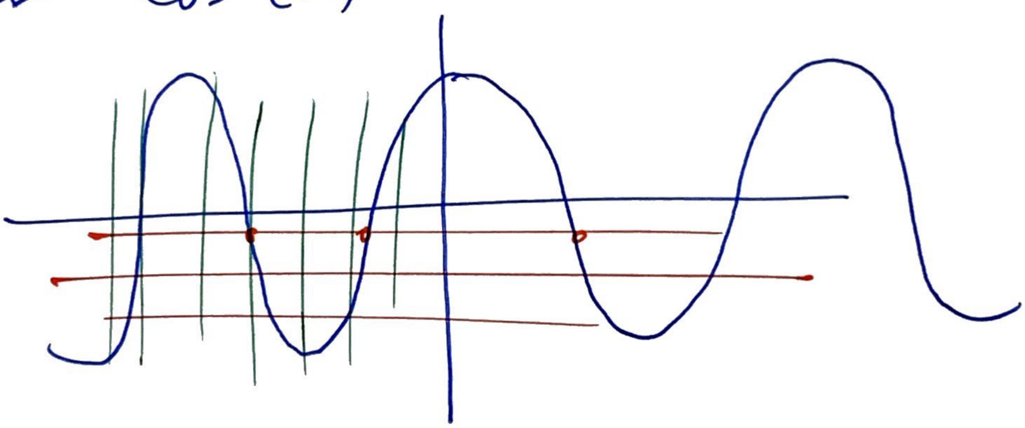
6.3 Inverse Trig Functions

- Recall the graphs of f^{-1} and f are reflections about the line $y=x$ { Basically an exchange of x and y }.

- So If $f(x)$ is a function (Vertical line Test) and is 1 to 1 (Horiz. Line Test) then $f(x)$ has inverse $f^{-1}(x)$ such that

$$f^{-1}(f(x)) = x \quad \text{and} \quad f(f^{-1}(x)) = x.$$

- Consider $\cos(x)$



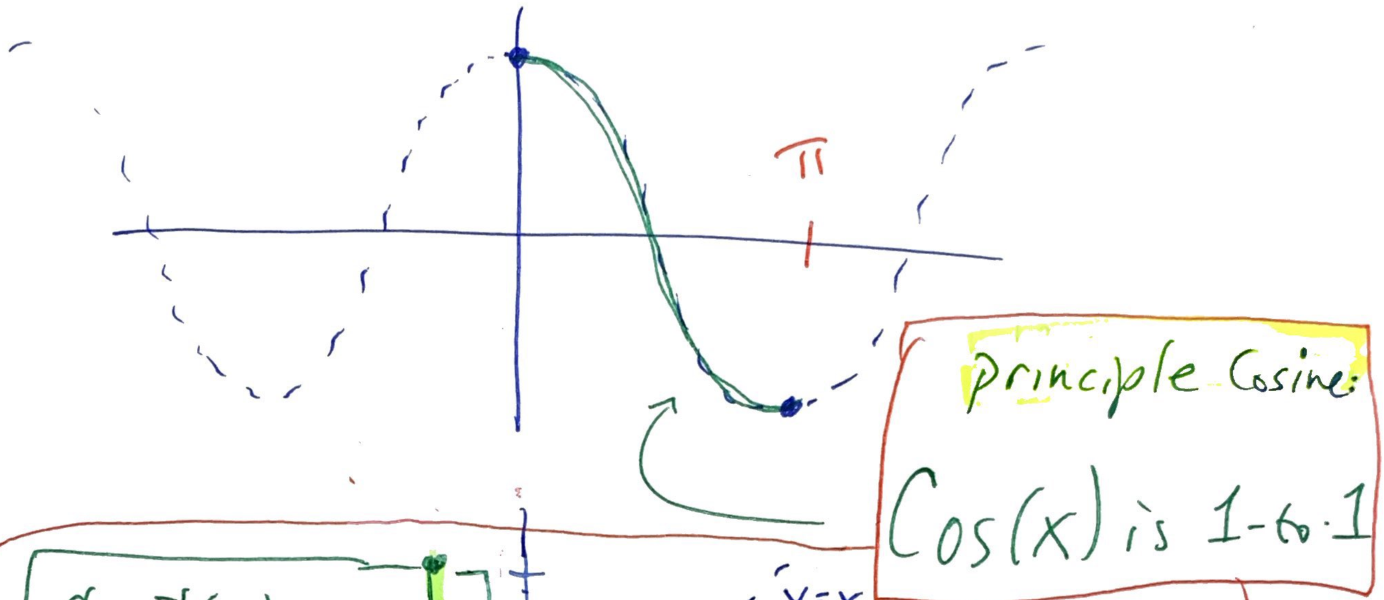
and we ask is $f(x) = \cos(x)$ a one-to-one function? yes

