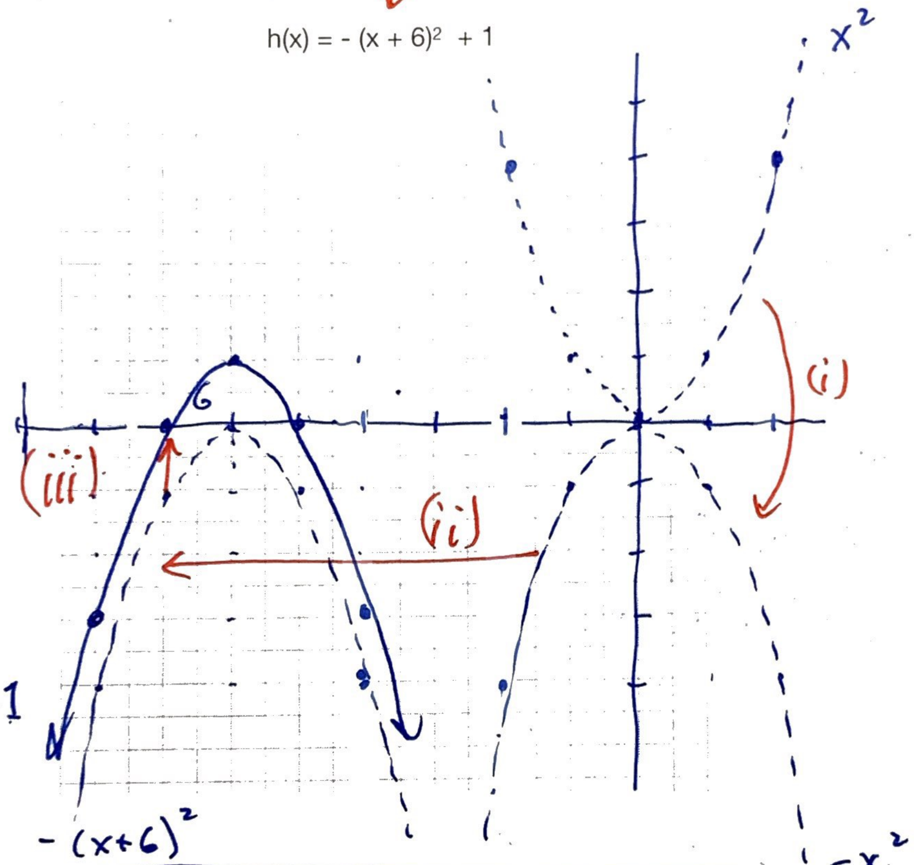
Math 104 Sp24 CoC 90/90 Test 2 Name: KEY

1. (5 pts) Transform $f(x) = x^2$, showing all intermediate transformations, to sketch

start at the inside and work out

$h(x) = - (x + 6)^2 + 1$

5



$h = -(x+6)^2+1$

2. (5 pts) If $f(x) = \sqrt{4x+4}$ and $g(x) = \sqrt{16x-4}$ find fg

"No No" proceedures: Loose all points

$\sqrt{a^2+b^2} \neq \sqrt{a^2} + \sqrt{b^2}$

$\sqrt{a+b} \neq \sqrt{a} + \sqrt{b}$

fg

$= f(x)g(x)$

$= \sqrt{4x+4}\sqrt{16x-4}$

5

$= 2\sqrt{x+1} \cdot 2\sqrt{4x-1}$

$= \boxed{4\sqrt{(x+1)(4x-1)}}$

$= 4 \cdot \sqrt{4x^2+3x-1}$

10

3. (5 pts) If $f(x) = -5x + 7$ and $g(x) = 6x + 4$ find

(a) $(f \circ g)(x)$

$= f(g(x))$

2.5 $= -5(g(x)) + 7$

$= -5(6x+4) + 7$

$= -30x - 20 + 7$

$\boxed{(f \circ g)(x) = -30x - 13}$

(b) $(g \circ f)(x)$

$= g(f(x))$

2.5 $= 6(f(x)) + 4$

$= 6(-5x+7) + 4$

$\boxed{(g \circ f)(x) = -30x + 46}$

4. (5 pts) Find the inverse function of the one-to-one function $f(x) = (x+8)^3$

1. $f(x) = (x+8)^3$
2. $y = (x+8)^3$
3. $x = (y+8)^3$
4. $\sqrt[3]{x} = y + 8$

$y = \sqrt[3]{x} - 8$

5. $\boxed{f^{-1}(x) = \sqrt[3]{x} - 8}$

5

10

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5. (5 pts) Simplify (-6 + 5i) + (7 + 6i) + (3 - 5i)

