$\boxed{9.6}$ # Gauss-Jordan Elimination

⊛ Augmented matrices for systems

we get tired of writing the x, y, z so we arrange the coefficients of x, y, z into a matrix called the <u>augmented matrix</u>:

$\boxed{Ex}$

$$\left.\begin{array}{l} 3x - 2y + 5z = 21 \\ 5x + 4y = 37 \\ x - 2y - 5z = 5 \end{array}\right\} \longrightarrow \left(\begin{array}{ccc|c} 3 & -2 & 5 & 21 \\ 5 & 4 & 0 & 37 \\ 1 & -2 & -5 & 5 \end{array}\right)$$

- we can use all the same row ops we did for the full eqns since the aug. matrix are those coefficients.

⊛ Strategy of Gaussian Elimination

$$\left(\begin{array}{ccc|c} \# & \# & \# & \# \\ \# & \# & \# & \# \\ \# & \# & \# & \# \end{array}\right) \xrightarrow{\text{peform row ops}} \left(\begin{array}{ccc|c} \# & \# & \# & \# \\ 0 & \# & \# & \# \\ 0 & 0 & \# & \# \end{array}\right)$$

- we then write the matrix as a system with x, y, z and = signs and Back substitute

⊛ Gauss-Jordan Elimination

we proceed to obtain this form

$$\xrightarrow{\text{row ops}} \left(\begin{array}{ccc|c} \# & 0 & 0 & \# \\ 0 & \# & 0 & \# \\ 0 & 0 & \# & \# \end{array}\right) \longrightarrow \left\{\begin{array}{l} \#x = \# \\ \#y = \# \\ \#z = \# \end{array}\right\} \longrightarrow \boxed{\begin{array}{l} x = \\ y = \#/ \\ z = \# \end{array}}$$

⊛ Elementary Row Ops

(i) Swap rows: $\boxed{r_i \longleftrightarrow r_j}$

(ii) mult. a row by a const: $\boxed{a(r_i) \to (ar_i)}$

(iii) mult. a row by a constant and add to another row, replacing the latter row with the results.

$$\boxed{a(r_i) + r_j \to r_j}$$

Ex Solve

$$\begin{aligned} 3x + y - 2z &= -7 \\ 2x + 2y + z &= 9 \\ -x - y + 3z &= 6 \end{aligned}$$

(i) augment

$$\left(\begin{array}{ccc|c} 3 & 1 & -2 & -7 \\ 2 & 2 & 1 & 9 \\ -1 & -1 & 3 & 6 \end{array}\right) \text{ swap}$$

(ii) row opers.

$$\rightarrow \left(\begin{array}{ccc|c} -1 & -1 & 3 & 6 \\ 2 & 2 & 1 & 9 \\ 3 & 1 & -2 & -7 \end{array}\right) *2\,;\ 3$$

$$\left(\begin{array}{ccc|c} -1 & -1 & 3 & 6 \\ 0 & 0 & 7 & 21 \\ 0 & -2 & 7 & 11 \end{array}\right) \rightarrow \left(\begin{array}{ccc|c} -1 & -1 & 3 & 6 \\ 0 & -2 & 7 & 11 \\ 0 & 0 & 7 & 21 \end{array}\right) \div 7$$

Back Subst. or Jordan Elimin

$$\left(\begin{array}{ccc|c} -1 & -1 & 3 & 6 \\ 0 & -2 & 7 & 11 \\ 0 & 0 & 1 & 3 \end{array}\right) *-7\,;\ *-3 \rightarrow \left(\begin{array}{ccc|c} -1 & -1 & 0 & -3 \\ 0 & -2 & 0 & -10 \\ 0 & 0 & 1 & 3 \end{array}\right) \div -2$$

$$\left(\begin{array}{ccc|c} -1 & -1 & 0 & -3 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 3 \end{array}\right) *1 \rightarrow \left(\begin{array}{ccc|c} -1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 3 \end{array}\right) *-1$$

(iii) eqn. space

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 3 \end{array}\right) \rightarrow \begin{aligned} x &= -2 \\ y &= 5 \\ z &= 3 \end{aligned}$$

(iv) soln.

$$\boxed{(x, y, z) = (-2, 5, 3)}$$