$\boxed{6.1 \text{ \& } 6.2}$ revisited w/ phase shifts

Part 2

Consider the general form of sin & cos

vertical shifts

$$y = A\sin(Bx - C) + D$$
$$y = A\cos(Bx - C) + D$$

amplitude

horz. stretching & squeezing

horizontal moves

Life is easier if you isolate x:

$$\boxed{\begin{aligned} y &= A\sin\left(B\left[x - \tfrac{C}{B}\right]\right) + D \\ y &= A\cos\left(B\left[x - \tfrac{C}{B}\right]\right) + D \end{aligned}}$$

true horizontal shifting

STEPS

(i) factor the "B" out so "x" is alone

$\Rightarrow$ the $\boxed{\text{horizontal shift is } C/B}$

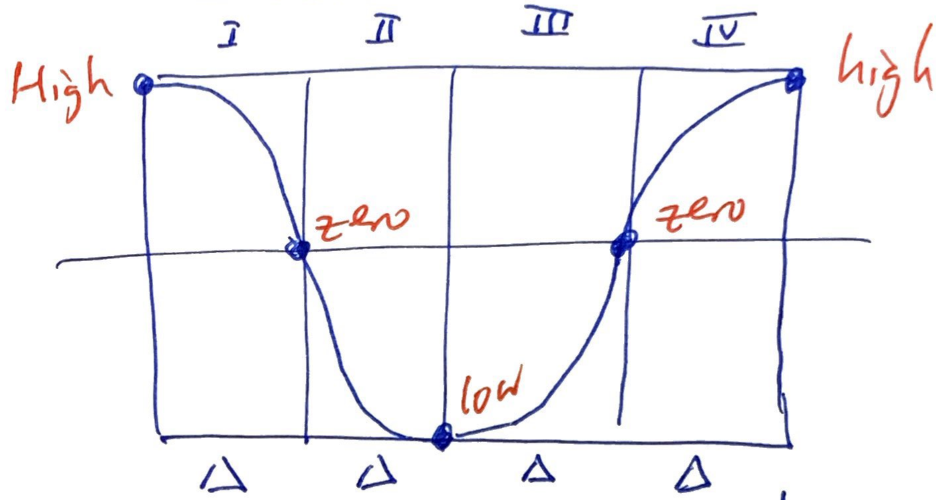
(ii) graph $\sin(Bx)$

(iii) shift the graph over by C/B

(iv) apply vertical shift D

(*) To graph $A\cos\left(B\left[x-\frac{C}{B}\right]\right)+D$

• Consider "the Box" for cos:



$\boxed{\text{Period} = \frac{2\pi}{B}}$

• Idetify the following

(i) period: $P = 2\pi/B$

(ii) ÷ period by 4 : $P/4 = \frac{2\pi/B}{4}$

(iii) Determin the location of the starting point for the Box, $\frac{C}{B}$.

(iv) Calculate $\Delta = P/4$

(v) List the 5 key points

$\boxed{\text{high}: \frac{C}{B}}$, $\boxed{\text{zero}: \frac{C}{B}+\Delta}$, $\boxed{\text{low}: \frac{C}{B}+2\Delta}$, $\boxed{\text{zero } \frac{C}{B}+3\Delta}$, $\boxed{\text{high } \frac{C}{B}+4\Delta}$

EX Graph $y = 2\cos\left(3x - \frac{\pi}{4}\right)$

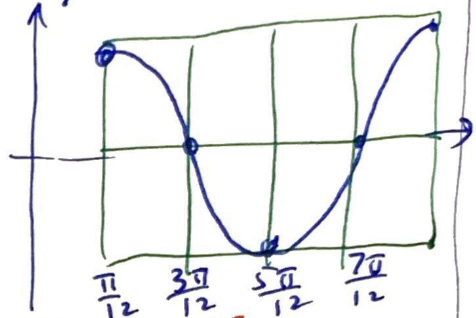
Form : $y = \boxed{2}\cos\left(3\left[x - \frac{\pi}{12}\right]\right)$

