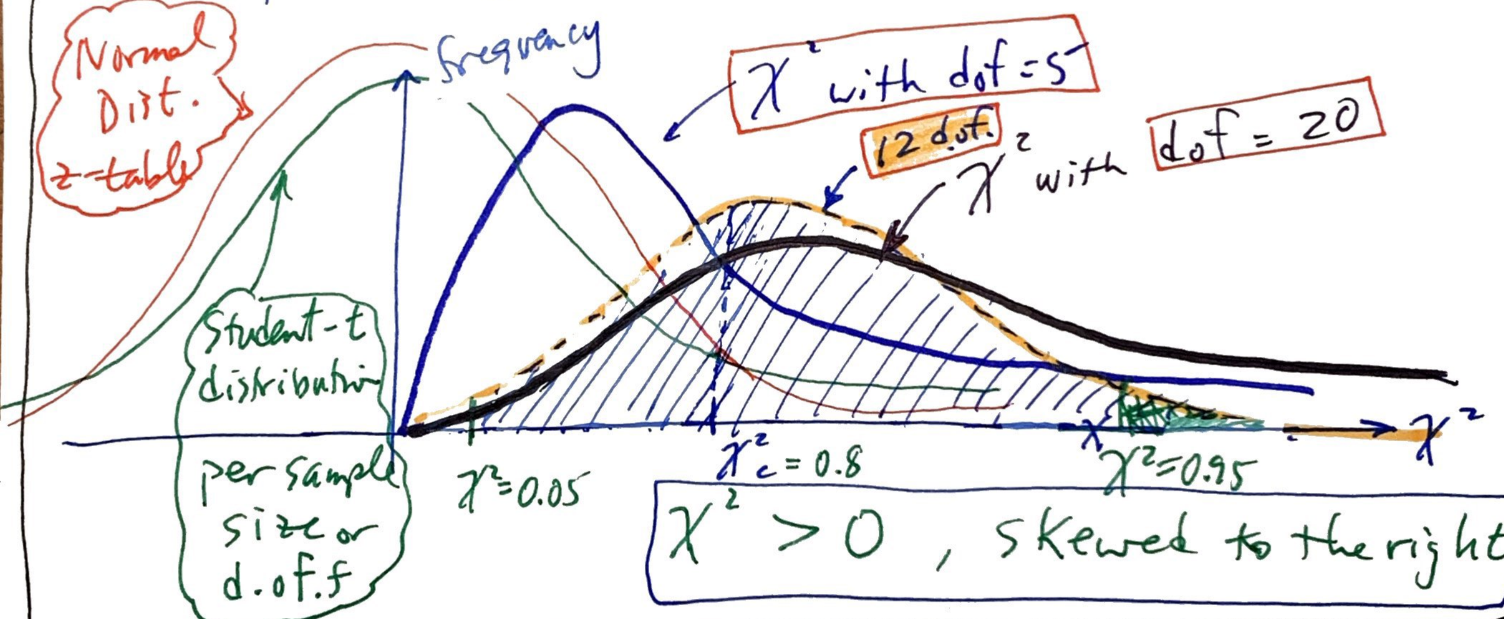
Chapter 12 Testing Qualitative Data

12.1 Goodness of Fit

In this chapter we will use a new table called the $\boxed{\chi^2 \text{ table}}$ (kigh-squared)

This table is not symmetric. The table relies on degrees_of_freedom like the t-table
All areas will be to the right

