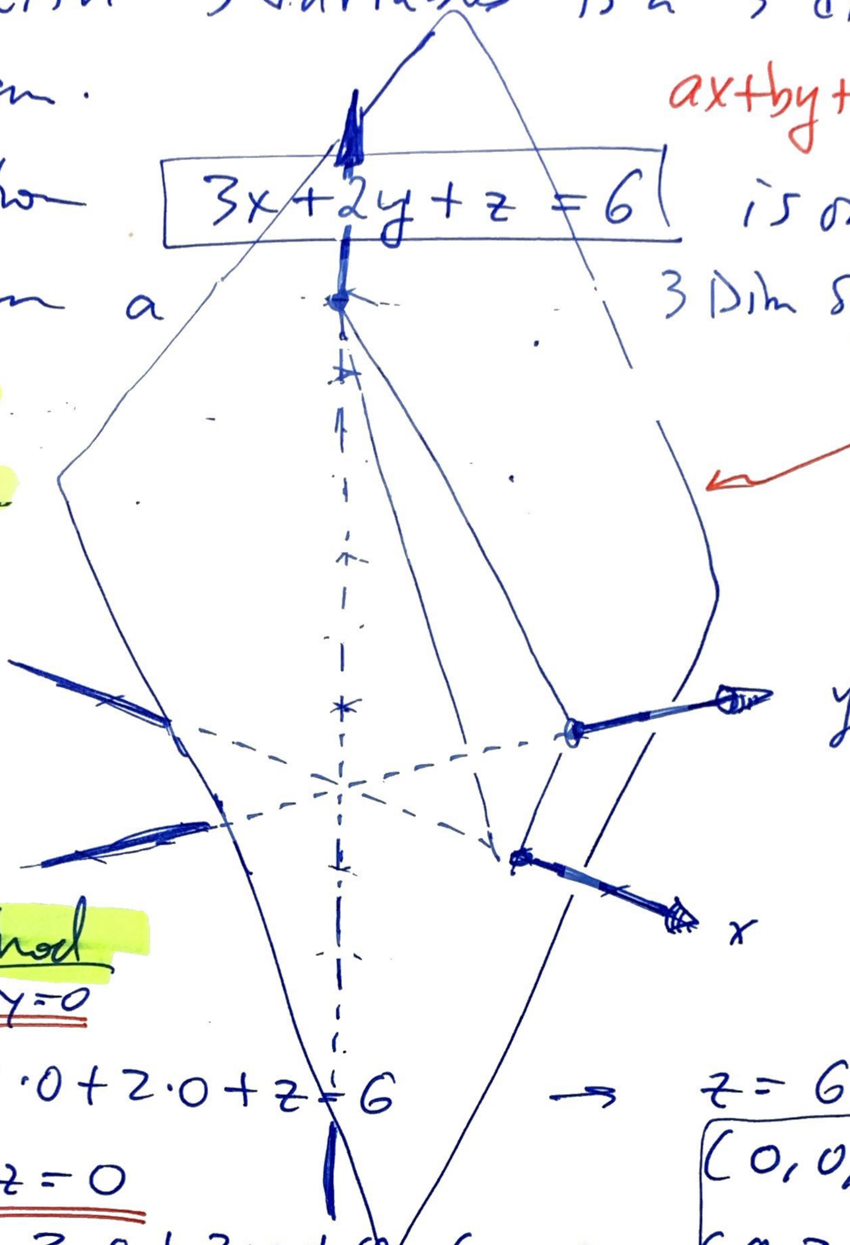
$\boxed{9.2}$ 3-Dim Systems

- A system with 3 variables is a 3-dimensional system.

$ax+by+cz=d$

- One equation $\boxed{3x+2y+z=6}$ is one such eqn. from a 3 Dim System.

