Math 104 Sp24 CoC Test 3 Name: KEY

1. **(3.7: 10 pts)** Consider $f(x) = \dfrac{x^2+2x-3}{x^2-4}$. Find the intercepts and horizontal and vertical asymptotes, and then use this knowledge to sketch the graph. {Factor top and bottom first.}

$$f(x) = \frac{(x-1)(x+3)}{(x+2)(x-2)}$$

(i) List the HA Sketch the HA on the graph below:

$\boxed{y=1}$ (deg on top $=1$) $=$ (deg on bottom $=1$)

(ii) List the VA's Sketch them:

$\boxed{x=-2,\ \text{odd}}$ & $\boxed{x=+2,\ \text{odd}}$

(iii) List the Zeros. Plot them:

7

@ $\boxed{x=1}$ and $\boxed{x=-3}$

(iv) Start the graph with a Helper Point (maybe x = 0?):

$f(0) = \dfrac{-3}{-4} = \boxed{3/4}$ $\boxed{(0, 3/4)}$

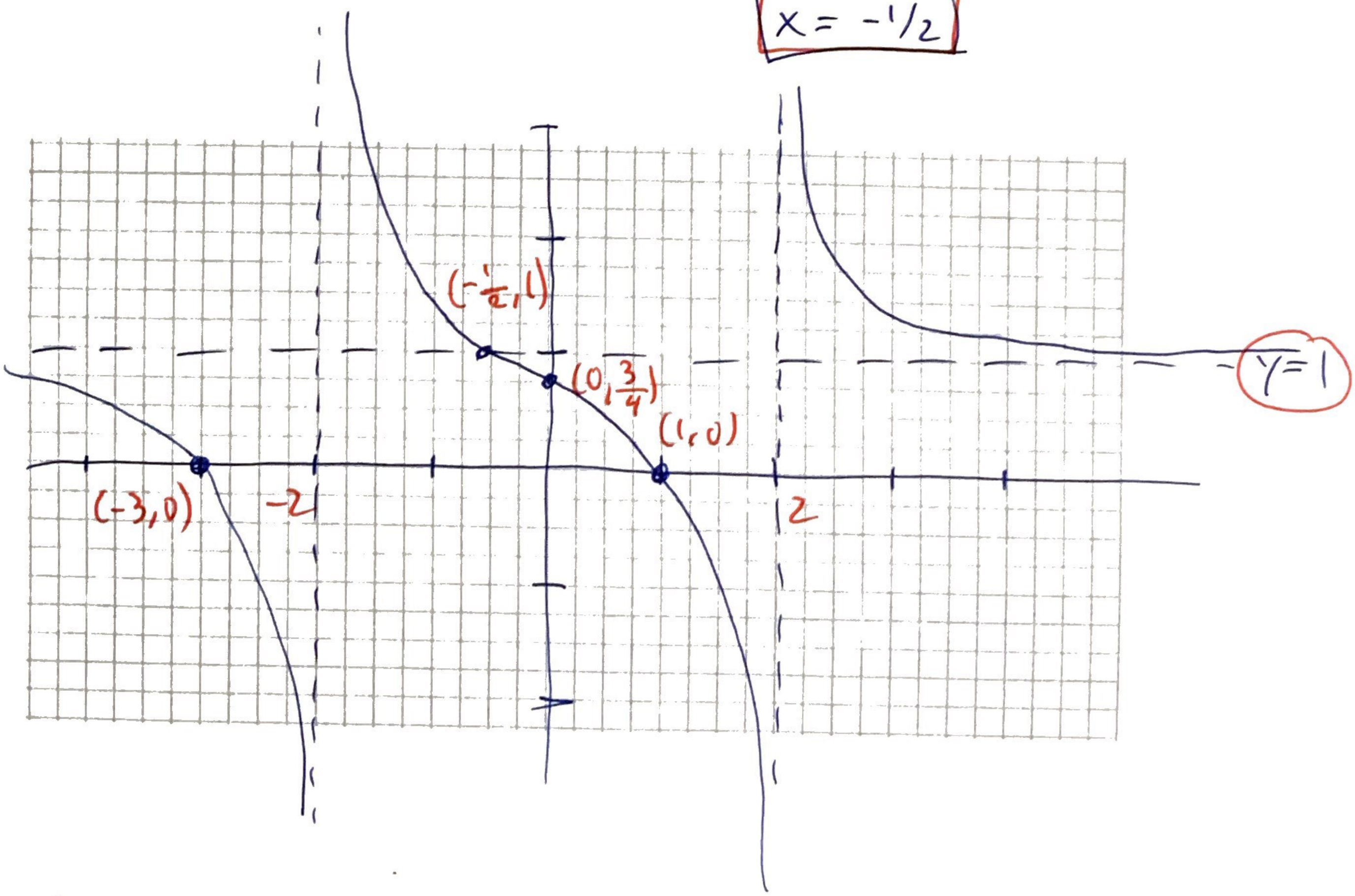
(v) Does the graph cross the HA. crossing? Calculate it by setting *f(x)* to the HA value?

