

4.5 Properties of Logarithms

①

* Equivalent Problem properties

$$a^x = b \iff \log_a(b) = x$$

• $a^0 = 1 \iff \log_a(1) = 0$ ~~1~~

• $a^1 = a \iff \log_a(a) = 1$

• $a^2 = a^2 \iff \log_a(a^2) = 2$

• $a^r = a^r \iff \log_a(a^r) = r$

* Inverse function / Function properties

let $f(x) = a^x$, then $f^{-1}(x) = \log_a(x)$

so $(f \circ f^{-1})(x) = x$

$\rightarrow f(f^{-1}(x)) = x$

$$a^{\log_a(x)} = x$$

• likewise

$(f^{-1} \circ f)(x) = x$

$$\log_a(a^x) = x$$

* General properties

(2)

$$a^{n+m} = a^n a^m \leftrightarrow \log_a(a^n a^m) = n+m$$

- let $a^n = N \leftrightarrow \log_a(N) = n$
- let $a^m = M \leftrightarrow \log_a(M) = m$

$$\log_a(NM) = \log_a(N) + \log_a(M)$$

{ Note $\log_a(N+M) \neq \log_a(N) + \log_a(M)$
 \Rightarrow the \log_a functions are non-linear }

Next

$$a^{n-m} = \frac{a^n}{a^m} \leftrightarrow \log_a\left(\frac{a^n}{a^m}\right) = n-m$$

$$\log_a\left(\frac{N}{M}\right) = \log_a(N) - \log_a(M)$$

Next

$$(a^m)^r = a^{mr} \leftrightarrow \log_a(a^{mr}) = mr$$

$$\log_a(M^r) = r \log_a(M)$$

⊗ change of base

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$$\log_a(M) = \frac{\log_b(M)}{\log_b(a)}$$

• derivation

[Faint handwritten notes and diagrams are visible in this section, including a coordinate system with axes and various mathematical expressions.]

⊗ Applications of the properties

(4)

To solve logarithmic eqns we seek to

Combine log terms

EX

Combine into one logarithm

$$\begin{aligned} & \log(7) + \log(x) + \log(y) \\ &= \boxed{\log(7 \cdot x \cdot y)} \end{aligned}$$

EX

$$\begin{aligned} & 3\ln(x) - 2\ln(x-1) + \ln(5) \\ &= 3\ln(x) + \ln(5) - 2\ln(x-1) \\ &= \ln(x^3) + \ln(5) - \ln(x-1)^2 \\ &= \ln(x^3 \cdot 5) - \ln(x-1)^2 \\ &= \boxed{\ln\left(\frac{5x^3}{(x-1)^2}\right)} \end{aligned}$$

⊕ Expand a single logarithm into sums & differences (5)

EX

$$\begin{aligned} & \log(\sqrt{x^3 y^{-4}}) \\ &= \log((x^3 y^{-4})^{1/2}) \\ &= \frac{1}{2} \log(x^3 y^{-4}) \\ &= \frac{1}{2} [\log(x^3) + \log(y^{-4})] \\ &= \frac{1}{2} [3 \log(x) + (-4) \log(y)] \\ &= \boxed{\frac{3}{2} \log(x) - 2 \log(y)} \end{aligned}$$

$$\sqrt{a} = a^{1/2}$$

* Change of Base application

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$$\log_a(x) = \frac{\log_b(x)}{\log_b(a)}$$

EX Use a calculator and evaluate

$$\begin{aligned} & \log_3(18) \\ &= \frac{\ln(18)}{\ln(3)} \\ &= \frac{2.89037}{1.09861} \\ &= \boxed{2.63093} \end{aligned}$$

18 [LN] [÷] 3 [LN] [=]

wait

$$\begin{aligned} & \log_3(18) \\ &= \frac{\log(18)}{\log(3)} \\ &= \boxed{2.63093} \end{aligned}$$

18 [LOG] [÷] 3 [LOG] [=]

the same.