5.3 Conditional Probability

we now look into another concept relating to probability.

Ex Consider the std. playing card deck

- Sample space
  - Clubs ♣ Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, King
  - Spades ♠ Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K
  - Hearts ♥ Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K
  - Diamonds ♦ Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K

$N(\text{Clubs}) = 13$ $\quad N(\text{Spades}) = 13$ $\quad N(\text{Hearts}) = 13$, $N(\text{diamonds}) = 13$

Total Cards $= 4 \times 13 = \underline{52 \text{ cards}}$

- "Without Replacement" means we will select a card for one event but not replace the card for further events.

- Consider two consecutive draws of cards

1 draw: $P(\text{draw an Ace}) = \dfrac{\text{specific}}{\text{generic}} = \boxed{\dfrac{4}{52}}$ keep that aside

2nd draw: $P(\text{draw an Ace } \underbrace{\text{after we drew one already}}_{\text{Condition}})$

$= \dfrac{\text{specific}}{\text{generic}} = \boxed{\dfrac{3}{51}}$ ← conditional probability

- "With Replacement"

1st draw: $P(\text{Ace}) = 4/52$ → put back & shuffle dec

2nd draw: $P(\text{Ace}) = \boxed{4/52}$

When the 1st event influences the outcome of the 2nd event we have $\boxed{\text{conditional Prob.}}$

- Terminology (nomenclature)

The probability of an event B occuring, given that event A has already occured, is denoted as $P(B|A)$

"prob. B occures" "given that", A has occured

mathematically

$$\boxed{P(B|A) = \frac{P(A \text{ and } B)}{P(A)}}$$

cross-multiply

or write as

$$\boxed{P(A \text{ and } B) = P(A) \cdot P(B|A)}$$

- $\boxed{\text{"and"}}$ means "multiply" probabilities

vs.

$$\boxed{P(A \text{ or } B) = P(A) + P(B)}$$

mutually exclusive

- $\boxed{\text{"or"}}$ means "add" probabilities

EX Consider a poll of 25 y.o. pple, or older, surveying Educational Levels (in millions). ③

	No HS Diploma	H.S. Dipl.	Some College	AS	BS	Adv. Deg.	
M	14.0	29.6	15.6	7.2	17.5	10.1	94.0
F	13.7	31.9	17.5	9.6	19.2	9.1	101.0
	27.2	61.5	33.1	16.8	36.7	19.2	195.0

If we randomly select a person from the database

(a) $P(\text{Male participant}) = \dfrac{\text{specific}}{\text{generic}} = \dfrac{94.0}{195.0} = \boxed{48.2\%}$

(b) $P(\text{B.S. deg}) = \dfrac{36.7}{195.0} = \boxed{18.8\%}$

(c) $P(M \text{ "and" with } BS) = \dfrac{17.5}{195.0} = \boxed{8.97\%}$

(d) P (select M given that they have a B.S.)

$= P(M|BS)$

$= \dfrac{P(M \text{ and } BS)}{P(BS)} = \dfrac{17.5/195.0}{36.7/195.00} = \boxed{\dfrac{17.5}{36.7}}$

$\dfrac{\frac{a}{b}}{\frac{c}{b}} = \dfrac{a}{c}$

The conditional probability limits our Universe to only B.S. ⇒ all numbers come from the B.S. column

$P(M|BS) = \boxed{\dfrac{17.5}{36.7}}$

(e) P (BS. given that they are M) ← New universe is just males. All numbers come from the M row.

$= P(BS|M)$

$= \boxed{\dfrac{17.5}{94.0}}$

④ Gen. Multiplication Rule

$$\boxed{P(A \text{ and } B) = P(A) \cdot P(B|A)}$$

EX what is the probability of drawing two consecutive ace's ? {w/o replacement}

$P(Ace_{\#2} | Ace_{\#1})$

$= P(Ace_{\#1}) \cdot P(Ace_{\#2} | Ace_{\#1})$

$= \left(\frac{4}{52}\right) \cdot \left(\frac{3}{51}\right)$

$= \frac{12}{2652} = 0.00452$ or $\boxed{0.45\%}$

1% is 1 in 100 experiment

0.5% 1 in 200 experiments.

Maybe we can do it in 200 attempts.

((5.3 not done yet.))

5% is 1 in 20	0.5% is 1 in 200

Ex • Among those who apply for a job, the probability of being granted an interview is 0.1 {1 in 10}.