- This eqn describes... a plane

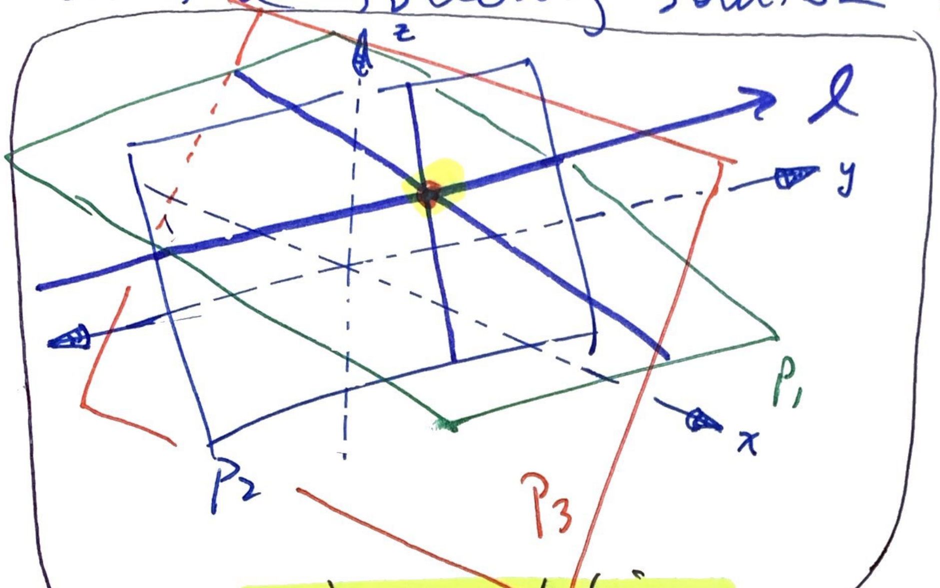


single plane

- Cover Up Method
  - let $x=0,\ y=0$
    then $3\cdot 0+2\cdot 0+z=6 \rightarrow z=6$
  - let $x=0,\ z=0$
    then $3\cdot 0+2y+0=6 \rightarrow$
  - let $y=0,\ z=0$
    then $3x+2\cdot 0+0=6 \rightarrow$

$$\boxed{\begin{array}{c}(0,0,6)\\(0,3,0)\\(2,0,0)\end{array}}$$

• The solution of a 3-Dim system with 3 eqn has the following solution configurations..