$1 \rightarrow y = \dfrac{x^2+2x-3}{x^2-4} \Rightarrow \cancel{x^2}-4 = \cancel{x^2}+2x-3$

$-4+3 = 2x$

$-1 = 2x$

$\boxed{x = -1/2}$



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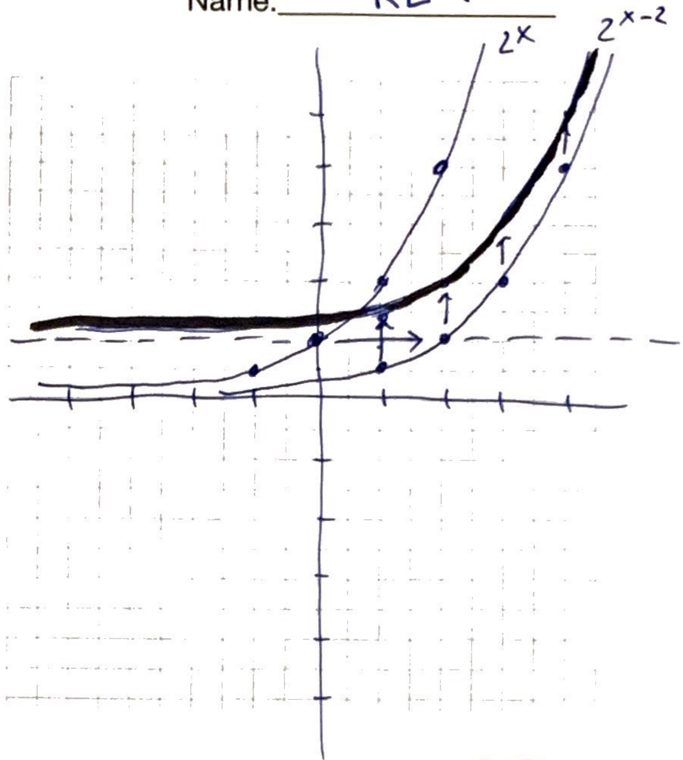
2. **(4.4: 10 pts)** (a) Starting by graphing $h(x) = 2^x$ then graph the intermediate transformations to finally sketch the graph of $g(x) = 2^{x-2} + 1$. Show the asymptotes along the way.