The probability distribution looks like this



* The χ^2-table

area to the right of χ^2_c

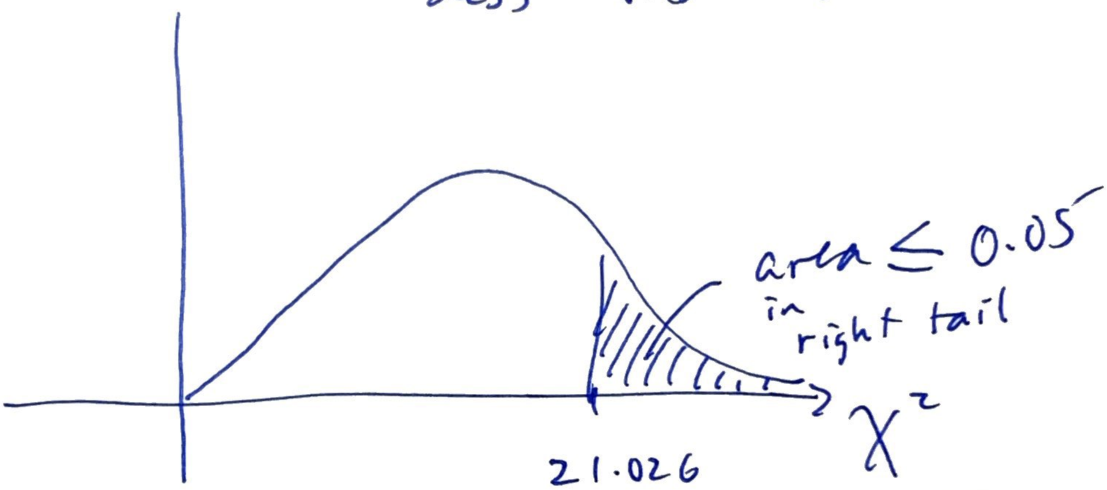
D.of F \ area	0.995	0.99	0.975	0.95	0.90	0.05	0.025	0.01	0.005
12				5.226		21.026			

χ^2_c value $\qquad$ χ^2_c

EX Find the $\alpha = 0.05$ critical value for the χ^2 distribution if dof $= 12$

	0.05
12	$\boxed{21.026}$

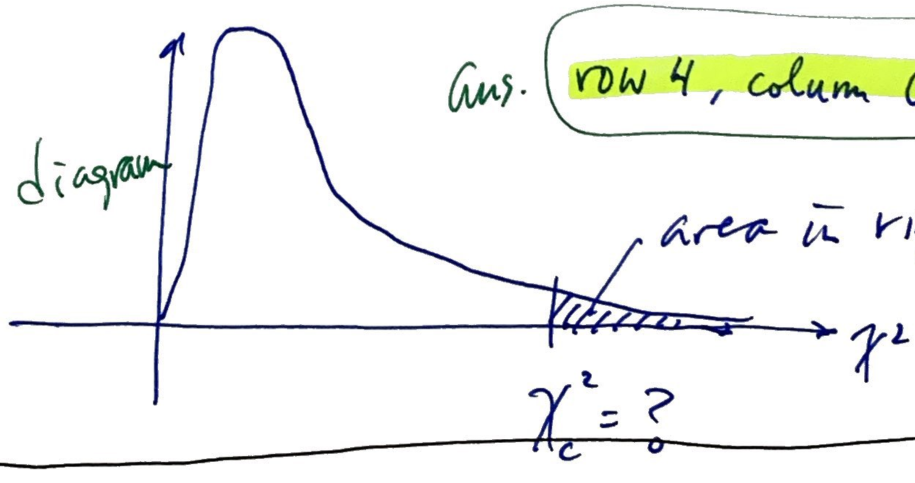
if $\chi^2 > 21.026$ then area to the right is less than $\alpha = 0.05$



EX Find the $\alpha = 0.10$ critical value for the chi-squared distribution if the d.o.f. is 4

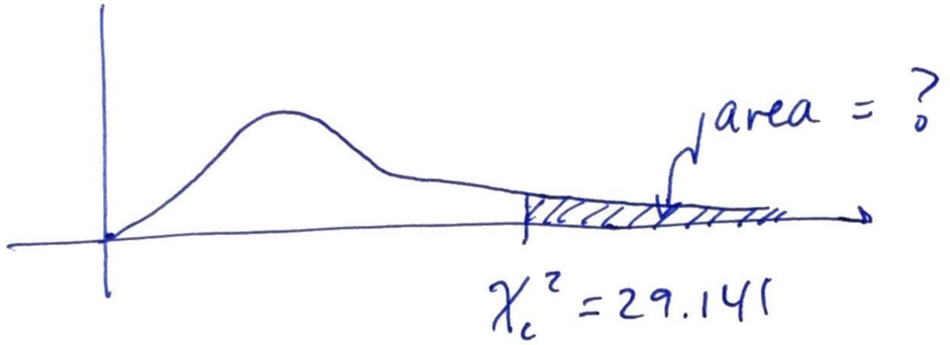
Ans. row 4, column 0.10 $\rightarrow$ $\boxed{7.779 = \chi_c^2}$

diagram



⊛ Reverse look up:

EX Find the area to the right of $\chi_c^2 = 29.14$ if we have 14 degrees of f.