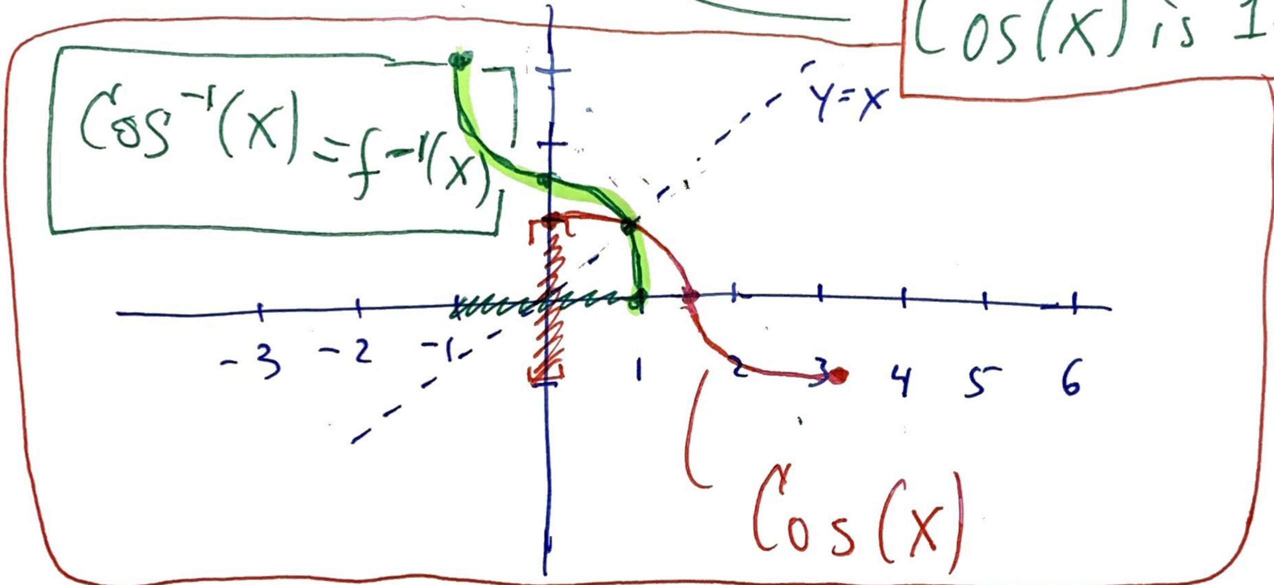
fails.

$\boxed{\text{We cannot form } f^{-1} \text{ for } \cos(x). \text{ What to do?}}$

