11.1 Two ind. groups H.T. on means σ-unknown

①

Ex A new postsurgical treatment was compared to the std treatment for incission recovery

- New Treatment : 12, 13, 15, 19, 20, 21, 24 ($n = 7$)
- old Treatment : 18, 23, 24, 30, 32, 35, 39 ($n = 7$)

not the same person

need NOT be the same sample sizes!

Q: Can we conclude that the new method improve healing times? $\alpha = 0.05$

Notations :

μ_1 & μ_2 pop. means for group 1 & 2

$\bar{X}_1$ & $\bar{X}_2$ sample means " " "

S_1 & S_2 sample std. deviations for group 1 and 2 $\boxed{\sigma_{n-1}}$

n_1 & n_2 sample sizes for groups 1 & 2

Conditions :

1. SRS in both groups.
2. Samples must be independent from the other group.
3. Also, there must be independence within each group.
4. Each sample must be bell shaped.

   OR

   $n_1 \geq 30$ and $n_2 \geq 30$

⊛ Hypotheses

$H_0 : \mu_1 = \mu_2$ ie $(\mu_1 - \mu_2 = 0)$

$H_A : \mu_1 - \mu_2 < 0$ left-tail ← one tail header

$\neq 0$ two-tail ← two-tail header

> 0 right-tail ← one-tail header

claim vs. null: $\boxed{\mu_1 < \mu_2 \quad \mu_1 \neq \mu_2 \quad \mu_1 > \mu_2}$

⊛ Test Statistic:

$$t_c = \frac{(\bar{x}_1 - \bar{x}_2) - \overbrace{(\mu_1 - \mu_2)}^{=0 \text{ null}}}{SE}$$

$$SE = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

$SE_{1\,pop} = \sqrt{\frac{s^2}{n}}$

⊛ sig. level

- $\alpha = 0.01$ (99%), 0.05 (95%), etc.
- one vs two tail header
- row (degree of freedom)

= ? ← messy formula

$$\boxed{\text{row} = \min(n_1 - 1, n_2 - 1)}$$

⊛ Critical t-value:

Row → • DoF $\min(n_1 - 1, n_2 - 1)$ } estimates only

Header → • One vs. two tail

Col → • α-level

$\Rightarrow t_{crit}$

* Follow the steps!

Ex The interpersonal Reactivity Index is a survey designed to asses empathy.
an example is to what degree does a person feel empathy for <u>people who are less fortunate</u>

Ranges go from 0 (no empathy) to 28 (excessive empathy)
The data is shown below. C

Q: Can we conclude that there is a difference in emptathy scores between men and women?
use $\alpha = 0.05$

Males: 13, 20, 12, 16, 13, 26, 21, 23, 8, 15, 18, 25, 15, 23, 17, 22

Females: 22, 20, 26, 25, 28, 24, 16, 19, 20, 23, 21, 23, 15, 26, 19, 25

$n_1 = 16, \quad n_2 = 16$

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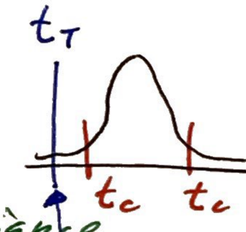
* ~~The Steps~~
((see worksheet))

* Statdisk:
- Data Editor
  - males → col 1
  - females → col 2

- Analysis → H. Testing → Two Ind. Samples
- "Use data", $\alpha = 0.05$
  - Sample 1 = col 1 (male)
  - Sample 2 = col 2 (female)
  - method of analysis: Unequal variance

plot



Results:

$t_c = \pm 2.05092$, DOF $= 27.3$, $t_{Test} = -2.971$

Assumptions

4

Proportions (z)

- **One sample**
  1. Individuals are independent. — 1. SRS and $n < 10\%$ of the population.
  2. Sample is sufficiently large. — 2. Successes and failures each ≥ 10.
- **Two Groups**
  1. Groups are independent. — 1. (Think about how the data were collected.)
  2. Data in each group are independent. — 2. Both are SRSs and $n < 10\%$ of populations OR random allocation.
  3. Both samples are sufficiently large. — 3. Successes and failures each ≥ 10 for both groups.