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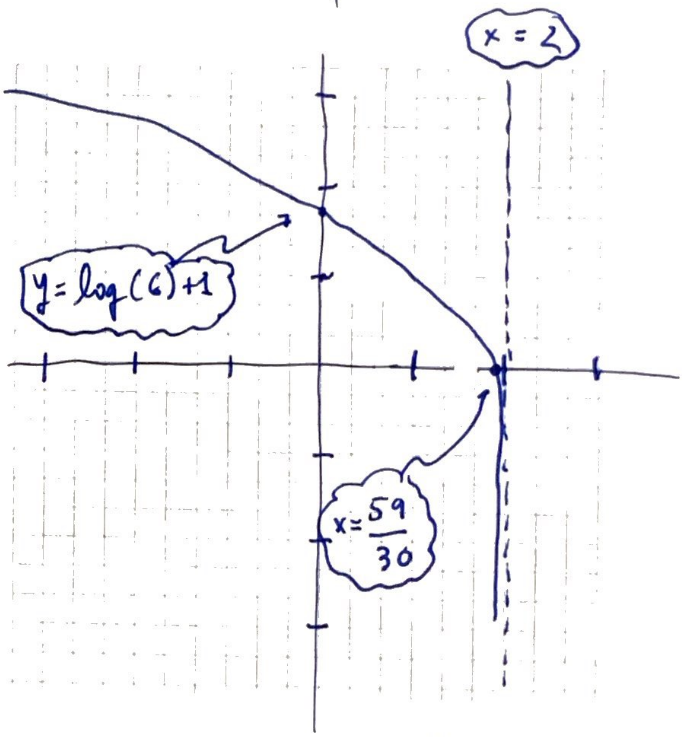
(b) Starting by graphing $g(x) = \log(x)$ then graph the intermediate transformations to finally sketch the graph of $g(x) = \log(6 - 3x) + 1$ Don't for get the asymptotes along the way.

base 10



$\log(3(2 - \cancel{x})) + 1$

$\log(-3(x-2)) + 1$

asymptote: $6 - 3x = 0$

$6 - 0 = 3x$

$\frac{6}{3} = x$ $\boxed{x = 2}$

5

x - zero: $0 = \log(6-3x) + 1$

$-1 = \log(6-3x)$

$10^{-1} = 10^{\log(6-3x)}$

$\frac{1}{10} = 6 - 3x$

$3x = \frac{60}{10} - \frac{1}{10}$

$x = \frac{59}{30}$

Y - zero: $y = \log(6 - 3\cdot 0) + 1$

$y = \log(6) + 1$

$\log(10) = 1$

so $y \approx 1.8$?

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$\boxed{3.}$ **(4.5: 10 pts)**

(a) Rewrite this expression as a sum, difference, or product of logs: $\log\left(x^2y^3\sqrt[3]{x^2y^5}\right)$

$$= \log(x^2) + \log(y^3) + \log\left(\sqrt[3]{x^2y^5}\right)$$

$$= 2\log(x) + 3\log(y) + \frac{1}{3}\log(x^2y^5)$$

$$= 2\log(x) + 3\log(y) + \frac{1}{3}\left[\log(x^2) + \log(y^5)\right]$$

3

$$= 2\log(x) + 3\log(y) + \frac{1}{3}\left[2\log(x) + 5\log(y)\right]$$

$$= \left[2 + \frac{2}{3}\right]\log(x) + \left[3 + \frac{5}{3}\right]\log(y)$$

$$= \boxed{\frac{8}{3}\log(x) + \frac{14}{3}\log(y)}$$

(b) Condense into a single logarithm using the properties: $\log(x) - \frac{1}{2}\log(y) + 3\log(z)$

