

4.6 Solving Exponential & Log Problems (1)

There are two general methods used to solve Exp. & Log equations

I. Equivalent Problems (already covered)

EX

$$e^{4x+1} = 2 \iff \log_e(2) = 4x+1$$

$$\Rightarrow \ln(2) = 4x+1$$

$$\frac{\ln(2) - 1}{4} = 4x \Rightarrow$$

$$x = \frac{\ln(2) - 1}{4} \text{ exact}$$

$$x \approx -0.0767 \text{ approx.}$$

EX

Solve via equiv. problem

$$\log(x^2+1) = 3 \iff 10^3 = x^2+1$$

$$\Rightarrow x^2 = 10^3 - 1$$

$$x^2 = 1000 - 1$$

$$x^2 = 999$$

$$x = \sqrt{999} \text{ exact}$$

$$\approx 31.60696 \text{ approx}$$

II use of inverse functions

When we see an exponential function,
Solve for the term with the exponent,
then take the log of both sides
{ any base works }

EX

Solve for x:

$$2e^{6x} = 13 \quad \downarrow \text{isolate}$$

$$e^{6x} = 13/2 \quad \downarrow \text{take log}$$

$$\ln (\quad)$$

$$\ln (e^{6x}) = \ln (13/2)$$

$$6x \cdot \ln(e) = \ln(13/2)$$

$$6x = \ln(13/2)$$

$$x = \frac{\ln(13/2)}{6} = \frac{\ln(13) - \ln(2)}{6}$$

$$\log_a(a) = 1$$

EX

Solve $\log_6 (4x-6) = \log_6 (2)$

• We see $\log = \log$ then we equate their arguments

$4x - 6 = 2 \Rightarrow \boxed{x = 2}$

EX

$\log_6 (4x-6) = \log_6 (2) + 1$

Since this is a log eqn lets exponentiate both sides ... raise both sides as powers of base 6

Exponentiation \rightarrow

$6^{\log_6 (4x-6)} = 6^{[\log_6 (2) + 1]}$ \rightarrow $a^{n+m} = a^n a^m$

$4x - 6 = 6^{\log_6 (2)} \cdot 6^1$

$4x - 6 = 2 \cdot 6$

$4x = 12 + 6$

$4x = 18$

$\boxed{x = 18/4}$

Alt: $\log_6 (4x-6) = \log_6 (2) + \log_6 (6)$

$\log_6 (4x-6) = \log_6 (2 \cdot 6)$

$4x - 6 = 2 \cdot 6$

$\boxed{x = 18/4}$

EX Solve $2^{x+1} = 5^{2x-1}$

Take the log of both sides: Since diff bases pick a base on your calculator:

$$\ln(2^{x+1}) = \ln(5^{2x-1})$$

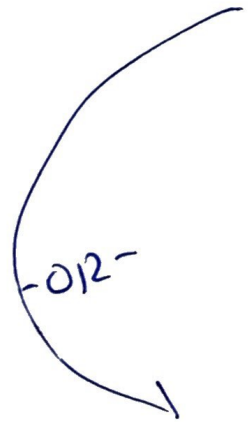
$$(x+1)\ln(2) = (2x-1)\ln(5)$$

$$x\ln(2) + \ln(2) = 2x\ln(5) - \ln(5)$$

$$x\ln(2) - 2x\ln(5) = -\ln(5) - \ln(2)$$

$$x[\ln(2) - 2\ln(5)] = -[\ln(5) + \ln(2)]$$

$$x = \frac{-[\ln(5 \cdot 2)]}{[\ln(2) - \ln(25)]}$$



$$x = \frac{-\ln(10)}{\ln(\frac{2}{25})}$$

$$x = \frac{\ln(10)}{\ln(25) - \ln(2)}$$

$$x = \frac{\ln(10)}{\ln(12.5)} \text{ exact ans.}$$

BTW $\frac{\ln(6)}{\ln(2)} \neq \ln(\frac{6}{2})$ No No

EX

Solve $10 - 4 \ln(9 - 8x) = 6$

Instead of Brute for $e^{(\quad)} = e^{(6)}$

lets isolate the ln:

$-4 \ln(9 - 8x) = 6 - 10$ $\downarrow \div -4$

$\ln(9 - 8x) = \frac{-4}{-4}$

$\ln(\underbrace{9 - 8x}_e) = 1$

Hence $\log_a(a) = 1$

$\Rightarrow 9 - 8x = e$
 $-8x = e - 9$
 $x = \frac{e - 9}{-8}$

$x = \frac{9 - e}{8}$

EX

Solve w/o a calculator:

$6 \log_8(2) + \frac{\log_8(64)}{3 \log_8(4)}$

Strategy: clean up logs:

$\log_8(2^6) + \frac{\log_8(8^2)}{\log_8(4^3)}$
 $= \log_8(64) + \frac{2 \log_8(8)}{\log_8(64)}$
 $= \log_8(8^2) + \frac{2}{\log_8(8^2)}$
 $= 2 + \frac{2}{2}$

$\rightarrow = 2 + 1$
 $= \boxed{3}$

Quadratic Like problems

6

EX

$$e^{2x} - e^x - 132 = 0$$

We cannot log both sides due to the addition/subtraction

$$\log(a+b) \neq \log(a) + \log(b)$$

Note that if $u = e^x$
then $u^2 = e^{2x}$

Then the problem becomes

$$u^2 - u - 132 = 0$$

which factors

$$(u - 12)(u + 11) = 0$$

$$u = 12$$

but $u = e^x$

$$e^x = 12$$

log both sides

$$\ln(e^x) = \ln(12)$$

$$x = \ln(12)$$

↑
only this

$$u = -11$$

$$e^x = -11$$

$$\ln(e^x) = \ln(-11)$$

$$x = \ln(-11) \quad \times$$