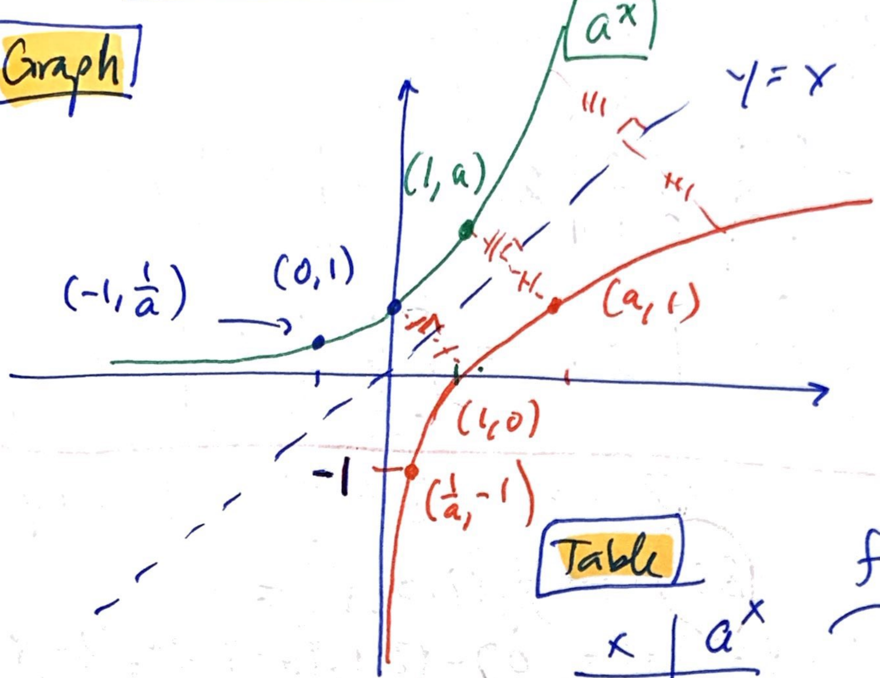
4.3 Inverse Functions of Exponentials

Graph



If a^x is a 1-to-1 function then we have a function that has an inverse function

Table

x	a^x
-1	$1/a$
0	1
1	a

$\xrightarrow{f^{-1}}$

x	$f^{-1}(x) =$
$1/a$	-1
1	0
a	1

analytical We know what the graph of the inverse of a^x looks like, but we then seek an analytical description of it:

1. $f(x) = a^x$
2. $y = a^x$
3. $x = a^y$
4. Solve for y ??? Hmm
   $\Rightarrow$ invent a new function $\boxed{f^{-1}(x) \equiv \log_a(x)}$
   $y = \log_a(x)$
5. $\boxed{f^{-1}(x) = \log_a(x)}$ ← each base, a^x, has its own inverse.

⊛ Base property of the logarithm:

- all function / function inverse pairs have the following property:

$$\boxed{\begin{gathered}(f \circ f^{-1})(x) = x \\ \text{and} \\ (f^{-1} \circ f)(x) = x\end{gathered}}$$

- specifically, if $f(x) = a^x$ & $f^{-1}(x) = \log_a(x)$

then $$\boxed{a^{\log_a(x)} = x}$$

and $$\boxed{\log_a(a^x) = x}$$

* Equivalent principle

exponent

$$\boxed{a^x = b \iff \log_a(b) = x}$$

logarithms are exponents!

Ex Write $4^x = y$ as a logarithm:

$a = 4$ $b = y$ $\Rightarrow$ $\log_4(y) = x$

Ex If $2^m = 128$ what is m?

$\log_2(128) = m$

Ex Write $a^3 = 8$ as a logarithm:

$\log_a(8) = 3$

* Nomenclature

- $\log_{10}(x)$ is just written as $\boxed{\log(x)}$
- $\log_e(x)$ is just written as $\boxed{\ln(x)}$

"Natural Log"

{warning: in some older & foreign texts $\log(x)$ is our $\ln(x)$.}

* writing logs as exponents

Ex Convert $\log_5(x) = 2$ into an exponent:

$a = 5$ b 2

$\Longrightarrow \boxed{5^2 = x}$ we just solved for "x"!

{Note: Hey 5^2 is 25 ... so

$\boxed{\log_5(25) = 2}$? yes }

Ex Solve for x: $\log_5(x) = 3$

equivalent problem: $5^3 = x \Longrightarrow x = 125$

{Note: $\log_5(125) = 3$ }

Ex Solve for x: $\log_2(x) = 128$

equiv. prob: $2^{128} = x$ Big #

Calc:

2 $\boxed{y^x}$ 128 $\boxed{=}$ 3.4028×10^{38}

* Solving log problems via Equiv. Prob.

EX Solve $\log_a(x) = b$

write equiv. prob. $a^b = x$

* Evaluating a log problem without a calculator

EX Evaluate $\log_{10}(100)$

w/ calculator: $\log(100)$ 100 $\boxed{\log}$ → 2

w/o calculator: equiv. problem

(i) but 1st assign "x" to $\log_{10}(100)$

So

$$\log_{10}(100) = x$$

(ii) equiv. Prob $10^x = 100$

(iii) State x from experience : $\boxed{x = 2}$

EX Eval $\log_3(81)$

(i) $\log_3(81) = x$

(ii) $3^x = 81$

(iii) $\boxed{x = 4}$ since $3^4 = (3^2)^2 = 9^2 = 81$

EX Solve for x: $\log_2(3x+4) = 4$

EProb: $\boxed{2^4 = 3x+4}$

$16 = 3x+4$

$12 = 3x$

$\boxed{x = 4}$

EX Evaluate $\log(100^8)$

(i) $\log(100^8) = x$

(ii) $10^x = 100^8$

(iii) $10^x = (10^2)^8$

$10^x = 10^{16}$

$\boxed{x = 16}$

100 $\boxed{y^x}$ 8 $\boxed{=}$ $\boxed{LOG}$

$\rightarrow 16$

EX Evaluate $\log(0.001)$

(i) $\log(0.001) = x$

(ii) $10^x = 0.001$

(iii) $10^x = 10^{-3}$

$\boxed{x = -3}$

0.001 $\boxed{LOG}$

$\rightarrow -3$

Ex Solve for x $\ln(2x+1) = 2$

(E.Prob.)

$$e^2 = 2x+1$$

$$e^2 - 1 = 2x$$

$$\boxed{x = \frac{e^2-1}{2}}$$ exact answer.

$x \approx 3.1945$ approx. answer

2 $\boxed{2^{nd}}$ $\boxed{e^x}$ $\rightarrow$ 7.38905...

$\boxed{-}$ 1 $\boxed{=}$

$\boxed{\div}$ 2 $\boxed{=}$ $\rightarrow$ 3.1945