4.6 Solving Exponential & Log Problems ①

There are two general methods used to solve Exp. & Log equations

I. Equivalent Problems (already covered)

Ex

$$e^{4x+1} = 2 \longleftrightarrow \log_e(2) = 4x+1$$

$$\Rightarrow \ln(2) = 4x+1$$

$$\frac{\ln(2)-1}{4} = \frac{4x}{4} \Rightarrow \boxed{x = \frac{\ln(2)-1}{4}} \text{ exact}$$

$$x \approx -0.0767 \text{ approx.}$$

Ex Solve via equiv. problem

$$\log(x^2+1) = 3 \Longleftrightarrow 10^3 = x^2+1$$

$$\Rightarrow x^2 = 10^3 - 1$$

$$x^2 = 1000 - 1$$

$$x^2 = 999$$

$$\boxed{x = \sqrt{999}} \text{ exact}$$

$$\approx 31.60696 \text{ approx}$$

$\boxed{\text{II}}$ use of inverse functions

When we see an exponential function, solve for the term with the exponent then take the log of both sides {any base works}

EX Solve for x:

$$2e^{6x} = 13$$

isolate

$$e^{6x} = 13/2$$

take log

$$\boxed{\ln(\quad\quad\quad)}$$

$$\ln(e^{6x}) = \ln(13/2)$$

$\log_a(a) = 1$

$$6x \cdot \ln(e) = \ln(13/2)$$

$$6x = \ln(13/2)$$

$$\boxed{x = \frac{\ln(13/2)}{6}} = \frac{\ln(13) - \ln(2)}{6}$$

EX Solve $\log_6(4x-6) = \log_6(2)$

- We see log = log then we equate their arguments

$$4x - 6 = 2 \Rightarrow \boxed{x = 2}$$

EX $\log_6(4x-6) = \log_6(2) + 1$

Since this is a log eqn lets exponentiate both sides ... raise both sides as powers of base 6

Exponentiation → $\boxed{6^{\log_6(4x-6)} = 6^{[\log_6(2) + 1]}}$ $\boxed{a^{n+m} = a^n a^m}$

$$4x - 6 = 6^{\log_6(2)} \cdot 6^1$$

$$4x - 6 = 2 \cdot 6$$

$$4x = 12 + 6$$

$$4x = 18$$

$$\boxed{x = 18/4}$$

Alt: $\log_6(4x-6) = \log_6(2) + \overbrace{\log_6(6)}^{1}$

$$\log_6(4x-6) = \log_6(2 \cdot 6)$$

$$4x - 6 = 2 \cdot 6$$

$$\boxed{x = 18/4}$$

EX Solve $2^{x+1} = 5^{2x-1}$

Take the log of both sides: Since diff'nt bases pick a base on your calculator:

$$\ln(2^{x+1}) = \ln(5^{2x-1})$$

$$(x+1)\ln(2) = (2x-1)\ln(5)$$

$$x\ln(2) + \ln(2) = 2x\ln(5) - \ln(5)$$

$$x\ln(2) - 2x\ln(5) = -\ln(5) - \ln(2)$$

$$x[\ln(2) - 2\ln(5)] = -[\ln(5) + \ln(2)]$$

$$x = \frac{-[\ln(5\cdot 2)]}{[\ln(2) - \ln(25)]}$$

$$\boxed{x = \frac{-\ln(10)}{\ln\left(\frac{2}{25}\right)}}$$

-OR-

$$x = \frac{\ln(10)}{\ln(25) - \ln(2)}$$

$$\boxed{x = \frac{\ln(10)}{\ln(12.5)}}$$ exact ans.

BTW $\frac{\ln(6)}{\ln(2)} \neq \ln\left(\frac{6}{2}\right)$ No No

Ex Solve $10 - 4\ln(9-8x) = 6$

Instead of Brute for $e^{(\quad)} = e^{(6)}$

Lets $\boxed{\text{isolate the ln}}$:

$$-4\ln(9-8x) = 6 - 10 \quad \Big\} \div -4$$

$$\ln(9-8x) = \frac{-4}{-4}$$

$$\ln(\underbrace{9-8x}_{e}) = 1$$

Hmm $\log_a(a) = 1$

$$\Longrightarrow \quad 9 - 8x = e$$

$$-8x = e - 9$$

$$x = \frac{e-9}{-8}$$

$$\boxed{x = \frac{9-e}{8}}$$

Ex Solve w/o a calculator:

$$6\log_8(2) + \frac{\log_8(64)}{3\log_8(4)}$$

Strategy: clean up logs:

$$\log_8(2^6) + \frac{\log_8(8^2)}{\log_8(4^3)}$$

$$= \log_8(64) + \frac{2\log_8(8)}{\log_8(64)}$$

$$= \log_8(8^2) + \frac{2}{\log_8(8^2)}$$

$$= 2 + \frac{2}{2}$$

$$= 2 + 1$$

$$= \boxed{3}$$

Quadratic Like problems

Ex $e^{2x} - e^{x} - 132 = 0$

we cannot log both sides due to the addition/subtraction $\log(a+b) \neq \log(a) + \log(b)$

Note that if $u = e^{x}$
then $u^2 = e^{2x}$

Then the problem becomes

$$\boxed{u^2 - u - 132 = 0}$$

which factors

$$(u-12)(u+11) = 0$$

$u = 12$

but $u = e^{x}$

$e^{x} = 12$

log both sides

$\ln(e^{x}) = \ln(12)$

$$\boxed{x = \ln(12)}$$

only this

$u = -11$

$e^{x} = -11$

$\ln(e^{x}) = \ln(-11)$

$$\boxed{x = \cancel{\ln}(-11)}$$

X