$$= \log(x) - \log(y^{1/2}) + \log(z^3)$$

$$= \boxed{\log\left(\frac{x \cdot z^3}{y^{1/2}}\right)}$$

3

(c) Rewrite in exponential form: $\log_{15}(a) = b$

$$\boxed{15^b = a}$$

2

(d) Rewrite in logarithmic form: $19^x = y$

$$\boxed{\log_{19}(y) = x}$$

2

4. **(4.6: 10 pts)**

(a) Use logarithms to solve $-5\boxed{e^{9x-8}} - 8 = -62$

isolate

$$-5e^{9x-8} = -62+8$$

$$-5e^{9x-8} = -54$$

$$\boxed{e^{9x-8}} = 54/5$$

$$\ln[\quad] = \ln[\quad]$$

5

$$9x-8 = \ln(54/5)$$

$$\boxed{x = \frac{\ln(54/5)+8}{9}}$$

✓

(b) Solve $\ln(x^2-10) + \ln(9) = \ln(10)$

$$e \cdot e = e$$

$$(x^2-10)\cdot 9 = 10$$

$$x^2-10 = 10/9$$

5

$$x^2 = 10 + \frac{10}{9}$$

$$x^2 = \frac{100}{9}$$

$$\boxed{x = \pm\frac{10}{3}}$$

—OR—

$$\ln(x^2-10) = \ln(10) - \ln(9)$$

$$\ln(x^2-10) = \ln\left(\frac{10}{9}\right)$$

$$x^2-10 = \frac{10}{9}$$

⋮

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$\boxed{5.}$ **(6.1: 10 pts)** Sketch two full periods of $f(t) = 4\cos(2(t + \frac{\pi}{4})) - 1$ State the amplitude, period, and midline. Indicate the maximum and minimum *y*-values and their corresponding *x*-values

(i) Period: $2\pi/2 = \boxed{\pi}$

(ii) Starting point: $\boxed{-\pi/4}$

(iii) P/4: $-\pi/4$

(iv) Lowest common multiple: $\boxed{\pi/4}$

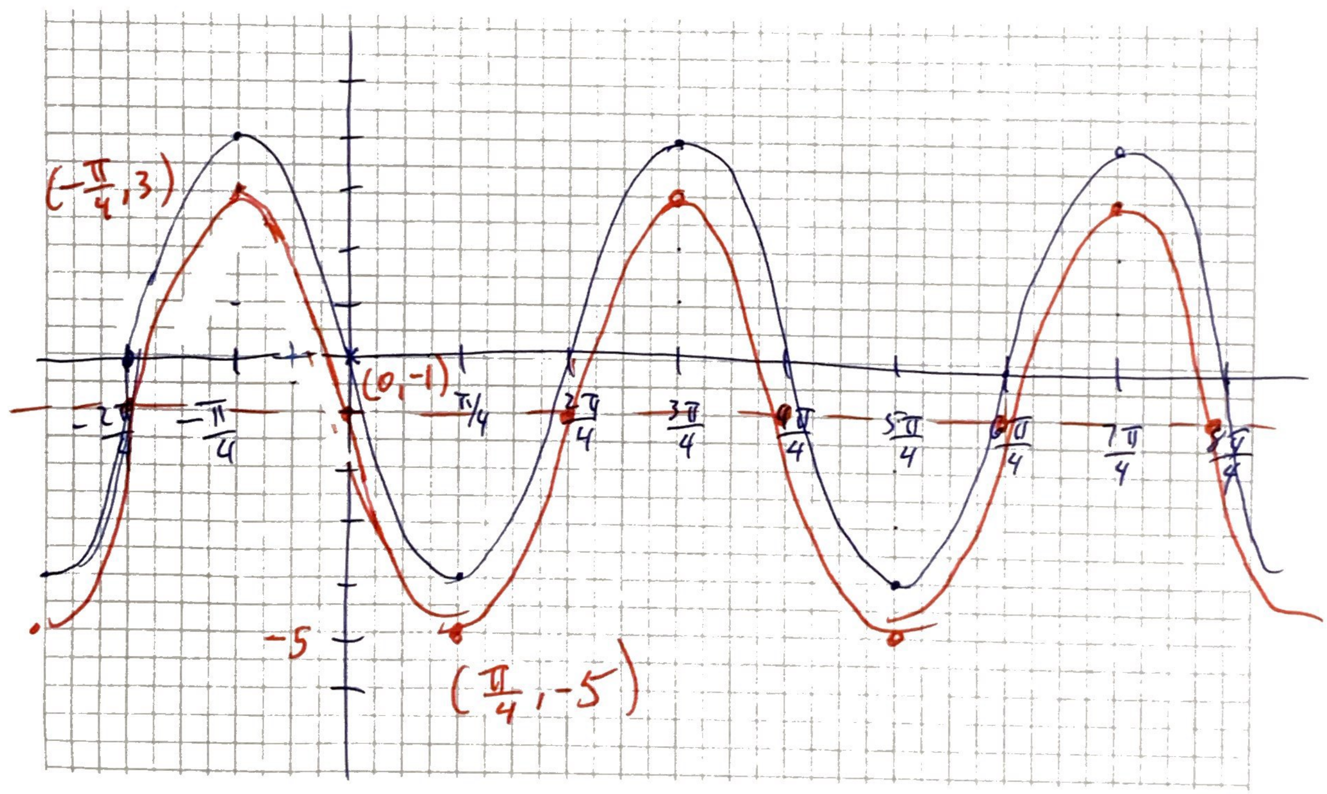
(v) List the location on the t-axis for the 5 key points:

5

High:	$-\pi/4$
Zero:	0
Low:	$\pi/4$
Zero:	$2\pi/4$
High:	$3\pi/4$

(vi) Sketch with appropriate t-axis hash marks and labels:

5



6. **(6.3: 10 pts)**

(a) Determine the amplitude, period, midline, then *write an equation* involving cosine for the function whose graph shown below.

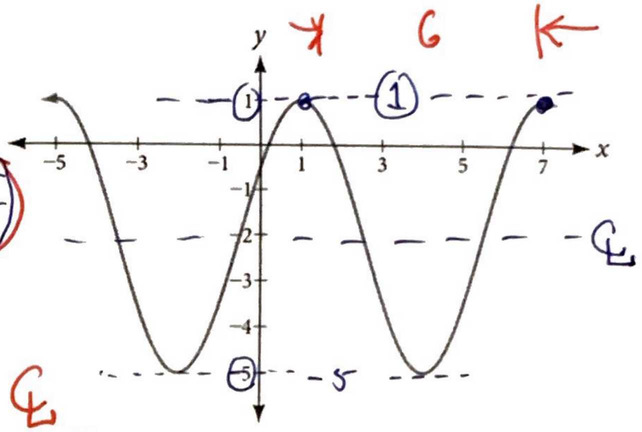
- $2A = 1-(-5) = 6$ so $\boxed{A=3}$
- period: 1 to 7 $\quad 7-1 = 6$

$\boxed{P = \frac{2\pi}{B}} \Rightarrow 6 = \frac{2\pi}{B} \Rightarrow \boxed{B = \frac{2\pi}{6} = \frac{\pi}{3}}$