$\boxed{\cos^{-1}(x)}$ ②

We get around the one-to-one failure by limiting the domain of $\cos(x)$

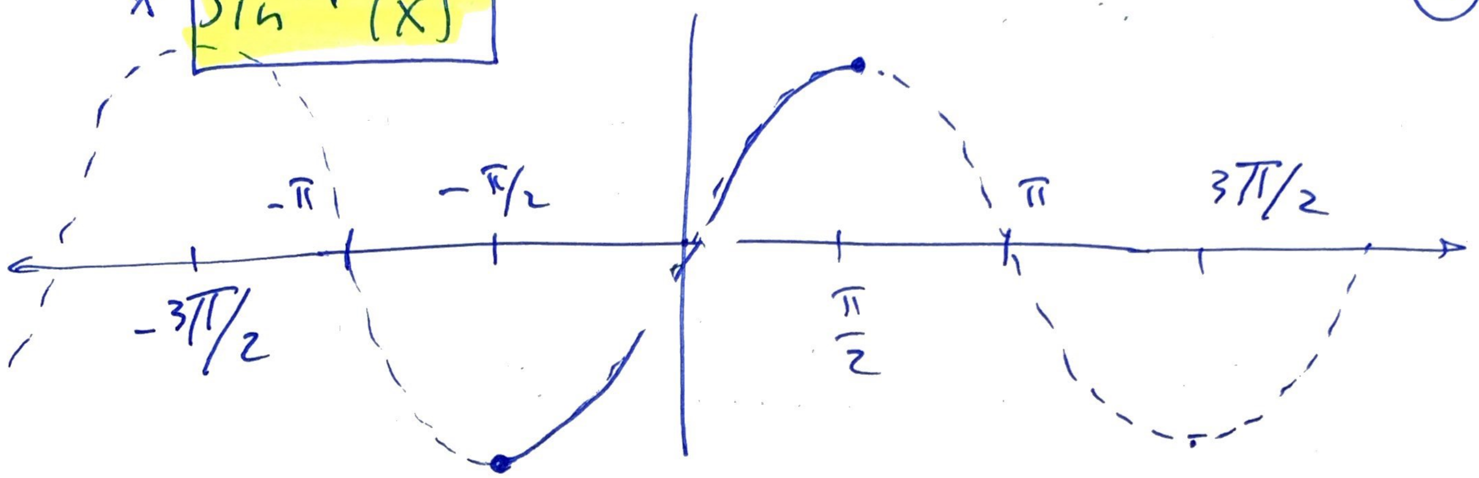




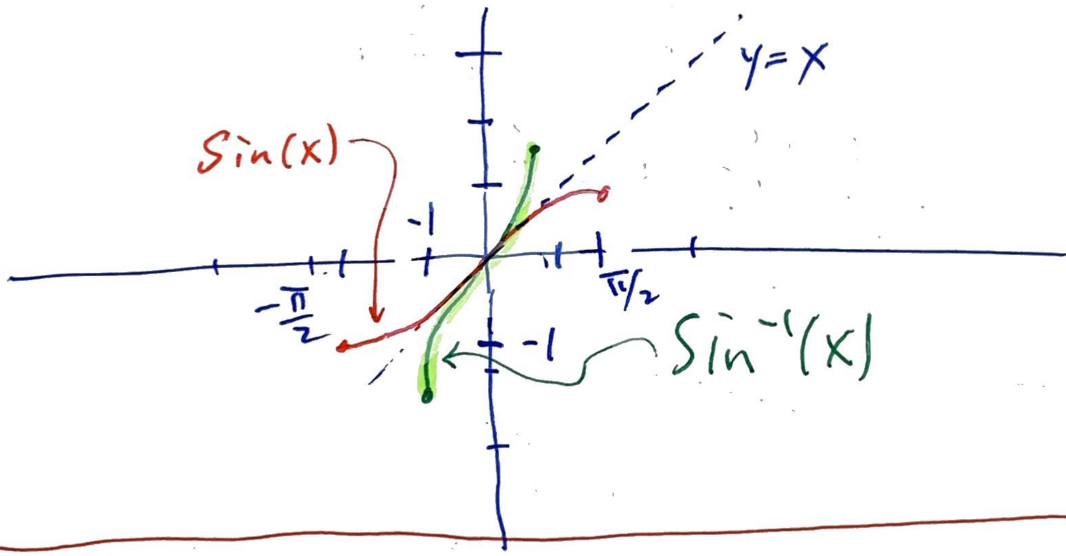
$f(x) = \cos(x)$ $\begin{cases} D_{\cos} : \{x \mid x \in [0, \pi]\} \\ R_{\cos} : \{y \mid y \in [-1,1]\} \end{cases}$

$f^{-1}(x) = \cos^{-1}(x)$ $\begin{cases} D_{\cos^{-1}} : \{x \mid x \in [-1,1]\} \\ R_{\cos^{-1}} : \{y \mid y \in [0, \pi]\} \end{cases}$

* $\sin^{-1}(x)$



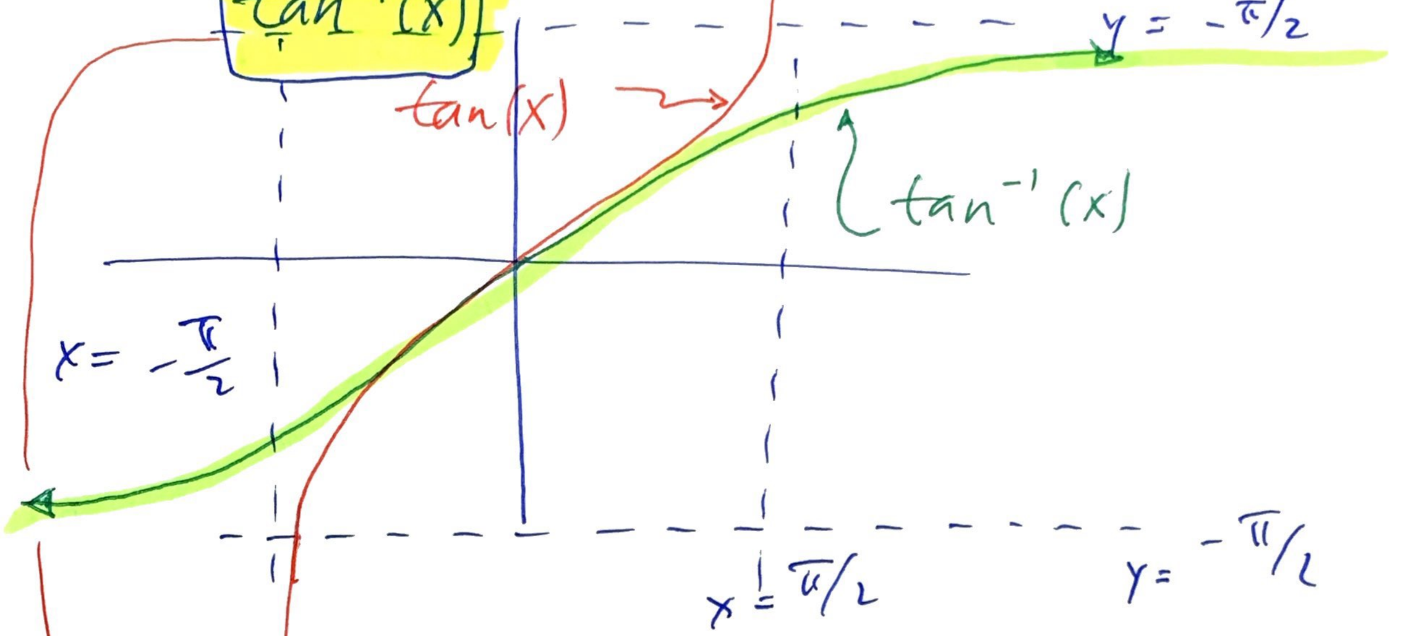
principle sin(x) = Sin(x) $-\frac{\pi}{2}$ to $\frac{\pi}{2}$



