

## 9.8 Determinants and Cramer's Rule

Recall in 9.7 we found a formula for the inverse of a  $2 \times 2$ . In the denominator we saw a common number  $D$ , which we called the determinant.

**2-D:** If  $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$  then its determinant is  $D = ad - cb$

**EX**  $\begin{vmatrix} 5 & -6 \\ 2 & 1 \end{vmatrix} = 5 \cdot 1 - (2 \cdot -6) = \underline{\underline{17}}$

- The determinant of a matrix is a single scalar number.

Notation:

$$\det(A) \text{ or } \begin{vmatrix} \# & \# \\ \# & \# \end{vmatrix}$$

Not  $\begin{bmatrix} \# & \# \\ \# & \# \end{bmatrix}$  as this is just a matrix

nor  $\det \begin{vmatrix} \# & \# \\ \# & \# \end{vmatrix}$  as this is duplicating meaning.

• For 3-D, if we went through the same process we used to solve for the 2x2 inverse formula,

namely solve for  $j, k, l, m, n, o, p, q, r$

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} j & k & l \\ m & n & o \\ p & q & r \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

we would see that the denominators of  $j, k, l, \dots, p, q, r$  are all the same.

• That denom is the Determinant of a 3x3.

• Fortunately we have a short cut in finding the determinant of a 3x3 matrix

3x3 only:

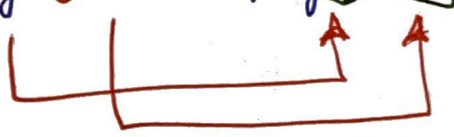
let  $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$  then

(i) multiply up the anti-diagonal, mult. up the diag's

$$\det(A) = [aei + bfg + cdh] - [gec + hfa + idb]$$

So  $\det(A) = \begin{vmatrix} a & b & c & a & b \\ d & e & f & d & e \\ g & h & i & g & h \end{vmatrix}$

(i) Repeat columns one & two

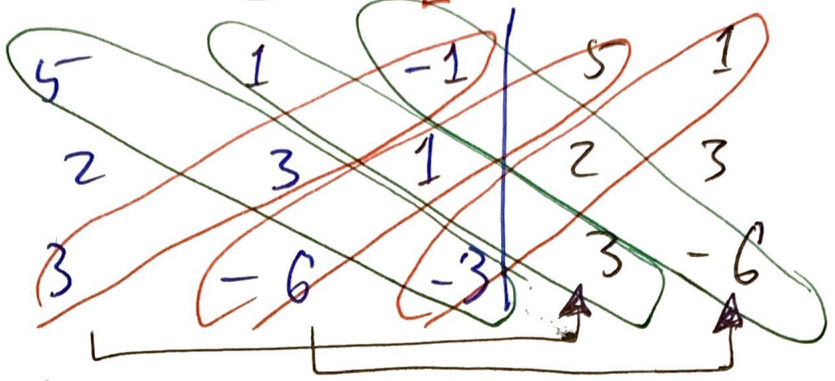




EX

Find the determinant of

$[-45 + 3 + 12] = [-9 - 30 - 6]$



$= [-30] - [-45]$

$= \boxed{15}$

\* Cramer's Rule for 2x2 Systems:

The solution of the system

$ax + by = e$

$cx + dy = f$

is

$$x = \frac{\begin{vmatrix} e & b \\ f & d \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a & e \\ c & f \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}}$$

EX

Solve  $x - 3y = 4$  via Cramer's  
 $2x + y = 1$

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$$x = \frac{\begin{vmatrix} 4 & -3 \\ 1 & 1 \end{vmatrix}}{\begin{vmatrix} 1 & -3 \\ 2 & 1 \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} 1 & 4 \\ 2 & 1 \end{vmatrix}}{\begin{vmatrix} 1 & -3 \\ 2 & 1 \end{vmatrix}}$$

$$= \frac{7}{7} = \frac{-7}{7}$$

$$= 1 = -1$$

$$(x, y) = (1, -1)$$

# ⊛ Cramer's Rule for 3-D System

(5)

The solution of  $ax + by + cz = j$   
 $dx + ey + fz = k$   
 $gx + hy + iz = l$

$$x = \frac{\begin{vmatrix} j & b & c \\ k & e & f \\ l & h & i \end{vmatrix}}{\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a & j & c \\ d & k & f \\ g & l & i \end{vmatrix}}{\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}}, \quad z = \frac{\begin{vmatrix} a & b & j \\ d & e & k \\ g & h & l \end{vmatrix}}{\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}}$$

So we need to evaluate four 3x3 matrices. A lot of work. Use the Gauss-Jordan (most efficient) or the Inverse matrix method (less efficient)

EX

Solve via Cramer's (set-up only)

6

$$-5x + 2y - 4z = -3$$

$$4x - 3y - z = -1$$

$$3x - 3y + 2z = 0$$

$$x = \frac{\begin{vmatrix} -3 & 2 & -4 \\ -1 & -3 & -1 \\ 0 & -3 & 2 \end{vmatrix}}{\begin{vmatrix} -5 & 2 & -4 \\ 4 & -3 & -1 \\ 3 & -3 & 2 \end{vmatrix}}$$

$$y = \frac{\begin{vmatrix} -5 & -3 & -4 \\ 4 & -1 & -1 \\ 3 & 0 & 2 \end{vmatrix}}{\text{ditto}}$$

$$z = \frac{\begin{vmatrix} -5 & 2 & -3 \\ 4 & -3 & -1 \\ 3 & -3 & 0 \end{vmatrix}}{\text{ditto}}$$