9.5 Matrix Operations

- In 9.6 we will convert systems of linear equations to matrices and then solve the system using an inverse matrix.
- Here in 9.5 we need to understand how matrices work.

(*) A matrix is an array of numbers.

Ex $A = \begin{bmatrix} 1 & 5 & -6 \\ 0 & 14 & -3 \end{bmatrix}$ 2 x 3 matrix (rows, columns)

Ex $B = \begin{bmatrix} 1 & 5 \\ 6 & -3 \end{bmatrix}$ 2x2 square matrix

Ex $C = [4]$ 1x1 matrix

(*) Matrix addition: if the dimensions of A & B match we can add them and do so entry by entry

Ex $A = \begin{bmatrix} 1 & 5 & -6 \\ 0 & 14 & -3 \end{bmatrix}$ & $B = \begin{bmatrix} 3 & 11 & 0 \\ 5 & 6 & -8 \end{bmatrix}$

then $A + B = \begin{bmatrix} 1+3 & 5+11 & -6+0 \\ 0+5 & 14+6 & -3+(-8) \end{bmatrix}$

$= \boxed{\begin{bmatrix} 4 & 16 & -6 \\ 5 & 20 & -11 \end{bmatrix}}$

Scalar multiplication

a scalar multiplied into a matrix results in a matrix of the dimension but whose entries are all multiplied by the scalar

Ex using $A = \begin{bmatrix} 1 & 5 & -6 \\ 0 & 14 & -3 \end{bmatrix}$

then $\boxed{4A = \begin{bmatrix} 4 & 20 & -24 \\ 0 & 56 & -12 \end{bmatrix}}$

Subtraction

$A - B$ is shorthand for $A + (-B)$

Ex If $A = \begin{bmatrix} 1 & -4 \\ 5 & 2 \\ 3 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 11 & 0 \\ -10 & 12 \\ 13 & 3 \end{bmatrix}$

then $A - B = \begin{bmatrix} 1-11 & -4-0 \\ 5-(-10) & 2-12 \\ 3-13 & 0-3 \end{bmatrix} = \boxed{\begin{bmatrix} -10 & -4 \\ 15 & -10 \\ -10 & -3 \end{bmatrix}}$

- Together

Ex $5A - 3B$

$$= \begin{bmatrix} 5(1)-3(11) & 5(-4)-3(0) \\ 5(5)-3(-10) & 5(2)-3(12) \\ 5(3)-3(13) & 5(0)-3(3) \end{bmatrix}$$

$$= \boxed{\begin{bmatrix} -28 & -20 \\ 55 & -26 \\ -24 & 9 \end{bmatrix}}$$

matrix multiplication

Two matrices can be multiplied if the column count on the first matrix is the same as the row count on the second matrix

$$\mathbb{A}_{n\times \underline{\underline{m}}}\ \mathbb{B}_{\underline{\underline{m}}\times p} = \mathbb{C}_{n\times p}$$

must match

Each entry in the 1st row of $\mathbb{A}$ gets multiplied by the corresponding entries in the 1st column of $\mathbb{B}$. their products are summed and occupy the 1st entry of the 1st row of the resultant matrix

Ex

$$\begin{bmatrix} 3 & 6 & 4 \\ -8 & 0 & 12 \end{bmatrix} \begin{bmatrix} 4 & 10 \\ -2 & 6 \\ 5 & 9 \end{bmatrix}$$

$2\times 3 \quad 3\times 2$

2×2

$$= \left[\begin{array}{c|c} 3\cdot 4 + 6(-2) + 4\cdot 5 & 3\cdot 10 + 6\cdot 6 + 4\cdot 9 \\ \hline -8\cdot 4 + 0\cdot(-2) + 12\cdot 5 & -8\cdot 10 \quad 0\cdot 6 \quad 12\cdot 9 \end{array}\right]$$

$$= \begin{bmatrix} 20 & 102 \\ 28 & 28 \end{bmatrix}$$

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1. let $\mathbb{A} = \begin{bmatrix} 6 & 5 \\ 4 & -4 \\ 8 & -2 \end{bmatrix}$ & $\mathbb{B} = \begin{bmatrix} 1 & 2 \\ 3 & 0 \\ 4 & -5 \end{bmatrix}$

Find $2\mathbb{A} - 4\mathbb{B} \equiv 2\mathbb{A} + (-4\mathbb{B})$

$$\begin{bmatrix} 12 & 10 \\ 8 & -8 \\ 16 & -4 \end{bmatrix} + \begin{bmatrix} -4 & -8 \\ -12 & 0 \\ -16 & 20 \end{bmatrix} = \begin{bmatrix} 12-4 & 10-8 \\ 8-12 & -8+0 \\ 16-16 & -4+20 \end{bmatrix} = \begin{bmatrix} 8 & 2 \\ -4 & -8 \\ 0 & 16 \end{bmatrix}$$

2. Multiply $\mathbb{A} = \begin{bmatrix} -2 & 3 & 6 \\ 1 & 4 & 1 \\ 0 & 5 & 0 \end{bmatrix}$ & $\mathbb{B} = \begin{bmatrix} 1 \\ 8 \\ -7 \end{bmatrix}$

3×3 $\quad$ $3 \times 1 = 3 \times 1$ $\begin{bmatrix} \cdot \\ \cdot \\ \cdot \end{bmatrix}$

i.e. Find $\mathbb{C} = \mathbb{A}\mathbb{B}$

3. Using $\mathbb{A}$ & $\mathbb{B}$ in #2 Find $\mathbb{C} = \mathbb{B}\mathbb{A}$.

$$\begin{bmatrix} 1 \\ 8 \\ -7 \end{bmatrix} \begin{bmatrix} -2 & 3 & 6 \\ 1 & 4 & 1 \\ 0 & 5 & 0 \end{bmatrix}$$

$3 \times \boxed{1 \quad 3} \times 3$

No

Not Defined

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$\boxed{\#4}$ multiply

$$\begin{bmatrix} 1 & 2 & 3 \\ -4 & 0 & 5 \\ 10 & -7 & 6 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$

$\boxed{\#5}$ multiply

$$\underbrace{\begin{bmatrix} 1 & 2 & 3 \\ -4 & 0 & 5 \\ 10 & -7 & 6 \end{bmatrix}}_{A} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 0 \\ -3 \end{bmatrix}$$

$$A \cdot \vec{x} = \vec{b}$$

$\boxed{\#6}$ multiply

$$\begin{bmatrix} 3 \\ -2 \\ 4 \end{bmatrix} \begin{bmatrix} 7 & 5 & -9 \end{bmatrix} = \begin{bmatrix} 3\cdot 7 & 3\cdot 5 & 3(-9) \\ -2\cdot 7 & -2\cdot 5 & (-2)(-9) \\ 4\cdot 7 & 4\cdot 5 & 4\cdot(-9) \end{bmatrix} = \begin{bmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix}$$

$3\times 1 \quad 1\times 3 \quad = 3\times 3$

$\boxed{\#7}$ multiply

$$\begin{bmatrix} 7 & 5 & -9 \end{bmatrix} \begin{bmatrix} 3 \\ -2 \\ 4 \end{bmatrix} = \begin{bmatrix} 7\cdot 3 + 5(-2) + (-9)4 \end{bmatrix}$$

$$= \begin{bmatrix} 21 - 10 - 36 \end{bmatrix}$$

$$= \begin{bmatrix} -46 + 21 \end{bmatrix}$$

$$= \begin{bmatrix} -25 \end{bmatrix}$$

$1\times 3 \quad 3\times 1 \quad = 1\times 1$