(i) $P = \frac{2\pi}{3}$

(ii) $\frac{P}{4} = \frac{2\pi}{12} = \frac{\pi}{6}$

(iii) Start @ $\boxed{\pi/12}$

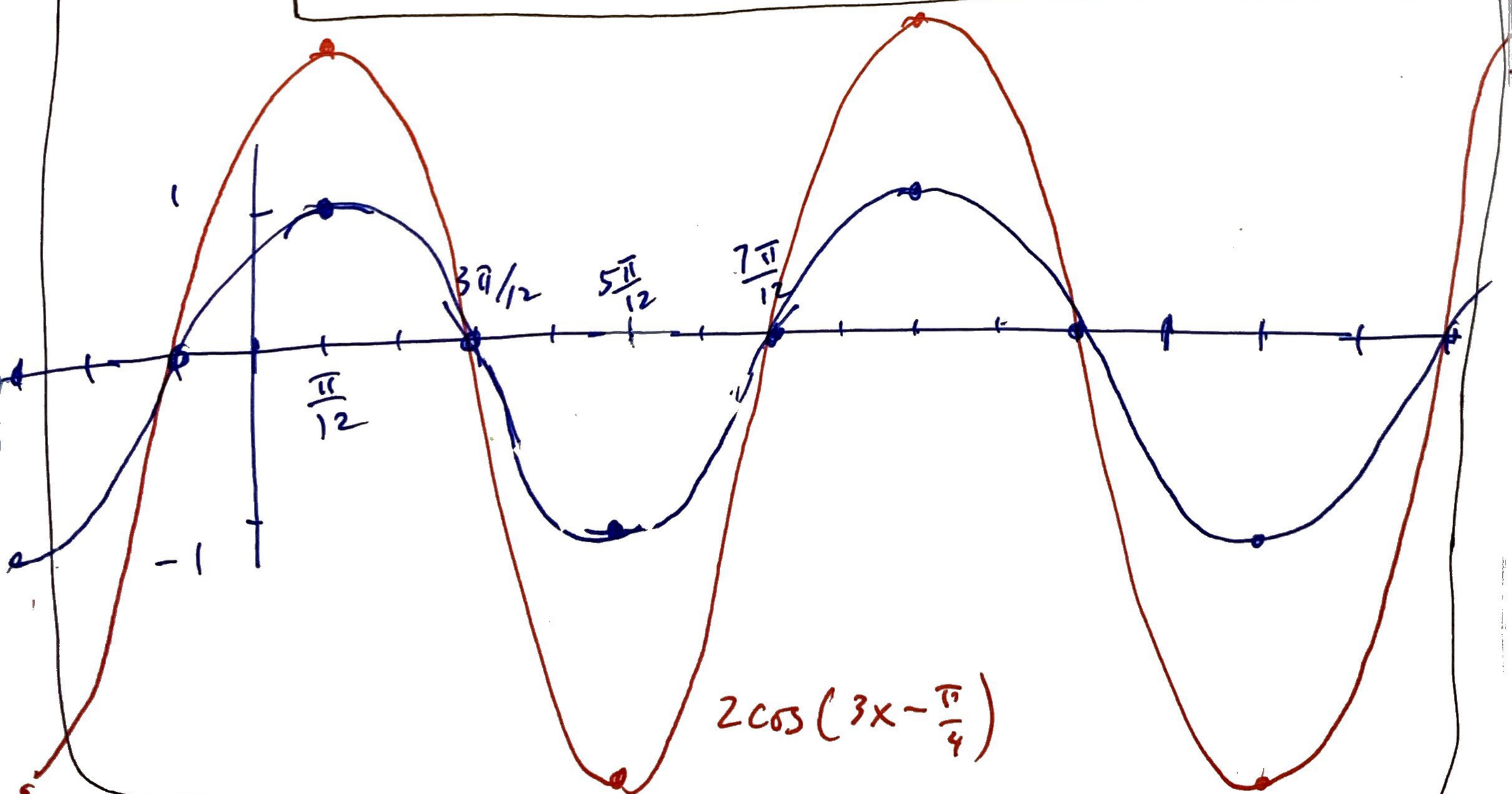
(iv) $\Delta = \boxed{\frac{\pi}{6}} = \boxed{\frac{2\pi}{12}}$



add $\Delta = \frac{2\pi}{12}$ add $\Delta = \frac{2\pi}{12}$ add Δ add Δ

(v) 5 Keypoints : $\frac{\pi}{12}$, $\frac{3\pi}{12}$, $\frac{5\pi}{12}$, $\frac{7\pi}{12}$, $\frac{9\pi}{12}$