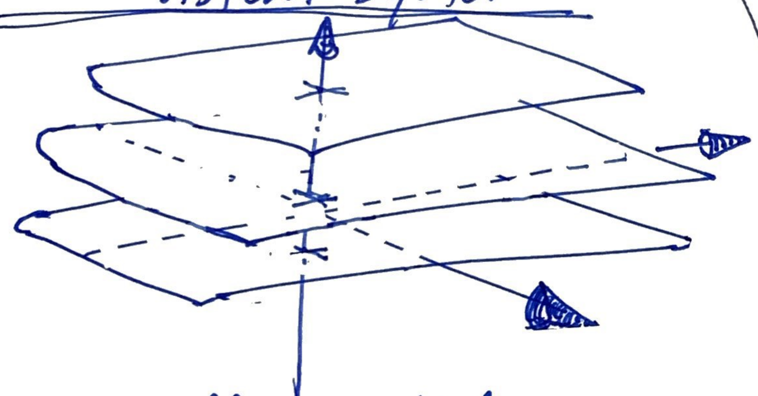


unique solution

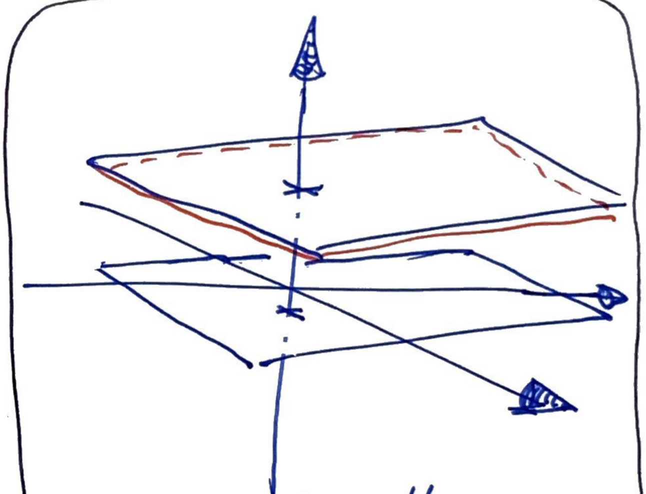
|| = parallel

11 eleven

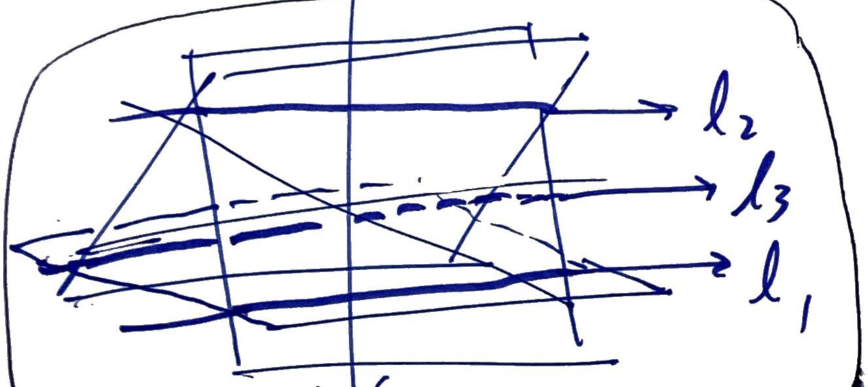
* Inconsistent Systems



all parallel
No - solution

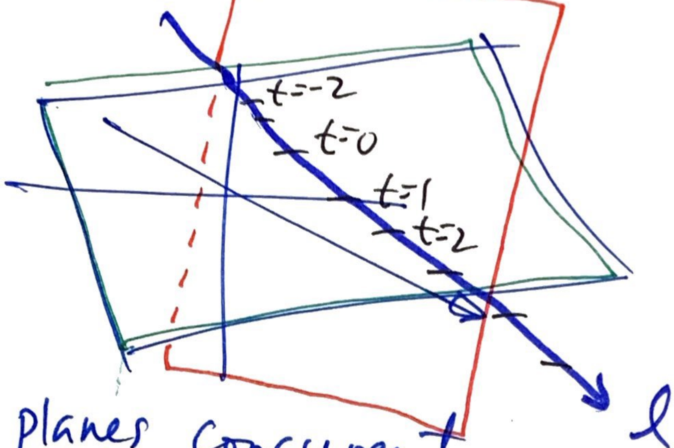


one plane || to two concurent planes
No solution



No Solution

* Consistent



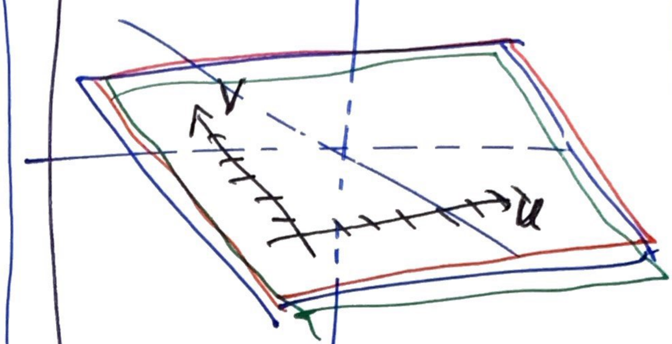
two planes concurrent.

∞-many solutions

(1-Dim Solution)

$\boxed{\text{1 parameter}}$ line

all planes concurrent



∞-many solution

(2-Dim soln)

$\boxed{\text{2 parameters}}$ plane

* Methods of Solutions

- Graphical, not convenient
- Subtitution
- Elimination
- Gaussian
- Gauss-Jordan
- Cramer's Rule
- matrix inverse

EX use substitution to Solve

Solve
$$5x - 2y + 3z = 20$$
$$2x - 4y - 3z = -9 \qquad (3\times3)$$
$$x + 6y - 8z = 21$$