Means (t)

- **One Sample** (df $= n - 1$)
  1. Individuals are independent. — 1. SRS and $n < 10\%$ of the population.
  2. Population has a Normal model. — 2. Histogram is unimodal and symmetric.*
- **Matched pairs** (df $= n - 1$)
  1. Data are matched. — 1. (Think about the design.)
  2. Individuals are independent. — 2. SRS and $n < 10\%$ OR random allocation.
  3. Population of differences is Normal. — 3. Histogram of differences is unimodal and symmetric.* or n>30
- **Two independent samples** (df from technology) by hand DOF = min(n_1, n_2)
  1. Groups are independent. — 1. (Think about the design.)
  2. Data in each group are independent. — 2. SRSs and $n < 10\%$ OR random allocation.
  3. Both populations are Normal. — 3. Both histograms are unimodal and symmetric.* or both n>30

Distributions/Association (χ^2)

- **Goodness of fit** (df = # of cells − 1; one variable, one sample compared with population model)
  1. Data are counts. — 1. (Are they?)
  2. Data in sample are independent. — 2. SRS and $n < 10\%$ of the population.
  3. Sample is sufficiently large. — 3. All expected counts ≥ 5.
- **Homogeneity** [df $= (r - 1)(c - 1)$; many groups compared on one variable]
  1. Data are counts. — 1. (Are they?)
  2. Data in groups are independent. — 2. SRSs and $n < 10\%$ OR random allocation.
  3. Groups are sufficiently large. — 3. All expected counts ≥ 5.
- **Independence** [df $= (r - 1)(c - 1)$; sample from one population classified on two variables]
  1. Data are counts. — 1. (Are they?)
  2. Data are independent. — 2. SRSs and $n < 10\%$ of the population.
  3. Sample is sufficiently large. — 3. All expected counts ≥ 5.

Regression (t, df $= n - 2$)

- **Association of each quantitative variable** ($\beta = 0$?)
  1. Form of relationship is linear. — 1. Scatterplot looks approximately linear.
  2. Errors are independent. — 2. No apparent pattern in residuals plot.
  3. Variability of errors is constant. — 3. Residuals plot has consistent spread.
  4. Errors have a Normal model. — 4. Histogram of residuals is approximately unimodal and symmetric, or Normal probability plot reasonably straight.*

(*less critical as n increases)

EX: Empathy

CoC MATH 140 Statistics Manual Hypothesis Testing Name "Mean Guy" (5)

STEP 0: **(a) Type of problem and table to use**

- HT for a proportion $\hat{p}$: 1- pop or 2 pop (circle) then use a z-test statistic & z-table
- HT for means μ (σ unknown): 1- pop or 2 pop (circle) then use a t-test & t-table
- HT for matched pairs means μ (σ unknown): 1- pop or 2 pop (circle) then use a z-test
- goodness-of-fit test then use a χ^2-test statistic & χ^2-table
- contingency tests (independence or homogeneity) then use a χ^2-test & χ^2-table

(b) Assumptions	**Justification**
independent groups	assumed - not stated
SRS (both groups)	assumed - not stated
n < 10%	yes
bell shaped samples (or over 30) → statdisk	

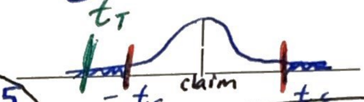
$\bar{x}_1 = 17.31$, $s_1 = 5.12$
$\bar{x}_2 = 22.00$, $s_2 = 3.69$

STEP 1: **State the Hypotheses and test-tail type (if appropriate)**

(a) H_0: $\mu_1 = \mu_2$ H_A: μ_1 >, <, ≠ μ_2 (circle)

(b) Tail: left | right | two-tail (circle)

(c) Sketch the tail(s):



t_T $-t_c$ claim t_c

STEP 2: **State the level of significance:** **α = 0.05**
Now look up the critical value in the appropriate table { revealed in STEP 0 (a) }
z_c or t_c or X^2 (circle) = 2.131 (row 15, Two tails, 0.05 column)

STEP 3: **Compute the test statistic.** {for contingency tests Exp Val = (Row Total)(Col Total) / Grand Total }

proportions → means

(a) SE Formula $\sqrt{\frac{p_0 q_0}{n}}$ $\sqrt{\frac{\hat{p}\hat{q}}{n_1} + \frac{\hat{p}\hat{q}}{n_2}}$, $\hat{p} = \frac{y_1 + y_2}{n_1 + n_2}$ | $\frac{s}{\sqrt{n}}$ $\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$ $\frac{s_d}{\sqrt{n}}$ (circle one):

$$SE = \sqrt{\frac{5.12^2}{16} + \frac{3.69^2}{16}} = \sqrt{1.6384 + 0.851} = \sqrt{2.489}$$