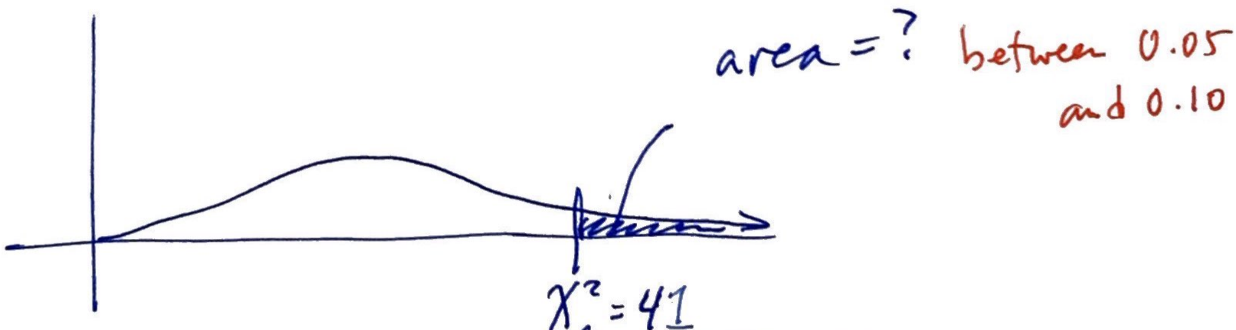


Here we look into the body of the table on

	$\alpha = 0.01$
row 14 :	29.141

so area to the right of $\chi_c^2 = 29.141$ is $\underline{\underline{0.01}}$

EX Find the area to the right of $\chi^2_c = 41$ if we have 30 d.o.f.



	...	0.10	0.05	0.025	...
row 30	...	40.256	43.773	46.979	...

41.0

we only have a range

The area to the right of $\chi^2 = 41.0$ is between 0.10 and 0.05

⊛ Goodness-of-Fit Tests

Chpt 9

we want to determine if a coin is fair.
We could toss it a number of times and compute $\hat{p}$, the proportion of heads (two choices)

$H_0: p = 0.5$; $H_A: p \neq 0.5$

we can take some data, toss a coin a bunch of times come up with a $\hat{p}$ and perform a single pop Hypothesis Test

Q: How do we test a fair die. There is more than two choices. Possible outcomes is 1, 2, 3, 4, 5, or 6.

X	1	2	3	4	5	6
P	1/6	1/6	1/6	1/6	1/6	1/6

} if die is fair.

• Lets Perform our own test on the die and toss the die 60 times. Lets say Freq. table looks this:

X	1	2	3	4	5	6
observed values	12	7	14	15	4	8

Q: is the die fair. I.E. are the data differences statistical variation OR statistical significance?

we expect on 60 tosses.

X	1	2	3	4	5	6
Expected	10	10	10	10	10	10

Our hypothesis would be come

$$\boxed{\begin{array}{l} H_0 : \quad P_1 = P_2 = P_3 = P_4 = P_5 = P_6 = \frac{1}{6} \\ H_A : \quad P_1 \neq P_2 \neq P_3 \neq P_4 \neq P_5 \neq P_6 \neq \frac{1}{6} \end{array}}$$

Def: Expected Frequencies

$$E_1 = n \cdot p_1 , \quad E_2 = n \cdot p_2 , \quad \cdots \quad E_n = n p_n$$

thus for our fair die

$$E_1 = (60)\frac{1}{6} = 10 , \quad E_2 = (60)\left(\frac{1}{6}\right) = 10 \ \cdots$$