High zero Low zero high



EX

Sketch $y = 4\cos\left(2t + \frac{\pi}{2}\right) - 3$

Form $y = 4\cos\left(2\left[t + \frac{\pi}{4}\right]\right) - 3$

$A = 4 \quad B = 2 \quad C = \pi/4 \quad D$

(i) $P = \frac{2\pi}{2} = \pi$

(ii) $\div 4$: $\pi/4$

(iii) start @ $-\pi/4$

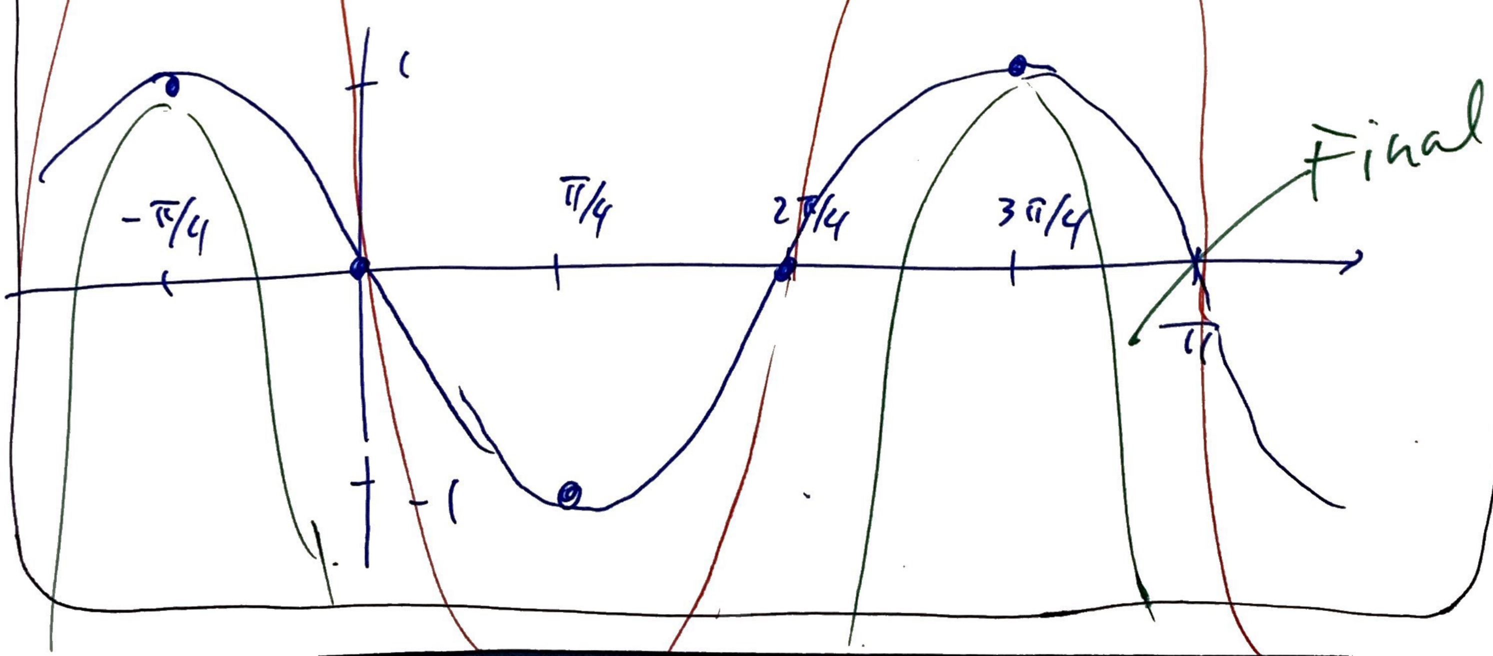
(iv) $\Delta = \boxed{\pi/4}$

– LCD $= \frac{\pi}{4}$ tick marks.

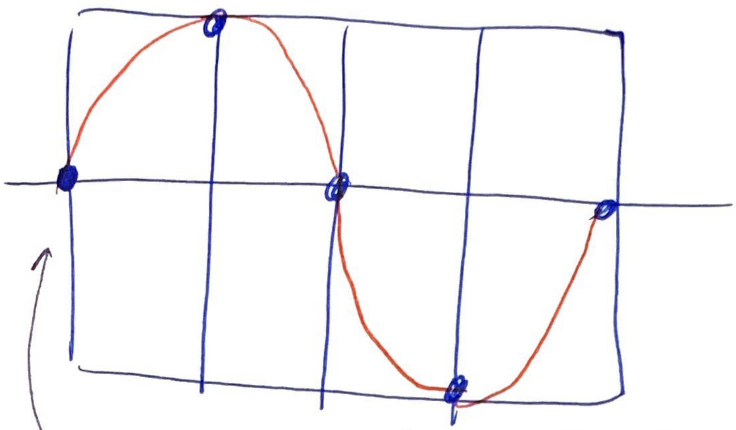
(v) 5-points

high @ $-\frac{\pi}{4}$, zero $= -\frac{\pi}{4} + \frac{\pi}{4} = 0$, low $= -\frac{\pi}{4} + 2\frac{\pi}{4} = \frac{\pi}{4}$

zero $= \frac{2\pi}{4}$, high $= \frac{3\pi}{4}$



Sine functions w/ phase shifts



the start point is a zero, then high, zero then low, then zero again

Ex Sketch $y = \sin\left(2x - \frac{\pi}{4}\right)$

(0) Form $y = \sin\left(2\left[x - \frac{\pi}{8}\right]\right)$

(i) Period $P = 2\pi/2 = \pi$

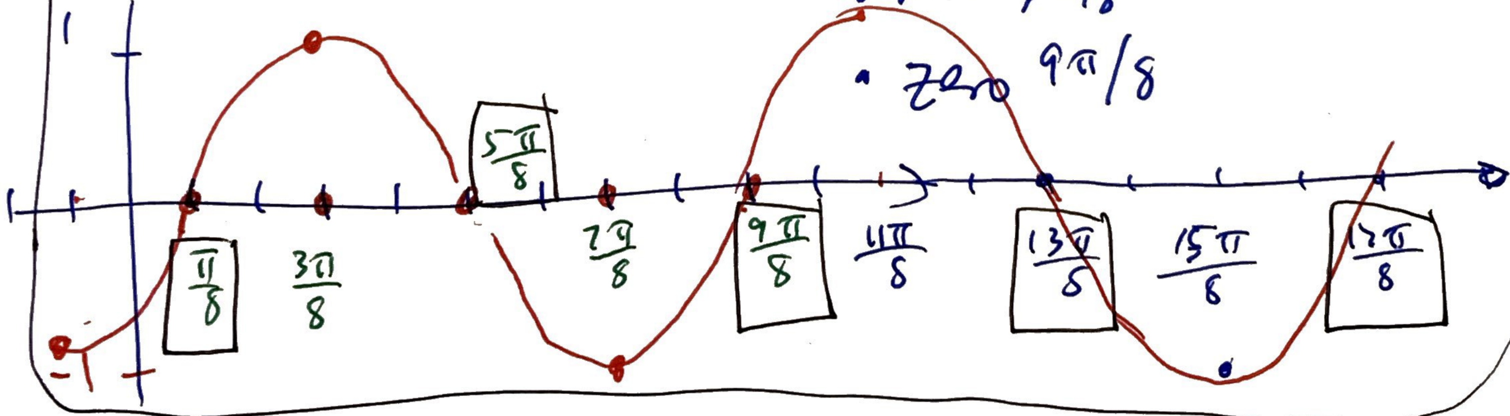
(ii) $P/4 = \pi/4 = \Delta$

(iii) start $\boxed{x = \pi/8}$

(iv) $\Delta = \frac{\pi}{4} = \boxed{\frac{2\pi}{8}}$

(v) 5 key points

- zero @ $\frac{\pi}{8}$
- max @ $\pi/8 + \frac{2\pi}{8} = \frac{3\pi}{8}$
- zero @ $5\pi/8$
- min $7\pi/8$
- zero $9\pi/8$