• Of those interviewed the probability of being offered the job is 0.25 {1 in 4}

Q: Find the probability an applicant will land a job

P (job offer)

= P (intview AND and offer)

= P(interview) • P(of an offer)

= (0.1) • (0.25)

= $\boxed{0.025}$ or 2.5% chance

$\boxed{1/40 \text{ is } 1 \text{ in } 40}$
applicants get a job

Independent Events

mutual exclusivity means that Event A and Event B have no overlap.

$$\boxed{P(A \text{ or } B) = P(A) + P(B)}$$

Independence means that the occurance of event A has no influence on Event B

i.e. $\boxed{P(B|A) = P(B)}$

then $\boxed{P(A \text{ and } B) = P(A) \cdot P(B)}$

Ex The prob. of drawing 2 Aces if we replace the 1st Ace and shuffle

$$P(Ace_1 \text{ and } Ace_2) = P(A_1) \cdot P(A_2 | A_1)$$
$$= P(A_1) \cdot P(A_2)$$
$$= \left(\frac{4}{52}\right) \cdot \left(\frac{4}{52}\right)$$
$$= 0.0059 \text{ or } \boxed{0.6\%}$$

with replacement

w/o replacement vs. 0.45%

(*) How to Detect Independence

Ex A fair coin is tossed twice in a row..

(a) $P(\text{second toss is Heads}) = \frac{\#\text{Heads}}{\#\text{outcomes}} = \boxed{\frac{1}{2}}$

did the outcome of the second toss depend on the the outcome of the 1st toss?

(b) consider all 2 coin tosses

$P(2^{nd}\text{ toss is Heads}) = \frac{\{HH, TH\}}{4} = \frac{2}{4} = \boxed{\frac{1}{2}}$

$S = \{HH, HT, TH, TT\}$

- A Test for Independence of events is

$$\boxed{P(B|A) = P(B)}$$ Test for Independence

- Then $P(A \text{ and } B) = P(A) \cdot P(B|A)$

but $\boxed{\text{independence of events}}$ then dictate

$$P(A \text{ and } B) \overset{?}{=} P(A) \cdot P(B)$$

Another Test

EX Determine if these two events are indgpedent

A college student is choosen at random and asked "are you a freshman?"

and

"are you 19 years or younger"

Q: Are these two events: "being a freshman" and "being 19 y.o. or younger" dependent?

or Does one have bering on the other?

Ans: Since someone starting college out of military service, for example, older than 19 but would still be a freshman we can see these events are indgpendant

⊛ Sampling and Independence

For inference to work properly we need to have independence in the sampling process...

Each person selected for polling must have an equally likely chance of being selected

Scenario

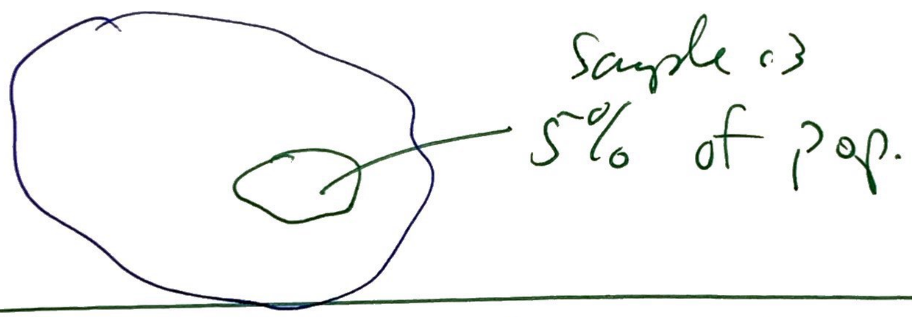
• select a person at random

"4×4" to get up to ask these people our polling questions AND if we choose to skip over them THEN members of the population are NOT equally likley to chosen (NOT an independant sample)

⊛ Sampling w/ Replacemant:

let the "people" polled be told that they may be approached again, if so please also answer the poll again

Sampling $\boxed{\text{w/o replaceplacemant}}$ (the person declines a second poll) can be considred indpent if the sample size is is less than 5% of the size of the population considred.



If your sample is larger than 5% then ask the respondents to answer a second or third poll the same way.

(Sampling with replacement)

Ex A pollster samples 1500 voters from a city of 1 million voters

Q: Can we treat this sample as independant if we assume the polled voter will decline a second interaction with the pollster?

A: $\frac{1500}{1{,}000{,}000} = 0.0015$ or $\underline{\underline{0.15\%}}$ of the pop. of the city so yes, $\overset{\text{L.T.}}{<} 5\%$ the sample can be <u>considered independant</u>

② Compound Probability Chart -