

9.1

2-Dimensional Linear Systems

* many many applications have more than two variables or more than one eqn.

• In this section we address two equations each of which has two variables

• Basic Form

$$ax + by = c$$

$$dx + ey = f$$

• Mathematically

$$a_{11}x_1 + a_{12}x_2 = b_1$$

$$a_{21}x_1 + a_{22}x_2 = b_2$$

{ math 214 Linear Algebra }

• when the power on x_i is 1 then these equations are linear eqns.

EX

$$2x^2 + 5y = 6$$

$$-x + \sin(y) = -3$$

} Non-Linear eqns,

⊗ Methods of Solving 2×2 systems: ^{← eqns} ^{← unknowns}

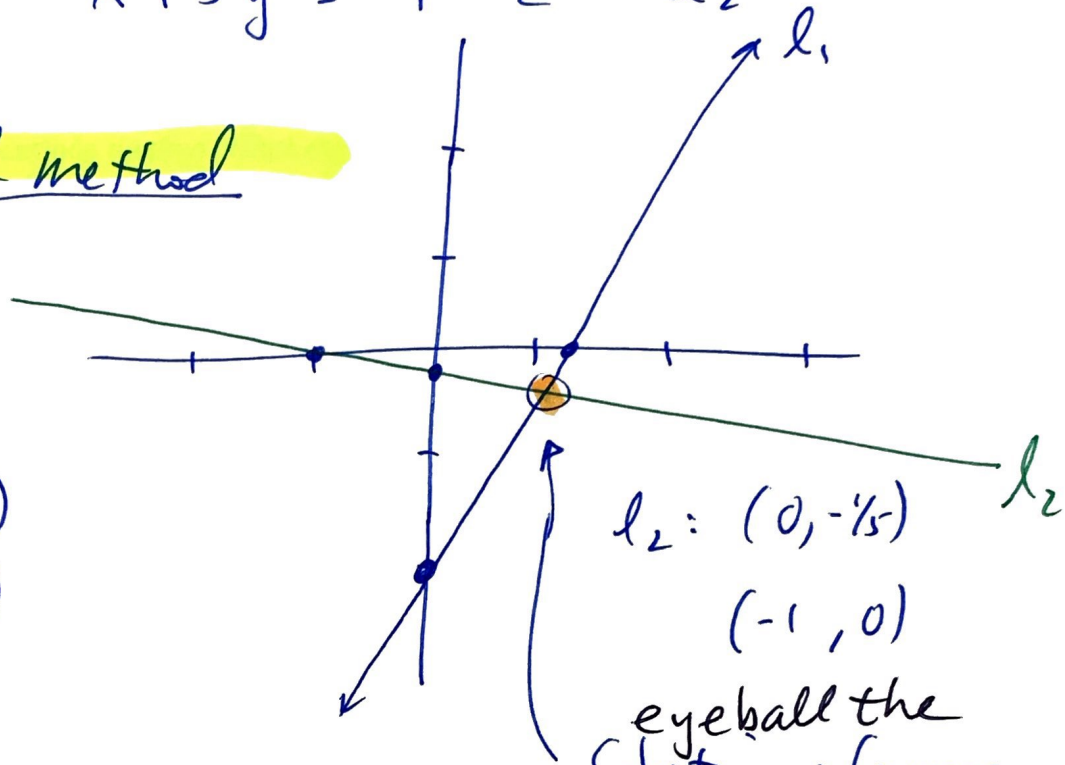
②

EX

$$3x - 2y = 4 \quad \leftarrow l_1$$

$$x + 5y = -1 \quad \leftarrow l_2$$

① Graphical method



$$l_1: (0, -2) \\ (\frac{4}{3}, 0)$$

$$l_2: (0, -\frac{1}{5}) \\ (-1, 0)$$

eyeball the solution: guess

$$(x, y) \approx (1.1, -0.3)$$

② Substitution (Football Coach)

3

$$3x - 2y = 4$$

$$x + 5y = -1$$

(i) Pick the simplest eqn.

