ways of being performed and a second activity has "n" ways of being performed, then there are

$\boxed{m+n}$ ways that

we can perform one or the other

Ex Sarah's accountant says she needs to start with only one car from the 3 form Toyota OR the 2 from Subaru OR the 6 from Lamborgini

Q: How many choices does she have now?

$$\underset{\text{Toyota}}{\underline{3 \text{ choices}}} \text{ OR } \underset{\text{Subaru}}{\underline{2 \text{ choices}}} \text{ or } \underset{\text{Lamborg.}}{\underline{6 \text{ choices}}}$$

$$= 3 + 2 + 6$$

$$= \boxed{11 \text{ choices}}$$

# ⊛ Combinations

We now address the instances where order is not important

**Ex** Instead of sending a Presidency to the ASG they ask for a committee of 3 students.

Q: How may different committees can be sent?

If order:

5 pple in the class

A B C D E

$$\underset{Pres}{5} \cdot \underset{VP}{4} \cdot \underset{Scty}{3} = \boxed{60}$$

But if we do not care about titles then $\boxed{ABC = ACB = BCA = BAC = CAB = CBA}$ is all one outcome, So we have to not count the ordering of those selected.

The number of ways to order the 3 selected is 3!

Lets divide out these orderings of those selected

So the number of combinations for a committee of 3 from a class of 5 is

$${}_5P_3 / 3! = \frac{5 \cdot 4 \cdot 3}{3 \cdot 2 \cdot 1} = \frac{60}{6} = \boxed{10 \text{ ways}}$$ to send a committee of 3 to the ASG

**Def:** A combination of selecting r objects from a collection of n non-alike objects can be performed ${}_nC_r$ ways, without regards to orderings

Here ${}_nC_r \equiv \dfrac{{}_nP_r}{r!}$

divide out the duplicates amongst the "r" chosen objects when order is NOT important

or $$\boxed{{}_nC_r = \frac{n!}{(n-r)!\,r!}}$$

**Ex:** American Airlines needs a crew of 2 pilots & one Navigator (also a pilot) to staff a flight from LA to Tokyo. If the available pool is 9 pilots how many ways can this task be performed?

- with order

$$\underset{\text{Pilot}}{9} \cdot \underset{\text{Co-P}}{8} \cdot \underset{\text{Nav}}{7} = {}_9P_3 = 504$$

- without order
{crew assigns themselves}

$${}_9C_3 = \frac{{}_9P_3}{3!} \equiv \frac{9!}{(9-3)!\,3!} = \frac{9\cdot8\cdot7\cdot\cancel{6!}}{\cancel{6!}\cdot3\cdot2\cdot1}$$

$$= \frac{\overset{3}{\cancel{9}}\cdot\overset{4}{\cancel{8}}\cdot7}{\cancel{3}\cdot\cancel{2}\cdot1}$$

$$= 3\cdot4\cdot7 = \boxed{84 \text{ ways to staff this flight w/o order}}$$

# ⊛ Probability w/ Combinations



**Ex** (a) In a 52 card deck if we pick 5 cards, how many ways can we select 5 face cards $\{\underbrace{J, Q, K}_{3}\}$

Q: Number of face cards : 3 * 4 suits = $\boxed{\text{12 face cards in a deck}}$

Q: How many ways can we possess a hand of 5 face cards from the deck?

$$_{12}P_5 = \frac{12}{\text{Card 1}} \cdot \frac{11}{\text{Card 2}} \cdot \frac{10}{\text{Card 3}} \cdot \frac{9}{\text{Card 4}} \cdot \frac{8}{\text{card 5}} = 95,040$$

But I do not care about their order:

$$\boxed{_{12}C_5} = \frac{_{12}P_5}{5!} = \frac{95040}{120} = \boxed{\text{792 face card Combinations}}$$

(b) what is the probability of selecting 5 face cards from a deck of 52 cards

P( select 5 face cards from deck of 52 cards)

$$= \frac{\text{specific ways to select only 5 face cards}}{\text{generic ways to select 5 cards in General}}$$

$$= \frac{_{12}C_5}{_{52}C_5}$$

$$= \frac{792}{2,598,960} = \boxed{0.000305 \text{ or } 0.031\text{ \% chance I select a hand of only Face Cards}}$$

## Repetitive Objects in Permutations



((mississippi Counting)) (order important)

- what are the number of ways to arrange, with order, n-objects (non-alike)? $\boxed{n!}$
- what are the number of ways to arrange r objects non-alike? $\boxed{r!}$
- If, of the "n-objects," "r" of them are actually duplicates, how many ways can we arrange "n" objects with r-alike objects? $\boxed{n!/r!}$

Ex

- How many ways can we rearrange the "word" mistjuvkpgl ← $\boxed{11!}$

1 2 3 4 5 6 7 8 9 10 11

- the word? missjuvkpgl ← $\boxed{11!/2!}$
- the word? missiuvkpgl ← $\boxed{\frac{11!}{2!\,2!}}$
- the word? mississkpgl ← $\boxed{\frac{11!}{4!\,2!}}$
- Finally the word mississippi ?

$$\boxed{\frac{11!}{4!\;4!\;2!}}$$ ways to rearrange the word mississippi

= 34,650 ways to rearrange mississippi

# Counting Outline



*summary of counting w/ multiple counting activites

**I** Identify the activities involved

**II** Identify the counting methods of each acti

**III** Identify the way the activities interact

**EX** Find the number of ways to award one 1st place, one 2nd place and three 3rd place ribbons to 12 contestants.

**I**
- activity 1 : select 1st place
- activity 2 : select a 2nd place
- activity 3 : select three 3rd places

**II** Count each activity:

12 ppl

$$\underbrace{\frac{12}{1^{st}}}_{A_1} \quad \underbrace{\frac{11}{2^{nd}}}_{A_2} \quad \underbrace{\frac{{}_{10}C_3}{\_\ \_\ \_}}_{3^{rd}\ A_3}$$

**III** all occur at once so multiply

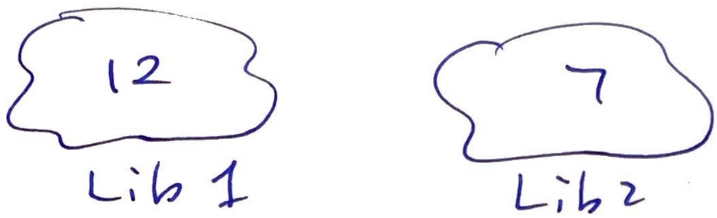
$$12 \cdot 11 \cdot {}_{10}C_3$$

$$= 12 \cdot 11 \cdot 120$$

$$= \boxed{15{,}840 \text{ ways to make these awards}}$$

EX A student is allowed the prrivikedge of choosing 4 books from either of two presidential Libraries. The 1st library has 12 available books the 2nd Library has 7 available book.

Q: How many different selections can be made?




I identify activities
- activity 1 pick 4 from 12
- activity 2 pick 4 from 7

II count each activity
- activity I : ${}_{12}C_4$
- activity 2 : ${}_{7}C_4$

III identify interactions
one or the other activity $\Rightarrow$ add

$$\boxed{{}_{12}C_4 + {}_{7}C_4}$$

$= 495 + 35$

$=$ $\boxed{530 \text{ choices}}$