Our tables can be combined to show

	1	2	3	4	5	6 ← $k=6$
Observed	12	7	14	15	4	8
Expected	10	10	10	10	10	10

Def: Test statistic for a Goodness-of-Fit problem

$$\chi^2 = \sum_{i=1}^{\text{"k Outcomes"}} \frac{(\text{Observed}_i - \text{Expected}_i)^2}{E_i}$$

Def: The degree of freedom for the G. of Fit test

$$\text{d. of. freedom} = k - 1$$

k = # of categories

Steps to do for a Goodness-of-fit

0. Conditions: each category must have 5 counts
1. State null and alt Hypothesis
   - Null, H_0, specifies the prob. for each category
   - The alt, H_A, specifies a deviation from the null.
2. Compute expected frequencies
3. Choose an α, significance level.
4. Compute the test statistic

$$\boxed{\chi^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i}}$$

5. Find the critical value from the χ^2 table using the k-1 d.of.f.
   - If χ^2 is greater than (to the right) or equal to the critical value we reject H_0.
   - If χ^2 is Less than the critical value we fail to reject
6. State a conclusion.

EX Use the data of our test to see if our die is fair:

outcome	"1"	"2"	"3"	"4"	"5"	"6"
observed	12	7	14	15	4	8
Expected	10	10	10	10	10	10

Step. 0 : Does each outcome have over 5 value?

Ans: No we only see 4 times we rolled a "5"

((But we will proceed knowing this shortcoming))

Step 1: H_0: all prob. = 1/6

H_A: Some prob. ≠ 1/6

Step 2: Expected Frequencies are shown in the table above.

Step 3: Lets use the std. level of significance $\alpha = 0.05$

Step 4: Test statistic

Category	O	E	O - E	$(O-E)^2$	$\frac{(O-E)^2}{E}$
1	12	10	2	4	4/10 = 0.4
2	7	10	-3	9	9/10 = 0.9
3	14	10	4	16	16/10 = 1.6
4	15	10	5	25	25/10 = 2.5
5	4	10	-6	36	36/10 = 3.6
6	8	10	-2	4	4/10 = 0.4
					$\sum = 9.4$

$$\boxed{\chi^2 = 9.4}$$

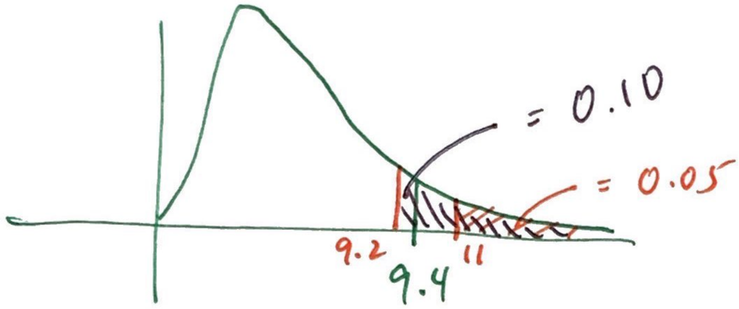
⟹

Step 5 : Row 6-1 = 5 we see that $\chi^2 = 9.4$ falls between

		0.10	0.05	
row 5	. . . .	9.236	11.071	. . . .

9.4

Our area to the right of $\chi^2 = 9.4$ is between 0.10 and 0.05



• So do we reject our null hypothesis?

Our sig. level was $\alpha = 0.05$.

We exceed that area in both boundaries

$\chi^2 = 9.236$ area > 0.10

$\chi^2 = 11.071$ area > 0.05

Step 6: Conclusion : Since our χ^2 score produced area to the right greater than $\alpha = 0.05$ we fail to reject the null and thus we cannot conclude the die is not fair

There is not enough evidence to conclude the die is altered.

Step 7 : Statdisk :

Step 7 : stat disk → analysis → Goodness of Fit → Equal Expected Freq.