5

$= (-6+7+3) + (5i+6i-5i)$

$= \boxed{4 + 6i}$

6. (5 pts) Find the product of (8 - 3i)(-2 - 3i)

5

$= (8-3i)(-2) + (8-3i)(-3i)$

$= [-16+6i] + [24i+9i^2]$

$= -16+6i-24i+9i^2$

$= \boxed{-25-18i}$

7. (5 pts) Divide and write the answer in standard form a + bi

$\left(\frac{9+4i}{4-8i}\right) \cdot \left(\frac{4+8i}{4+8i}\right)$

5

$= \dfrac{36+16i+72i+32i^2}{4^2-(8i)^2}$

$= \dfrac{36-32+88i}{16+64}$

$= \dfrac{4+88i}{80}$

$= \dfrac{4}{80} + \dfrac{88}{80}i$

$= \boxed{\dfrac{1}{20} + \dfrac{11}{10}i}$

8. (5 pts) Solve via the Quadratic Formula: $4x^2 - 3x + 1 = 0$

a b c

$x = \dfrac{-(-3) \pm \sqrt{(-3)^2-4(4)(1)}}{2(4)}$

$= \dfrac{3 \pm \sqrt{9-16}}{8}$

5

$= \dfrac{3}{8} \pm \dfrac{\sqrt{-7}}{8}$

$= \boxed{\dfrac{3}{8} \pm \dfrac{\sqrt{7}}{8}i}$

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9. (10 pts) (a) Complete the square for $f(x) = -x^2 - 8x - 7$ and write the function in vertex form $f(x) = a(x-h)^2 + k$

$y = -[x^2 + 8x] - 7$

$y = -[x^2 + 8x + \left(\frac{8}{2}\right)^2 - \left(\frac{8}{2}\right)^2] - 7$

$y = -[x^2 + 8x + \left(\frac{8}{2}\right)] + 4^2 - 7$

$= -[x+4]^2 + 16 - 7$

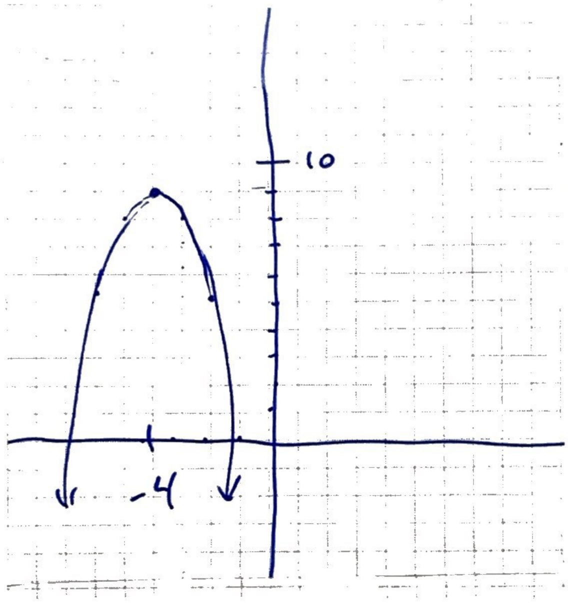
$\boxed{y = -(x+4)^2 + 9}$

4

(b) State the vertex (x, y) = (-4 , 9)

1

(c) Graph



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10. (5 pts) Factor by grouping { factor a common factor out of the first two terms and then factor a common factor out of the last two terms then factor the remaining equation}

$$f(x) = x^3 + 6x^2 - x - 6$$

$= x^2[x+6] - [x+6]$

5

$= (x^2 - 1)[x+6]$

$\boxed{f = (x+1)(x-1)(x+6)}$

1

11. (5 pts) List the zeros and their multiplicity for $f(x) = x^2(x^2-9)(x-3)$

$(x+3)(x-3)$

$= x^2(x+3)(x-3)^2$

zero	mult
-3	1
0	2
3	2

5

12. (5 pts) Divide by Long Division: $\dfrac{x^4+3x^3-6x^2-13x+45}{x^2+2x-3}$

$$
\begin{array}{r}
x^2 + x - 5 \\
x^2+2x-3 \enclose{longdiv}{x^4+3x^3-6x^2-13x+45} \\
-(x^4+2x^3-3x^2) \\
\hline
x^3-3x^2-13x \\
-(x^3+2x^2-3x) \\
\hline
-5x^2-10x+45 \\
-(-5x^2-10x+15) \\
\hline
30
\end{array}
$$