(i) Solve the bottom eqn for x:

$$\boxed{x = 21 - 6y + 8z}$$

$x = 21 - 6(4) + 8(1)$

$x = 21 - 24 + 8$

$\boxed{x = 5}$

(ii) Sub that into the unused eqns (top 2 eqns):

$$\begin{cases} 5(21-6y+8z) - 2y + 3z = 20 \\ 2(21-6y+8z) - 4y - 3z = -9 \end{cases}$$

$$\longrightarrow \begin{cases} -16y + 13z = -51 \\ -32y + 43z = -85 \end{cases} \qquad (2\times2)$$

(iii) Solve the top eqn in the 2x2 for y:

$$y = -\frac{51}{-16} - \frac{13}{-16}z \rightarrow \boxed{y = \frac{13}{16}z + \frac{51}{16}} = \frac{64}{16} = \underline{\underline{4}}$$

(iv) Sub into the unused 2x2 eqn:

$$-32\left[\frac{13z}{16} + \frac{51}{16}\right] + 43z = -85 \qquad (1\times1)$$

$$-26z - 102 + 43z = -85$$
$$17z = 17$$
$$\boxed{z = 1}$$

(v) answer:

$$\boxed{(x, y, z) = (5, 4, 1)}$$

Ex Solve via Elimination

$$\begin{cases} 5x - 2y + 4z = -13 \\ 5x - 8y - 4z = -7 \\ 2x + 3y + 2z = 5 \end{cases}$$

eliminate these

- **Strategy** is to eliminate the x positions of the bottom two eqns. • After that we eliminate the y position of the bottom eqn.

$$\left.\begin{aligned} \# x + \# y + \# z &= \# \\ \# y + \# z &= \# \\ \# z &= \# \end{aligned}\right\} \text{final form}$$

Reverse Substitution

- from here we solve the bottom eqn for z and subst. into the middle to solve for y, then subst. the z & y into the top eqn to get x.

ex Solve via Gaussian Elimination

$$5x - 2y + 4z = -13$$
$$5x - 8y - 4z = -7$$
$$2x + 3y + 2z = 5$$

"Mult. R_1 by -1 & add to R_2 & replace R_2 with the results"

→ in code: $\boxed{-R_1 + R_2 \Rightarrow R_2}$

$$5x - 2y + 4z = -13 \quad * -1$$
$$5x - 8y - 4z = -7 \quad +$$
$$2x + 3y + 2z = 5$$

Top row remains unchanged

$$5x - 2y + 4z = -13$$
$$0x - 6y - 8z = 6$$
$$2x + 3y + 2z = 5$$

skip: $* -2/5$, $+$

use: $* -2$, $* 5$

But to avoid fractions seek least common multiples.

$$-10x + 4y - 8z = 26 \quad * 1$$
$$0x - 6y - 8z = 6$$
$$10x + 15y + 10z = 25$$

$$-10x + 4y - 8z = 26 \quad \div -2$$
$$-6y - 8z = 6 \quad \div -2$$
$$19y + 2z = 51$$

$$5x - 2y + 4z = -13$$
$$3y + 4z = -3$$
$$38y + 4z = 102$$

"By the Book" the lowest common multiple of 3 & 38 is 114

$$5x - 2y + 4z = {}^{-}13$$
$$3y + 4z = -3 \quad * 38$$
$$38y + 4z = 102 \quad * -3$$

120
32

$$5x - 2y + 4z = {}^{-}13$$
$$114y + 152z = -114 \quad * 1$$
$$-114y - 12z = -306$$

$$5x - 2y + 4z = {}^{-}13$$
$$3y + 4z = -3$$
$$140z = -420$$

gone

$\boxed{x = 1}$

$\boxed{y = 3}$

$\boxed{z = -3}$

Back Substitute

• check the top eqn...