$$f(x) = \operatorname{Sin}(x) \begin{cases} D_{\operatorname{Sin}(x)} : \{x \mid x \in [-\frac{\pi}{2}, \frac{\pi}{2}]\} \\ \boxed{R_{\operatorname{Sin}(x)} : \{y \mid y \in [-1, 1]\}} \end{cases}$$

$$f^{-1}(x) = \operatorname{Sin}^{-1}(x) \begin{cases} \boxed{D_{\operatorname{Sin}^{-1}(x)} : \{x \mid x \in [-1, 1]\}} \\ R_{\operatorname{Sin}^{-1}(x)} : \{y \mid y \in [-\frac{\pi}{2}, \frac{\pi}{2}]\} \end{cases}$$

$\tan^{-1}(x)$



Note: $\cos^{-1}(x)$ is also known as $\arccos(x)$
$\sin^{-1}(x)$ is aka $\arcsin(x)$
$\tan^{-1}(x)$ is a.k.a. $\arctan(x)$

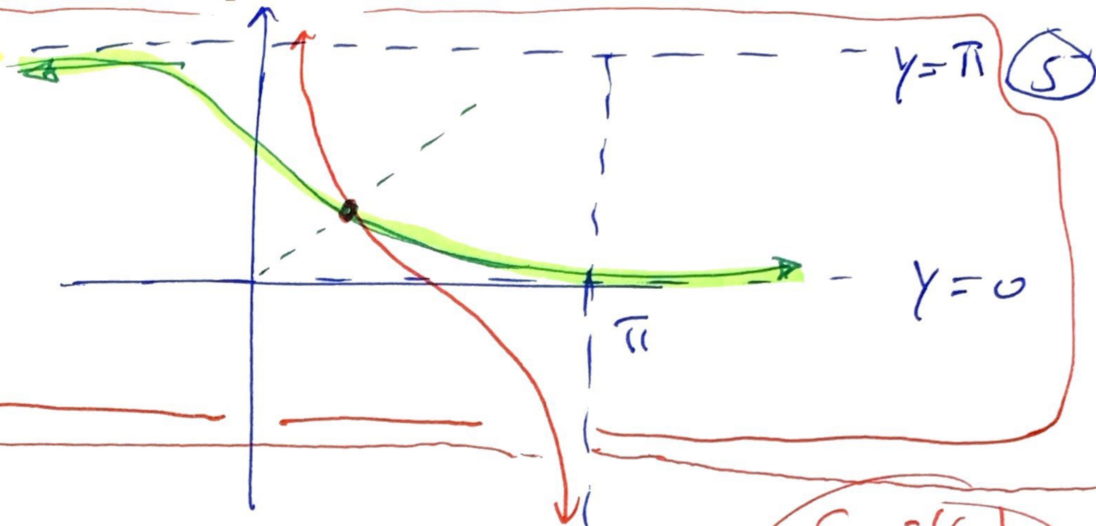
$$D_{\tan(x)} : \left\{ x \mid x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \right\}$$