(ii) pick the cleanest variable:

(iii) Solve for that variable

$$x = -5y - 1$$

(iv) substitute into the unused eqn.

$$3(-5y - 1) - 2y = 4$$

(v) Solve for the single variable:

$$-15y - 2y = 4 + 3$$

$$-17y = 7$$

$$\rightarrow y = -7/17$$

(vi) Insert this into the eqn of the chosen var.

$$x = -5\left(-\frac{7}{17}\right) - 1$$

$$x = \frac{+35 - 17}{17} = 18/17$$

Soln

$$(x, y) = \left(\frac{18}{17}, -\frac{7}{17}\right)$$

exact soln

$$\approx \begin{pmatrix} 1.06 & 0.41 \\ 1.1 & -0.4 \end{pmatrix} \text{ approx ans.}$$

③ Elimination Method

④

This method adds and subtracts the eqns to eliminate a variable.

Ex

$$3x - 2y = 4$$

$$(x + 5y = -1) \times -3$$

$$\Rightarrow \begin{array}{r} 3x - 2y = 4 \\ -3x - 15y = 3 \end{array}$$

add these eqns

$$\begin{array}{r} 3x - 2y = 4 \\ \oplus -3x - 15y = 3 \\ \hline -17y = 7 \end{array}$$

$$\rightarrow \boxed{y = -\frac{7}{17}}$$

Intert into the simplest looking eqn & solve for x

$$\text{Bot: } x + 5\left(-\frac{7}{17}\right) = -1$$

$$x = -1 + \frac{35}{17} = \frac{18}{17}$$

$$\boxed{(x, y) = \left(\frac{18}{17}, -\frac{7}{17}\right)}$$

Ex Solve

$$2x - 3y = 4$$

$$-5x + 6y = 7$$

(row 1) * 2

$$4x - 6y = 8$$

$$\underline{-5x + 6y = 7}$$

$$-x = 15$$

$$x = -15$$

put into top eqn:

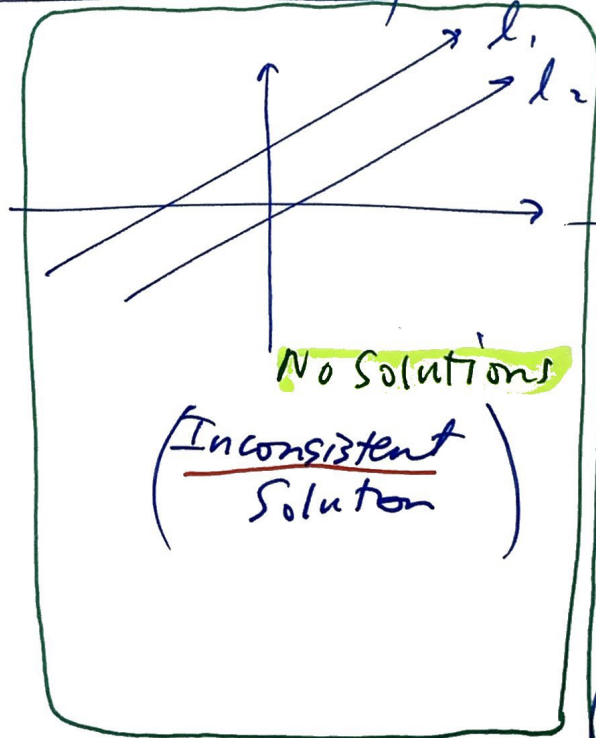
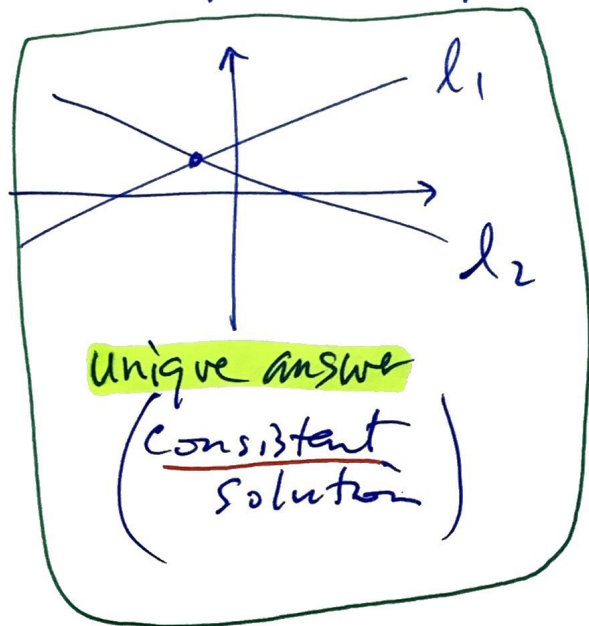
$$2(-15) - 3y = 4$$