$$= \boxed{x^2+x-5+\frac{30}{x^2+2x-3}}$$

5

13. (5 pts) Divide by Synthetic Division: $\dfrac{x^5+x^3-5}{x+3} = \boxed{x^4-3x^3+10x^2-30x+90-\frac{275}{x+3}}$

	x^5	x^4	x^3	x^2	x^1	x^0
-3	1	0	1	0	0	-5
		-3	9	-30	90	-270
	1	-3	10	-30	90	-275
	x^4	x^3	x^2	x^1	x^0	

5

1

14. (10 pts) Find the zeros of $f(x) = 3x^3 - 22x^2 + 29x + 30$ given that 3 is a zero.

5

$$\begin{array}{r|rrrr} & x^3 & x^2 & x^1 & x^0 \\ 3 & 3 & -22 & 29 & 30 \\ & & 9 & -39 & -30 \\ \hline & 3 & -13 & -10 & 0 \\ & x^2 & x^1 & x^0 & \end{array}$$

$f(x) = 3x^3 - 22x^2 + 29x + 30$

$= \boxed{(x-3)(3x^2 - 13x - 10)}$

3, 1 2, 5

1, 10

5

$= (x-3)(\cancel{3x - 10})(\cancel{x+1})$

$= (x-3)\boxed{(3x+2)(x-5)}$

$x = -\frac{2}{3}$ $x = 5$

$$\boxed{\text{roots are } 3, -\tfrac{2}{3}, 5}$$

$x = \dfrac{-(-13) \pm \sqrt{(-13)^2 - 4(3)(-10)}}{2(3)}$

$= \dfrac{13 \pm \sqrt{169 + 120}}{6}$

$= \dfrac{13 \pm \sqrt{289}}{6}$

messy ...

$= \dfrac{13 \pm 17}{6}$

$= \dfrac{13+17}{6}, \dfrac{13-17}{6}$

$= \dfrac{30}{6}, \dfrac{-4}{6}$

$= 5, -\frac{2}{3}$

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15. (10 pts) Find all the zeros of $f(x) = x^4 + 5x^3 - 4x^2 - 16x - 8$

1 (a) How many zeros will there be: $\boxed{4}$ b/c deg. (power) = 4

1 (b) How many positive real zeros can we expect (explain): $\boxed{1}$

only one sign change

1 (c) How many negative real zeros can we expect (explain): $\boxed{3 \text{ or } 1}$

$f(-x) = x^4 - 5x^3 - 4x^2 + 16x - 8$

(d) List the candidates for any rational zeros that f(x) may have (please order them from smallest to largest):

$\pm \frac{\text{factors of } 8}{\text{factors of } 1} = \pm \frac{8,4,2,1}{1} =$

3

$\Rightarrow \boxed{\{-8,-4,-2,-1,1,2,4,8\}}$

(e) Use synthetic division to find the zeros of f(x) (use the back sheet for more room).

-4	1	5	-4	-16	-8
		-4	-4	32	-64
	1	1	-8	16	-72

4	1	5	-4	-16	-8
		4	36	128	
	1	9	32	112	+ ← upper bound

4

2	1	5	-4	-16	-8
		2	14	20	8
	1	7	10	4	$\boxed{0}$!!!

2	1	7	10	4
		2	18	56
	1	9	28	60 ← upper Bd.

1	1	7	10	4
		1	8	18
	1	8	18	22 ← upper Bd for reduced poly.

-1	1	7	10	4
		-1	-6	-4
	1	6	4	$\boxed{0}$ ✓

(x^2+6x+4)

(2X 2X

1 4

use Q. Formula

$x = \frac{-(6) \pm \sqrt{6^2-4\cdot1\cdot4}}{2\cdot1}$

$= -\frac{6}{2} \pm \frac{\sqrt{20}}{2}$

$= -3 \pm \sqrt{5}$

• zeros:

$\boxed{2, -1, -3-\sqrt{5}, -3+\sqrt{5}}$

10