$$5x = -13 + 2y - 4z$$
$$= -13 + 2(3) - 4(-3)$$
$$= -13 + 6 + 12$$
$$= -1 + 6$$
$$5x = 5 \quad \div 5$$
$$\boxed{x = 1}$$

$$\boxed{(x, y, z) = (1, 3, -3)}$$

* Check your answer at matrixcalc.org

Math 104 CW 11a Name ______

1. Use Gaussian Elimination to solve

$$\begin{aligned} x - 2y + 3z &= 9 \quad *1\ ;\ *-2 \\ -x + 3y - z &= -6 \quad + \\ 2x - 5y + 5z &= 17 \quad + \end{aligned}$$

$\#x + \#y + \#z = \#$

$\#y + \#z = \#$

$\#z = \#$

$$\begin{aligned} x - 2y + 3z &= 9 \\ y + 2z &= 3 \quad *1 \\ -y - z &= -1 \end{aligned}$$

$\parallel$

$$\boxed{\begin{aligned} x - 2y + 3z &= 9 \\ y + 2z &= 3 \\ z &= 2 \end{aligned}}$$

Gaussian form

$\parallel$

- Back substitute
  - Bottom: $z = 2$
  - middle: $y = 3 - 2z = 3 - 2(2) = -1$
  - top $x = 9 + 2y - 3z = 9 + 2(-1) - 3(2)$

$x = 9 - 2 - 6$

$x = 1$

$$\boxed{(x, y, z) = (1, -1, 2)}$$

⊛ Gauss–Jordan Elimination

Jordan desired to continue row operations to obtain the form

zero this out now

$$\begin{aligned} \#x \quad\quad\quad &= \# \\ \#y \quad\quad &= \# \\ \#z &= \# \end{aligned}$$

(Continued from Classwork)

Ex In the CW we went from

$$\begin{cases} x - 2y + 3z = 9 \\ -x + 3y - z = -6 \\ 2x - 5y + 5z = 17 \end{cases}$$

to

$$\begin{cases} x - 2y + 3z = 9 \\ \quad\quad y + 2z = 3 \\ \quad\quad\quad\quad z = 2 \end{cases} \quad * -2 \; ; \; * -3$$

• Now instead of back substitution lets zero out the upper triangle of numbers

$$\begin{aligned} x - 2y + 0z &= 3 \\ y + 0z &= -1 \quad\quad *2 \\ z &= 2 \end{aligned}$$

$$\begin{aligned} x + 0y + 0z &= 1 \\ y + 0z &= -1 \\ z &= 2 \end{aligned} \quad \Rightarrow \quad \left.\begin{aligned} x &= 1 \\ y &= -1 \\ z &= 2 \end{aligned}\right\} \quad \boxed{(1, -1, 2)}$$

* Tips • Clean up your system before beginning

Ex

$$0.3x + 0.3y + 0.5z = 0.6$$
$$0.4x + 0.4y + 0.4z = 1.8$$
$$0.4x + 0.2y + 0.1z = 1.6$$

• multiply all rows by 10 to eliminate the decimals

$$\Rightarrow \quad 3x + 3y + 5z = 6$$
$$4x + 4y + 4z = 18 \qquad \div 2$$
$$4x + 2y + z = 16$$

$$\boxed{\begin{aligned} 3x + 3y + 5z &= 6 \\ 2x + 2y + 2z &= 9 \\ 4x + 2y + z &= 16 \end{aligned}} \qquad *-1$$

• make a "one" $-1 \cdot R_2 + R_1 \rightarrow R_1$

pivot

$$x + y + 3z = -3 \qquad *-2 \; ; \; *-4$$
$$2x + 2y + 2z = 9$$
$$4x + 2y + z = 16$$

• Now start Gauss-Jordan Process:

$$x + y + 3z = -3$$
$$-4z = 15$$
$$-2y - 11z = 28$$

swap rows

$$x + y + 3z = -3$$
$$-2y - 11z = 28 \qquad * 4$$
$$0 \qquad -4z = 15 \qquad * -11$$