Ex Sketch $p(t) = 35\sin\left(\frac{\pi}{6}t + \frac{\pi}{3}\right)$

(a) Form $35\sin\left(\frac{\pi}{6}[t+2]\right)$

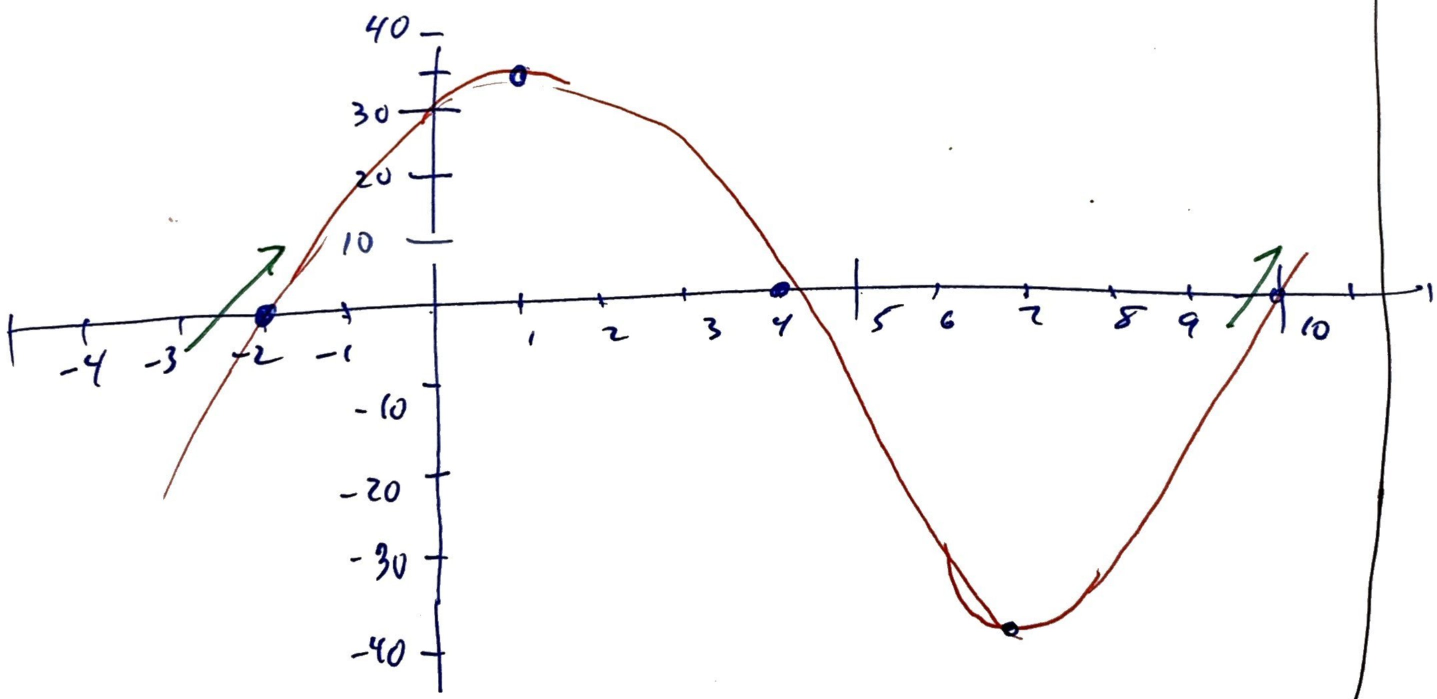
(i) $P = \frac{2\pi}{\pi/6} = \frac{2\pi \cdot 6}{\pi} = \boxed{12}$ No π!!

(ii) $P/4 = 12/4 = \boxed{3} = \Delta \leftarrow$ features @ ...

(iii) start @ $\boxed{t=-2}$ $\leftarrow$ start

(iv) $1 \longleftarrow$ LCD

(v) $-2, 1, 4, 7, 10$



EX

Graph $y = -2\cos\left(4x + \frac{\pi}{5}\right)$

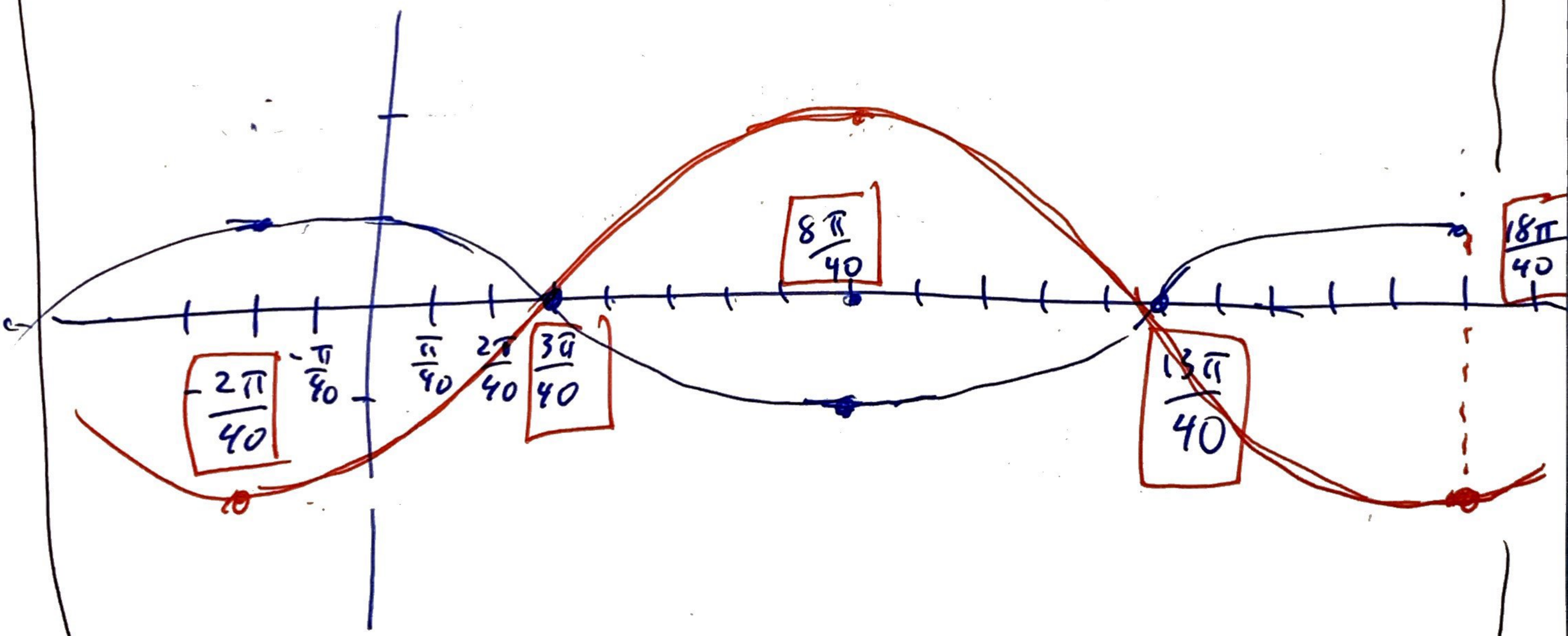
Form $y = -2\cos\left(4\left[x + \frac{\pi}{20}\right]\right)$