$$R_{\tan(x)} : \left\{ y \mid y \in (-\infty, \infty) \right\}$$

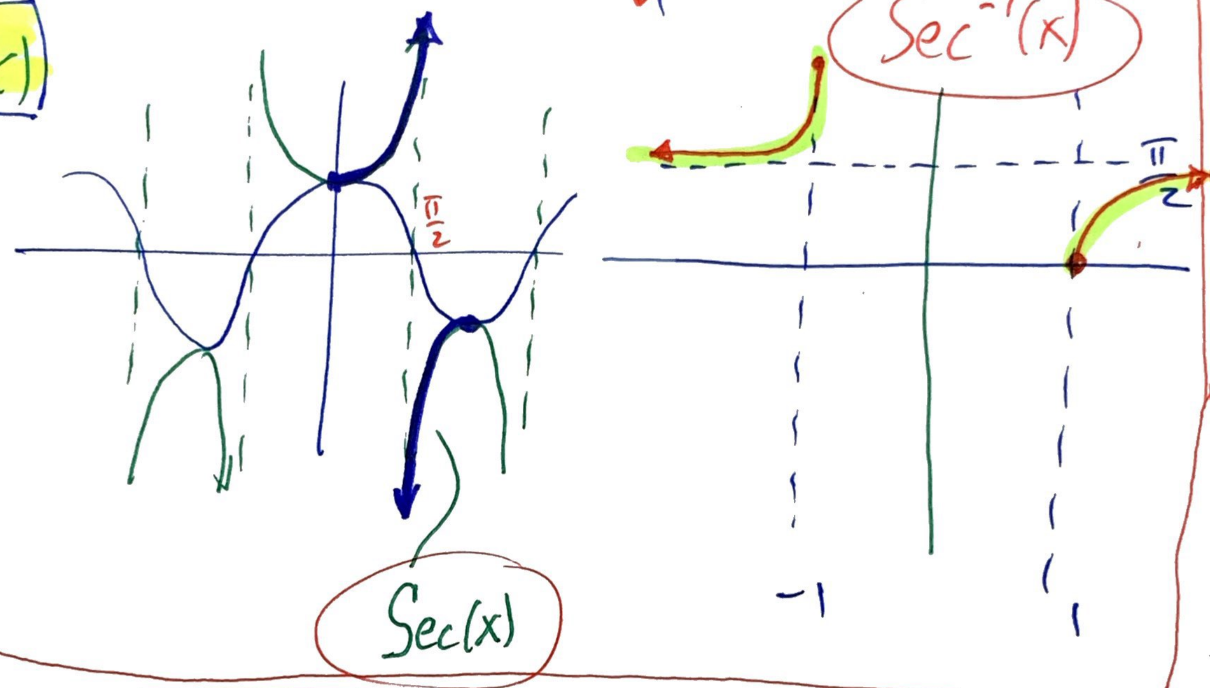
$$D_{\tan^{-1}(x)} : \left\{ x \mid x \in (-\infty, \infty) \right\}$$

$$R_{\tan^{-1}(x)} : \left\{ y \mid y \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \right\}$$