- Start point: 1 $\Rightarrow$ $\boxed{x = +1}$

6 • Vertical Drop: $\frac{1+(-5)}{2} = \frac{-4}{2} = \boxed{-2}$ ℄

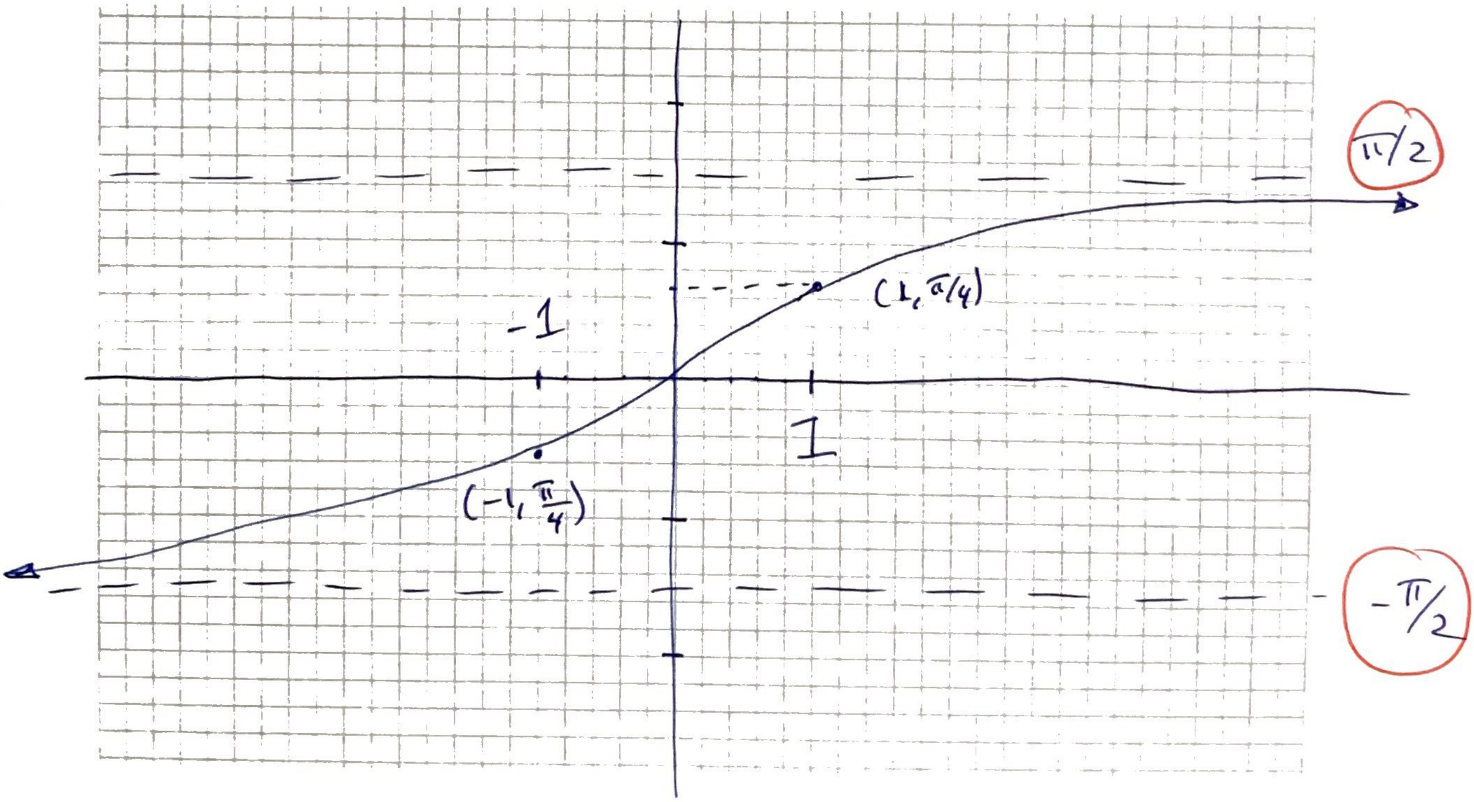
$$\boxed{y(x) = 3\cos\left[\frac{\pi}{3}(x-1)\right]-2}$$



center Line

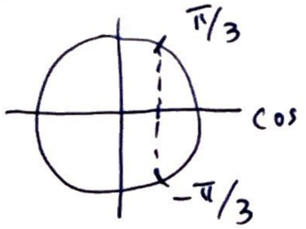
(b) Sketch $f(x) = \tan^{-1}(x)$

4



7. **(7.5: 10 pts)**

(a) List all the families of solutions for the equation: $\cos(\ 2\theta) = \frac{1}{2}$



$(+): 2\theta = \frac{\pi}{3} \Rightarrow \boxed{\theta = \pi/6}$

$(-): 2\theta = -\frac{\pi}{3} \Rightarrow \boxed{\theta = -\pi/6}$

4

- Families

$2\theta = \frac{\pi}{3} + 2\pi k \rightarrow \boxed{\pi/6 + \pi k}$

$2\theta = -\frac{\pi}{3} + 2\pi k \qquad \boxed{-\pi/6 + \pi k}$

$\left(\ \frac{5\pi}{3}\right.$ $\qquad$ ↳ equivalently $\cancel{\frac{11\pi}{6}}$ $\frac{5\pi}{6} + \pi k$

(b) List all the families of solutions for the equation: $\sec(x)\sin(x) - 2\sin(x) = 0$

$\sin(x)\left[\sec(x) - 2\right] = 0$

$\sin(x) = 0$

$\boxed{x = \pi k}$

$\sec(x) - 2 = 0$

$\sec(x) = 2$

$\cos(x) = \frac{1}{2}$

$\boxed{x = \frac{\pi}{3} + 2\pi k}$

$\boxed{x = -\frac{\pi}{3} + 2\pi k}$

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