(i) $P = \frac{2\pi}{4} = \frac{\pi}{2}$

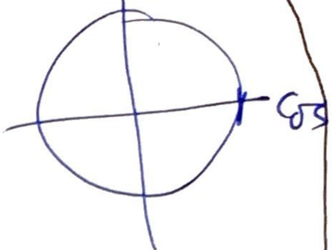
(ii) $\Delta = \pi/8 \rightarrow \frac{5\pi}{40}$

(iii) Start $= -\frac{\pi}{20} = \frac{-2\pi}{40}$

(v) High $-\frac{2\pi}{40}$, zero $\frac{3\pi}{40}$, low $\frac{8\pi}{40}$, zero $\frac{13\pi}{40}$, $\frac{18\pi}{40}$.



(*) "zero" approach to graphing: Graph $y = \sin\left(2x - \frac{\pi}{4}\right)$



Ex

Q: when is $\boxed{\sin\left(2x - \frac{\pi}{4}\right) = 0}$

A: when $2x - \frac{\pi}{4} = n\pi$

- Solve for x: $2x = n\pi + \frac{\pi}{4}$

$$x = \frac{n\pi}{2} + \frac{\pi}{8}$$

$$x = \frac{4n\pi + \pi}{8}$$

$$\boxed{x = (4n+1)\pi/8}$$ zeros

- Try consecutive "n" values:

$n=-1$: $\frac{-3\pi}{8}$

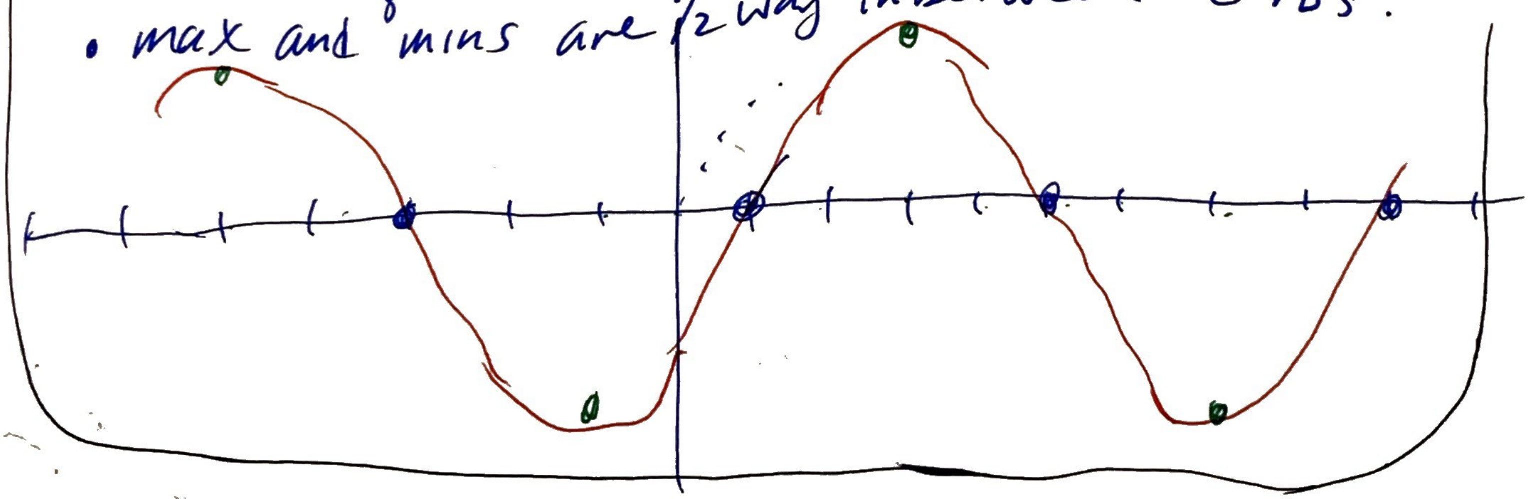
$n=0$: $\frac{\pi}{8}$

$n=1$: $\frac{5\pi}{8}$

$n=2$: $\frac{9\pi}{8}$

$n=3$: $\frac{13\pi}{8}$

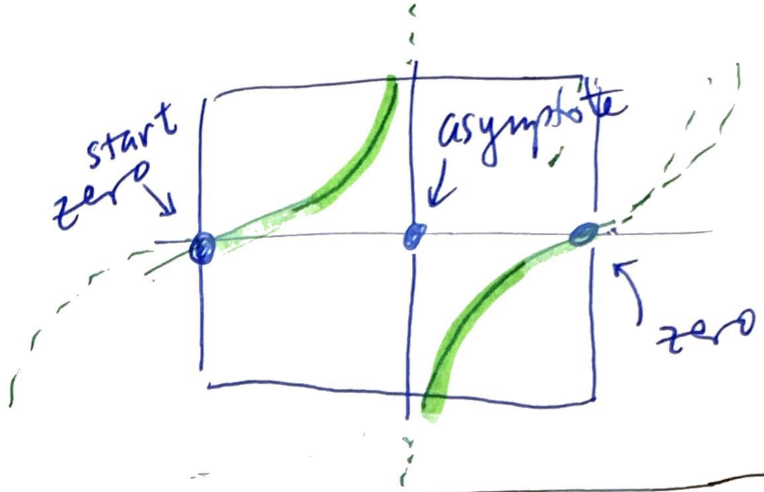
- max and mins are 1/2 way inbetween zeros.



(*) **Tangent**

"The Box"

- only 3 pts
- Period is $\boxed{P = \frac{\pi}{B}}$



Ex Graph $y = 2\tan\left(\frac{1}{5}t\right) + 1$

(0) already proper form

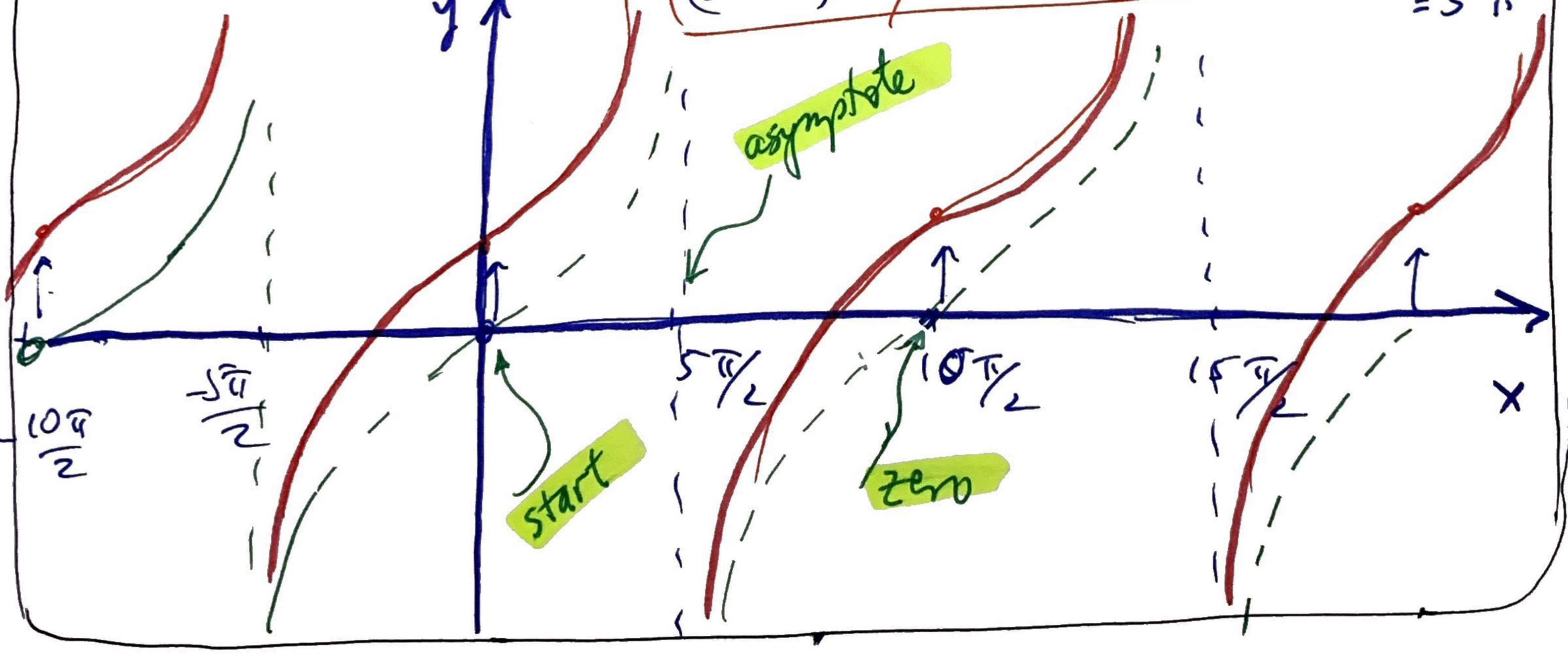
(i) Period $\boxed{P = \frac{\pi}{B}} = \frac{\pi}{1/5} = \boxed{5\pi}$ tan and cot

(ii) $\Delta = P/2 = \boxed{5\pi/2}$

(iii) start $= 0$ {no shift}

(iv) LCM $= \rightarrow$ LCM $= \frac{5\pi}{2}$

(v) 3 points : start (zero) 0 / asympt $= \frac{5\pi}{2}$ / end zero $\frac{10\pi}{2} = 5\pi$