SE Value = 1.578

(b) test statistic = $\frac{\text{sample data} - \text{pop claim}}{SE}$, For tables use $\sum \frac{(Obs - Exp)^2}{Exp}$

z_{test} or t_{test} or $X^2{}_{test}$ (circle) = $\frac{(17.31 - 22.00) - 0}{1.578} = \frac{-4.69}{1.578} =$

test statistic = $\boxed{-2.97}$ take absolute value

STEP 4: **Compare the test statistic to the critical value:**
the test-statistic is < > (circle) than the critical value of -2.131

STEP 5: **We therefore Reject | Fail-to-reject (circle) the** null

STEP 6: **State a conclusion:** or "fail to reject the claim" of there being a diff'nce

The evidence (data) supports the claim that there is a difference between men and womens empathy scores.

t_c

TABLE A-3 *t* Distribution: Critical *t* Values

	Area in One Tail				
	0.005	0.01	0.025	$\alpha =$ 0.05	0.10
Degrees of Freedom	**Area in Two Tails**				
	0.01	0.02	0.05	0.10	0.20
1	63.657	31.821	12.706	6.314	3.078
2	9.925	6.965	4.303	2.920	1.886
3	5.841	4.541	3.182	2.353	1.638
4	4.604	3.747	2.776	2.132	1.533
5	4.032	3.365	2.571	2.015	1.476
6	3.707	3.143	2.447	$\boxed{1.943}$	1.440
7	3.499	2.998	2.365	1.895	415
8	3.355	2.896	2.306	1.860	1.397
9	3.250	2.821	2.262	1.833	1.383
10	3.169	2.764	2.228	1.812	1.372
11	3.106	2.718	2.201	1.796	1.363
12	3.055	2.681	2.179	1.782	1.356
13	3.012	2.650	2.160	1.771	1.350
14	2.977	2.624	2.145	1.761	1.345
15	2.947	2.602	2.131	1.753	1.341
16	2.921	2.583	2.120	1.746	1.337
17	2.898	2.567	2.110	1.740	1.333
18	2.878	2.552	2.101	1.734	1.330
19	2.861	2.539	2.093	1.729	1.328
20	2.845	2.528	2.086	1.725	1.325
21	2.831	2.518	2.080	1.721	1.323
22	2.819	2.508	2.074	1.717	1.321
23	2.807	2.500	2.069	1.714	1.319
24	2.797	2.492	2.064	1.711	1.318
25	2.787	2.485	2.060	1.708	1.316
26	2.779	2.479	2.056	1.706	1.315
27	2.771	2.473	2.052	1.703	1.314
28	2.763	2.467	2.048	1.701	1.313
29	2.756	2.462	2.045	1.699	1.311
30	2.750	2.457	2.042	1.697	1.310
31	2.744	2.453	2.040	1.696	1.309
32	2.738	2.449	2.037	1.694	1.309
34	2.728	2.441	2.032	1.691	1.307
36	2.719	2.434	2.028	1.688	1.306
38	2.712	2.429	2.024	1.686	1.304
40	2.704	2.423	2.021	1.684	1.303
45	2.690	2.412	2.014	1.679	1.301
50	2.678	2.403	2.009	1.676	1.299
55	2.668	2.396	2.004	1.673	1.297
60	2.660	2.390	2.000	1.671	1.296
65	2.654	2.385	1.997	1.669	1.295
70	2.648	2.381	1.994	1.667	1.294
75	2.643	2.377	1.992	1.665	1.293
80	2.639	2.374	1.990	1.664	1.292
90	2.632	2.368	1.987	1.662	1.291
100	2.626	2.364	1.984	1.660	1.290
200	2.601	2.345	1.972	1.653	1.286
300	2.592	2.339	1.968	1.650	1.284
400	2.588	2.336	1.966	1.649	1.284
500	2.586	2.334	1.965	1.648	1.283
750	2.582	2.331	1.963	1.647	1.283
1000	2.581	2.330	1.962	1.646	1.282
2000	2.578	2.328	1.961	1.646	1.282
Large	2.576	2.326	1.960	1.645	1.282

• Sample Sizes:
$n_1 = 7$
$n_2 = 7$
$\min(n_1, n_2) = 7$

• Dof $= n-1$
$= 7-1$
$= \boxed{6}$