(10)

$$x + y + 3z = -3$$
$$-8y - 44z = 112$$
$$44z = -165$$

+

$$\Downarrow$$

$$x + y + 3z = -3 \quad * \; 4$$
$$-8y = -53 \quad * \; -1$$
$$-4z = 15 \quad * \; 3$$

$$\Downarrow$$

$$4x + 4y + 12z = -12$$
$$8y = 53$$
$$-12z = 45$$

+

$$\Downarrow$$

$$4x + 4y = 33 \quad * \; -2$$
$$8y = 53$$
$$-4z = 15$$

$$\Downarrow$$

$$-8x - 8y = -66$$
$$8y = 53$$
$$-4z = 15$$

+

$$\Downarrow$$

$$-8x = -13 \longrightarrow x = 13/8$$
$$8y = 53 \longrightarrow y = 53/8$$
$$-4z = 15 \longrightarrow z = -15/4$$

$$\boxed{(x, y, z) = \left(\frac{13}{8}, \frac{53}{8}, \frac{-30}{8}\right)}$$

EX 3x3 with ∞ many one-dim solutions

$$\begin{aligned} x + y + z &= 7 \quad * -3 \,;\ * -1 \\ 3x - 2y - z &= 4 \\ x + 6y + 5z &= 24 \end{aligned}$$

$$\begin{aligned} x + y + z &= 7 \\ -5y - 4z &= -17 \\ 5y + 4z &= 17 \quad + \end{aligned}$$

$$\begin{aligned} x + y + z &= 7 \\ -5y - 4z &= -17 \\ 0z &= 0 \end{aligned}$$

$\leftarrow$ row of zeros

$\Rightarrow$ ∞ many solutions

• let z be the parameter "t" : $\boxed{z = t}$

2nd Row : $-5y - 4t = -17$

$-5y = -17 + 4t$

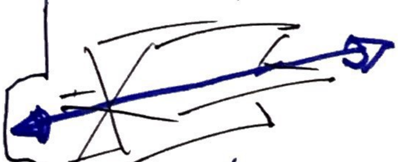
$\boxed{y = \frac{17}{5} - \frac{4}{5}t}$

Top Row : $x + \left(\frac{17}{5} - \frac{4}{5}t\right) + (t) = 7$

$x + \frac{17}{5} + \frac{1}{5}t = 7$

$x = 7 - \frac{17}{5} - \frac{1}{5}t$

$\boxed{x = \frac{18}{5} - \frac{1}{5}t}$



ANS:

$\boxed{(x(t), y(t), z(t)) = \left(\frac{18}{5} - \frac{1}{5}t, \frac{17}{5} - \frac{4}{5}t, t\right)}$ a line in 3D

Desmos 3D

EX 2-parameter ∞ solutions System

$$\begin{aligned} 4x+6y-2z&=8 && \div 2 \\ 6x+9y-3z&=12 && \div 3 \\ -2x-3y+z&=-4 && \div -1 \end{aligned}$$

$$\begin{aligned} 2x+3y-z&=4 && *-1\ ;\ *-1 \\ 2x+3y-z&=4 \\ 2x+3y-z&=4 \end{aligned}$$

$$\begin{aligned} 2x+3y-z&=4 \\ 0x+0y+0z&=0 \\ 0x+0y+0z&=0 \end{aligned}$$

2 rows of zeros the Solution is a 2-Dim Surface (plane)

- let $z=t$ & $y=s$

then top eqn: $2x+3(s)-(t)=4$

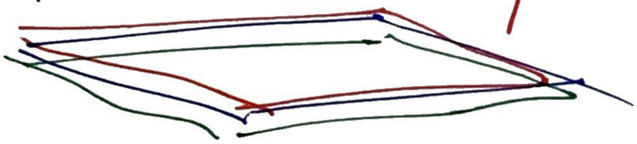
so $x = \frac{4}{2}-\frac{3s}{2}+\frac{t}{2}$

Solution:

$$\boxed{\left(x(t,s),\ y(t,s),\ z(t,s)\right)=\left(2-\tfrac{3}{2}s+\tfrac{1}{2}t,\ s,\ t\right)}$$

since all "t" & "s" are power = 1 this surface is a plane

* all three planes are concurrent *



9.2 is completed