$$-3y = 34$$

$$y = -34/3$$

$$(x, y) = (-15, -34/3)$$

* Graphical representations of possible outcomes



ex Solve

$$3x - 2y = 4$$

$$18x - 12y = 24$$

use elimination (top eq) * (-6)

$$\begin{array}{r} \Rightarrow -18x + 12y = -24 \\ + 18x - 12y = 24 \\ \hline \end{array}$$

$0x + 0y = 0 \rightarrow$

∞ many solutions

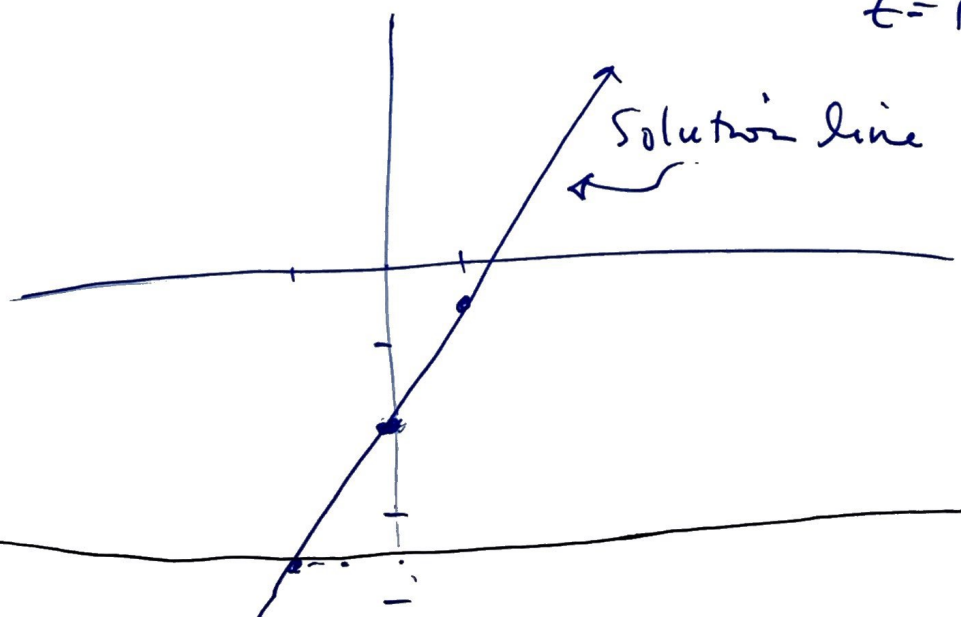
$$(x, y) = \left(x, \frac{4-3x}{-2}\right)$$

$$(x, y) = \left(x, -2 + \frac{3}{2}x\right) \text{ for all } x$$

let $x=t$, where "t" is called a parameter

$$(x(t), y(t)) = \left(t, -2 + \frac{3}{2}t\right)$$

- $t = -1: (-1, -7/2)$
- $t = 0: (0, -2)$
- $t = 1: (1, -1/2)$



Ex

7

$$3x - 2y = 5$$

$$18x - 12y = 24$$

(Top eqn) * (-6) \Rightarrow $-18x + 12y = -30$

$$\oplus \quad 18x - 12y = 24$$

$$0x + 0y = -6$$

$$0 = -6 \quad *$$

No solutions, parallel lines

An inconsistent system