EX Sketch $y = \cot\left(\frac{1}{2}t\right)$

(i) $P_0 = \pi$, $P = \pi/B = \pi/\frac{1}{2} = \boxed{2\pi}$

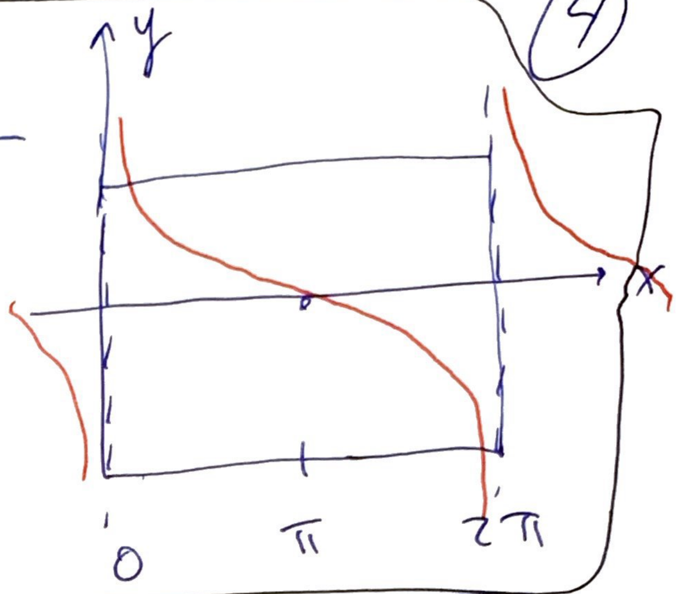
(ii) $\Delta = P/2 = \frac{2\pi}{2} = \pi$

(iii) start @ 0

$\}$ LCM $= \pi$

(iv) LCM $= \pi$

(v) 3-keypoints: $(0, \pi, 2\pi)$ with $+\Delta$, $+2\Delta$



EX Sketch $y = 8\cot(3t-2) = 8\cot\left[3\left(t-\frac{2}{3}\right)\right]$

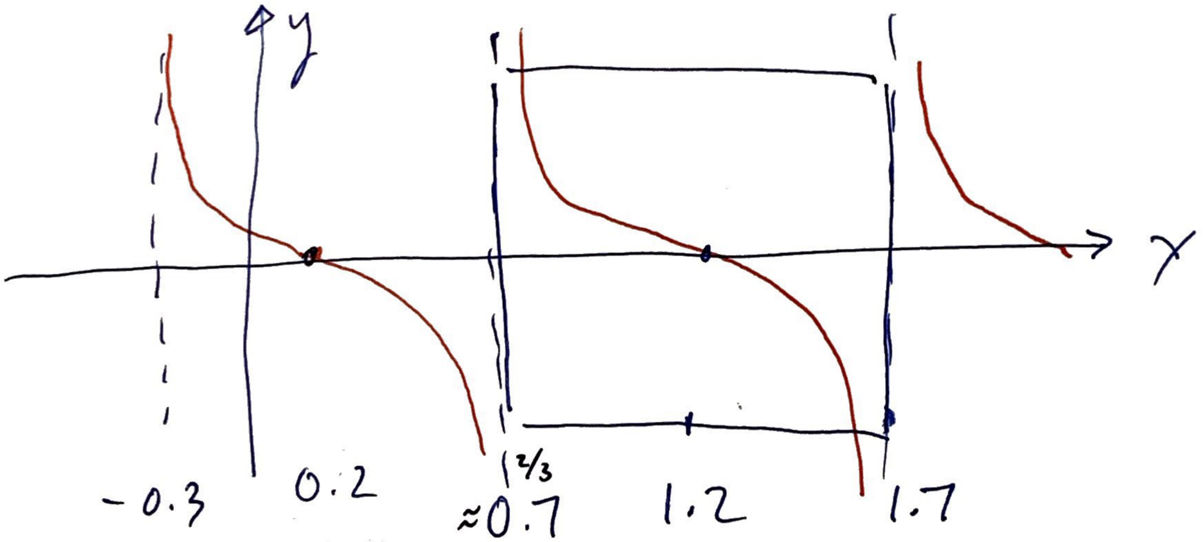
(i) $P = \pi/3$

(ii) $\Delta = P/2 = \pi/6$

(iii) start $= 2/3$

$\}$ LCM : no LCM, use the box & shift

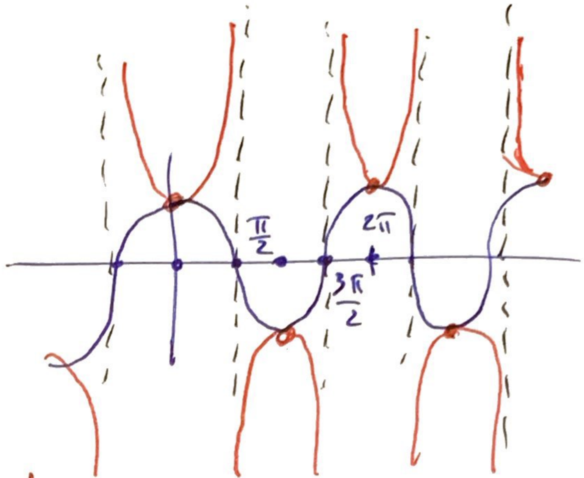
(v) 3-keypoints: $\left(\frac{2}{3}, \frac{2}{3}+\frac{\pi}{6}, \frac{2}{3}+2\cdot\frac{\pi}{6}\right) = \left(\frac{2}{3} \approx 0.7, \approx 1.2, 1.7\right)$