- Sig. Level = 0.05 (Keep)
- obs. data in col $\boxed{1}$

$\boxed{\text{Evaluate}}$

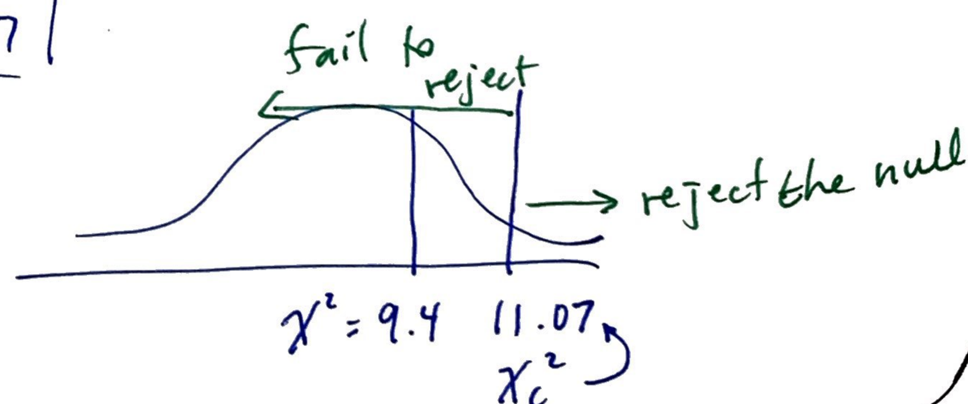
{ Statdisk will realize there are 6 categories and 60 data points thus know Expected outcomes }

Results are: $\chi^2 = 9.40000$ $\boxed{p\text{-value} : 0.09413}$

since p-value is > sig. level of 0.05 $\boxed{\text{we fail to reject}}$

-or- $\chi^2 = 9.4$ smaller than the associate

$\boxed{\chi^2_c = 11.07}$



⊛ Unequal Expected Outcomes

EX A poll shows that of 1155 people the follow results on wether people with higher incomes should be taxed more, less compared to low income people.

Category 2023	Observed	2015 - 2020 Historical
Should pay more	218	18.5%
Should pay a little more	497	39.2%
Pay the same	425	41.2%
Pay less	15	1.1%

Q: Can we conclude that the current percentages in these categories have changed from the historical percentage?

Refine Table and convert % to Expected values (11)

	observed	Expected ← Historical %
① More	218	$1155 \cdot (0.185) = 213.675$
② Little more	497	$1155 \cdot (0.392) = 452.76$
③ Same	425	$1155 \cdot (0.412) = 475.86$
④ Less	15	$1155 \cdot (0.011) = 12.705$

Steps 0: we see each observed categorie is > 5 counts

Step 1: $H_0: P_1 = 0.185,\ P_2 = 0.392,\ P_3 = 0.412,\ P_4 = 0.011$
H_A: the prob. are not the same

Step 3: Told to use $\alpha = 0.05$ $\quad \chi^2 = \sum_i \frac{(O_i - E_i)^2}{E_i}$

Step 4:

$$\chi^2 = \frac{(218 - 213.675)^2}{213.675} + \frac{(497 - 452.76)^2}{452.76} + \frac{(425 - 475.86)^2}{475.86} + \frac{(15 - 12.705)^2}{12.705}$$

$$\boxed{\chi^2 = 10.261}$$

Step 5: χ^2 table, row $= 4\text{cat} - 1 = \underline{\underline{3}}$

	0.025	0.01
row 3	9.348	11.345

10.261

the area to the right of $\chi^2 = 10.261$ is

between $\boxed{0.025 \text{ and } 0.01}$ ⟹

Our α is 0.05 so both these limits are below the α — Reject H_0: 2023 has different %'s

Step 6: We reject the null hyp. since our p-value is somewhere between 0.025 and 0.01 & is less than the significance level of 0.05.

Step 7: Statdisk:

Analysis → Good. of F. → unequal cat.

- Keep $\alpha = 0.05$
- select proportions
- Choose col 1. for observations and col 2 for proportions

col 1	col 2
218	0.185
497	0.392
425	0.412
15	0.011

Evaluate

Results: 4 cat. 3 D.of.F.

$\chi^2 = 10.26080$

Critical χ^2 associated with $\alpha = 0.05$ is 7.81474

{our χ^2 is beyond 7.8 so reject H_0}

p-value = 0.01647 We used the table to estimate p-value between 0.025 & 0.01