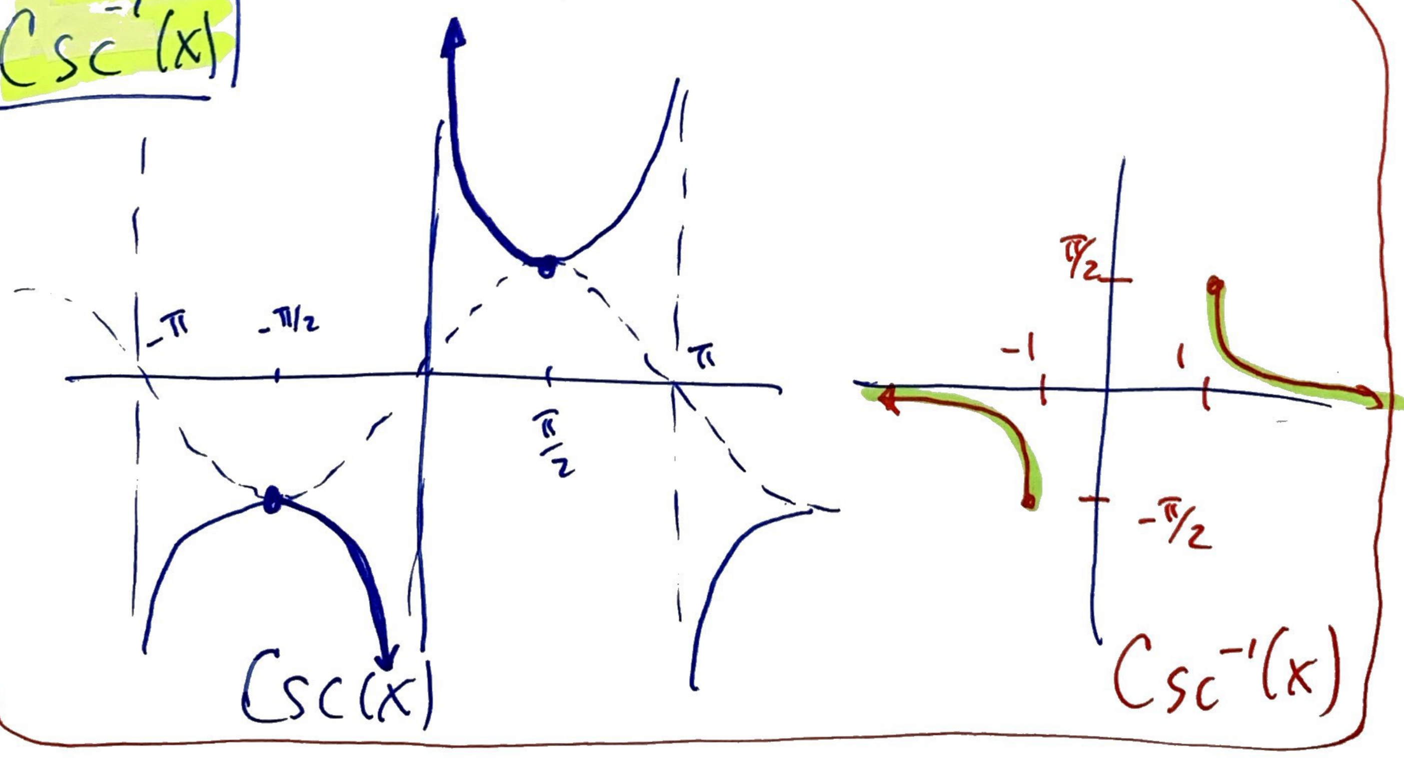
$Cot^{-1}(x)$



$Sec^{-1}(x)$



$Csc^{-1}(x)$



Examples of Inv. Trig functions

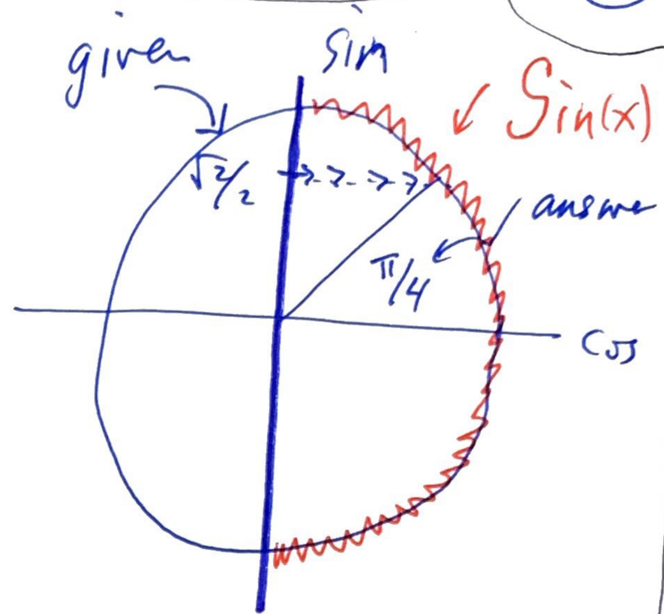
Ex Find $\sin^{-1}\left(\frac{\sqrt{2}}{2}\right)$

- let $\sin^{-1}(\sqrt{2}/2) = y$
- apply sin

$$\sin\left(\sin^{-1}\left(\frac{\sqrt{2}}{2}\right)\right) = \sin(y)$$

$$\frac{\sqrt{2}}{2} = \sin(y)$$

$\boxed{y \text{ must } \pi/4}$

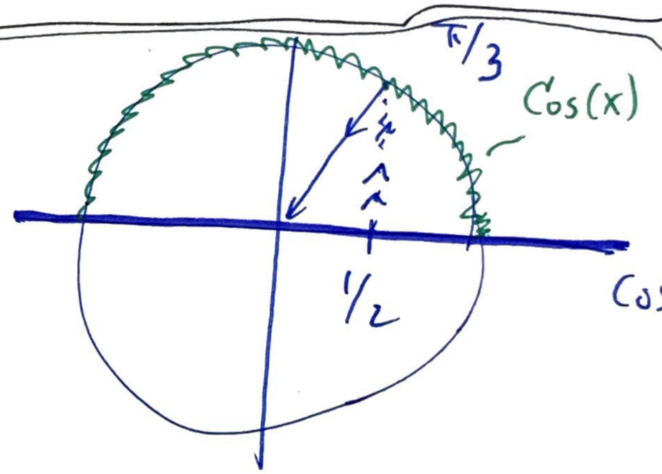


Ex Find $\cos^{-1}\left(\frac{1}{2}\right)$

- let $\cos^{-1}\left(\frac{1}{2}\right) = y$
- apply cos( )