⊕ sec

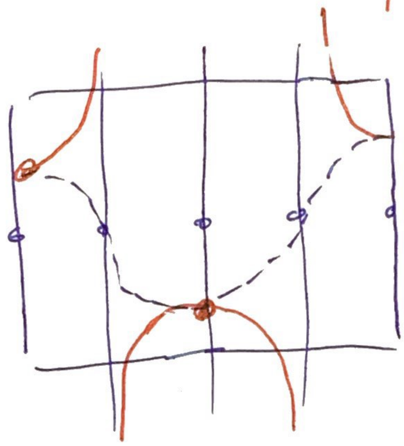
5

$\boxed{\sec(x)} = \dfrac{1}{\cos(x)}$



$P = 2\pi$

Box



EX Sketch $y = 2\sec\left(3t + \frac{\pi}{4}\right) = 2\sec\left(3\left[t + \frac{\pi}{12}\right]\right)$

start

(i) $P = \dfrac{2\pi}{3}$

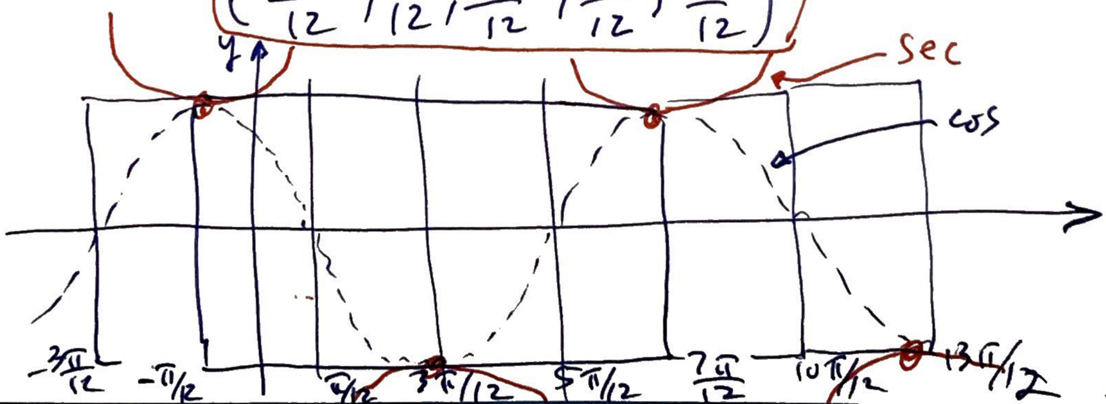
(ii) $P/4 = \dfrac{2\pi/3}{4} = \dfrac{2\pi}{12} = \dfrac{\pi}{6} = \dfrac{2\pi}{12}$

(iii) start $= -\dfrac{\pi}{12} = -\dfrac{\pi}{12}$

(iv) LCM : $2\pi/12$

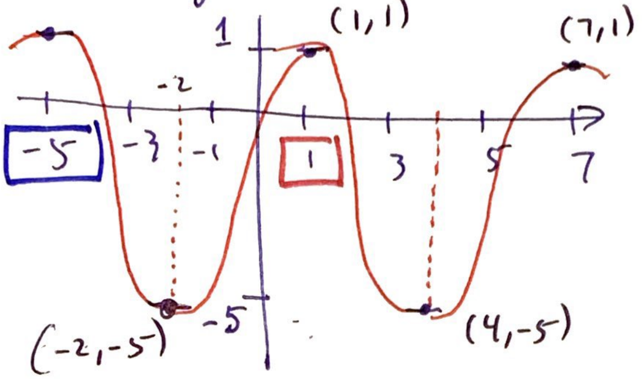
(v) 5-key point : $\left(-\dfrac{\pi}{12}, -\dfrac{\pi}{12} \overset{+\Delta}{+} \dfrac{\pi}{6}, -\dfrac{\pi}{12} + \dfrac{2\pi}{6}, -\dfrac{\pi}{12} + \dfrac{3\pi}{6}, -\dfrac{\pi}{12} + \dfrac{4\pi}{6}\right.$

$\left(-\dfrac{\pi}{12}, \dfrac{\pi}{12}, \dfrac{3\pi}{12}, \dfrac{5\pi}{12}, \dfrac{7\pi}{12}\right)$



Given a graph work backwards

Ex Write down the equation whose graph is given below



amplitude: subtract max−min

- $A = \frac{(1)-(-5)}{2} = 3$

vertical shift: average max + min

- $D = \frac{(1)+(-5)}{2} = -2$
- $P = |\boxed{1} - \boxed{-5}| = 6$

$\Rightarrow B = \frac{2\pi}{P} = \frac{2\pi}{6} = \frac{\pi}{3}$

- C ...

Choices $y = 3\sin\left(\frac{\pi}{3}x - C\right) - 2$

or $y = 3\cos\left(\frac{\pi}{3}x - C\right) - 2$ ← cosine is advised

$= 3\cos\left(\frac{\pi}{3}\left(x - \frac{C}{B}\right)\right) - 2$

phase shift "ϕ" $= C/B$ Horiz shift.

$\Rightarrow \frac{C}{B} = 1$

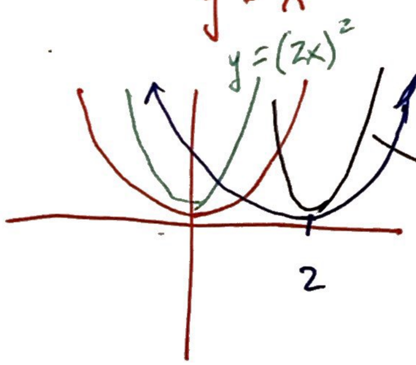
$\Rightarrow C = B = \frac{\pi}{3}$

$\Rightarrow \boxed{y = 3\cos\left(\frac{\pi}{3}x - \frac{\pi}{3}\right) - 2}$

$y = 3\sin\left(\frac{\pi}{3}x - \frac{\pi}{3} - \frac{\pi}{2}\right) - 2$ since $\sin\left(x - \frac{\pi}{2}\right) = \cos(x)$

$\boxed{y = 3\sin\left(\frac{\pi}{3}x - \frac{5\pi}{6}\right) - 2}$

$y = x^2$ $\quad y = (x-2)^2$



$y = (2[x-2])^2$