$$\cos\left(\cos^{-1}\left(\frac{1}{2}\right)\right) = \cos(y)$$

$$\frac{1}{2} = \cos(y)$$

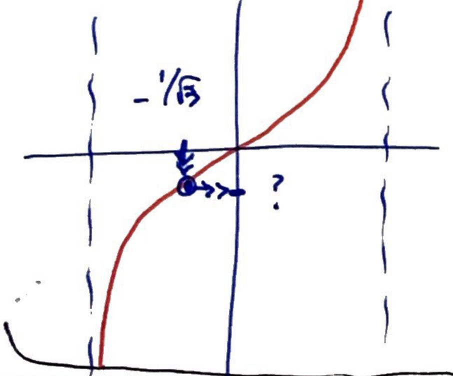


$\boxed{y = \pi/3}$

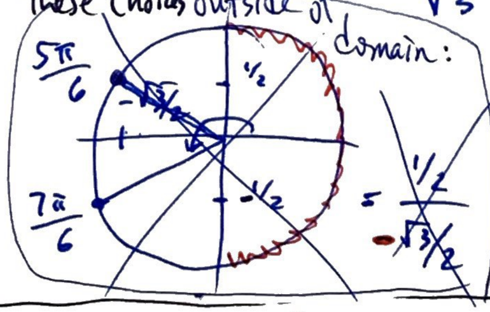
Ex $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)$

- let $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) = \theta$) apply tan

So $-\frac{1}{\sqrt{3}} = \tan(\theta)$



These Choices outside of domain:

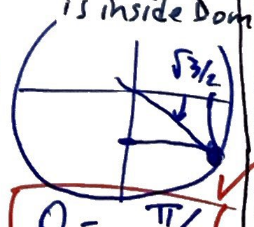


$$= \frac{\sin(\theta)}{\cos(\theta)}$$

$$= \cancel{\frac{1/2}{-\sqrt{3}/2}}$$

$$= \frac{-1/2}{\sqrt{3}/2}$$

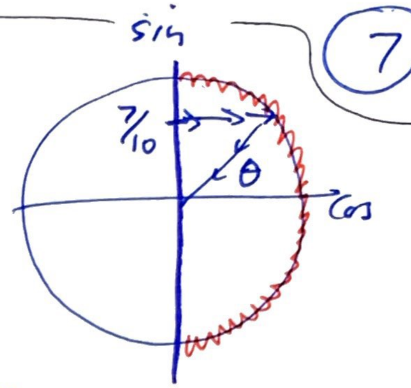
This choice is inside Dom



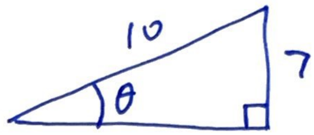
$\boxed{\theta = -\pi/6}$

EX Find $\sin^{-1}(7/10)$

- let $\theta = \sin^{-1}(7/10)$
- $\sin(\theta) = 7/10$



But $\sin\theta = \frac{opp}{hyp}$



No special numbers here w/ this triangle

So use Calculator: $7 \boxed{\div} 10 \boxed{=} \boxed{2^{nd}} \boxed{\sin^{-1}} \rightarrow \boxed{44.43°}$

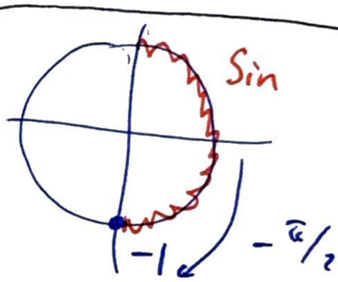
DRG → "rad" $0.7 \boxed{2^{nd}} \boxed{\sin^{-1}} \rightarrow 0.7754$ rad

⊛ Crossover trig inv (trig)

EX $\sin^{-1}(\cos(\pi))$

$= \sin^{-1}(-1)$

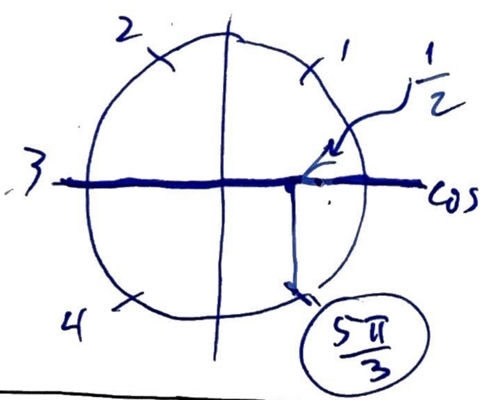
$= \boxed{-\pi/2}$

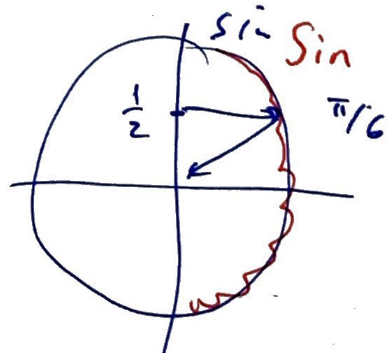


EX $\sin^{-1}\Big(\underbrace{\cos\left(\tfrac{5\pi}{3}\right)}\Big)$

$= \sin^{-1}(1/2)$

$= \boxed{\pi/6}$





⊛ Non-Crossover $\text{trig}^{-1}(\text{trig})$

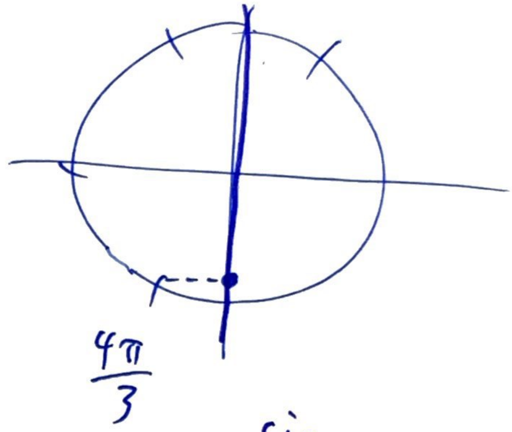
Ex $\sin^{-1}\left(\sin\left(\frac{4\pi}{3}\right)\right)$

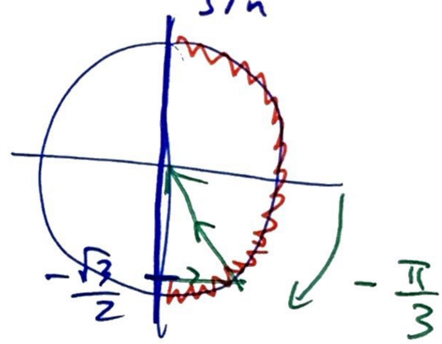
- We might think it is → $\cancel{4\pi/3}$ No!!
- But let $y = \sin\left(\frac{4\pi}{3}\right)$

  $= -\sqrt{3}/2$
- Now analyze this...

  $= \sin^{-1}\left(-\sqrt{3}/2\right)$

  $= \boxed{-\pi/3}$





Summary

$$\boxed{\sin^{-1}\left(\sin\left(\frac{4\pi}{3}\right)\right) = -\frac{\pi}{3}}$$

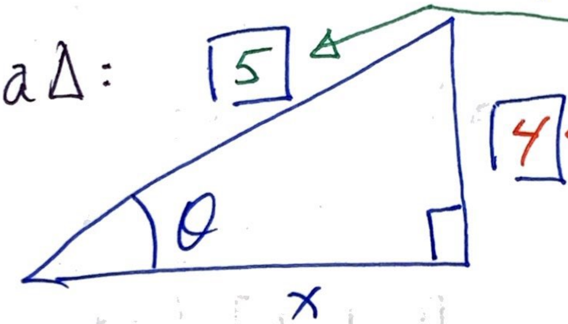
We must be careful when we see a $\text{trig}^{-1}(\text{trig}(x))$ as it may not be x.

Non-standard angles : trig(trig^{-1})

EX Find $\cos\Big(\underbrace{\sin^{-1}\left(\tfrac{4}{5}\right)}_{\theta}\Big)$

- So let $\sin^{-1}(4/5) = \theta$
- then $\sin(\theta) = 4/5$
- but $\sin\theta = \dfrac{opp}{hyp}$

  So Draw a Δ:



fill in sides

- However we seek $\cos(\theta)$

  where $\cos(\theta) = \dfrac{adj}{hyp} = \dfrac{x}{5}$

- So using the pythagorean identity: $a^2+b^2=c^2$

  we have $x^2+4^2=5^2$ or $x=\sqrt{25-16}$

  $x=\sqrt{9}$

  $x=3$

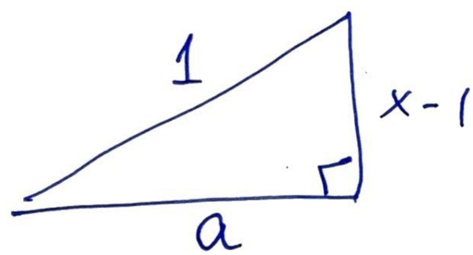
thus $\boxed{\cos\left(\sin^{-1}\left(\tfrac{4}{5}\right)\right) = \dfrac{3}{5}}$

Variables

In Calc II we use these relations with variables

Ex Simplify $\tan\big(\underbrace{\sin^{-1}(x-1)}_{\theta}\big)$

- So let $\sin\theta = x-1 = \frac{opp}{hyp} = \frac{x-1}{1}$ · assign sides
- Draw Δ:



- But we seek $\tan(\theta)$ which
  i) $\frac{opp}{adj}$

$$\Rightarrow \tan(\theta) = \frac{x-1}{a}$$

- So use pythagorea identity $a^2 + (x-1)^2 = 1^2$

So $a = \sqrt{1-(x-1)^2}$
$= \sqrt{\cancel{1} - x^2 + 2x \cancel{-1}}$

Summary: $\boxed{\tan\big(\sin^{-1}(x-1)\big) = \frac{x-1}{\sqrt{1-(x-1)^2}}}$