3.5 Dividing Polynomials

Review 5th grade division

Turn $\frac{37}{3}$ into a proper fraction

$$
\begin{array}{r}
12 \\
3\,\overline{)\,37} \\
-(3)\downarrow \\
\hline
07 \\
-(6) \\
\hline
1
\end{array}
$$

$\frac{37}{3} = 12 \text{ r } 1 = 12 + \frac{1}{3}$

$= \boxed{12\frac{1}{3}}$

- we do the same for polynomials:

Divide $\frac{2x^2 - 9x - 5}{x-5}$

$$
\begin{array}{r}
2x + 1 \\
x-5\,\overline{)\,2x^2 - 9x - 5} \\
-(2x^2 - 10x)\downarrow \\
\hline
x - 5 \\
-(x-5) \\
\hline
0
\end{array}
$$

- Answer:

$$\boxed{\frac{2x^2 - 9x - 5}{x-5} = 2x + 1 + \frac{0}{x-5}}$$

remainder

mult

$* \, x-5$

BTW: $2x^2 - 9x - 5 = (2x+1)(x-5)$ Factored!!

Ex long divide $\dfrac{x^3-126}{x-5}$

Fill in missing powers

$$
\begin{array}{r l}
 & x^2 + 5x + 25 \\
\cline{2-2}
x-5 \,\big) & x^3 + 0x^2 + 0x - 126 \\
 & -(x^3 - 5x^2) \\
\cline{2-2}
 & \quad 5x^2 + 0x \\
 & \quad -(5x^2 - 25x) \\
\cline{2-2}
 & \qquad 25x - 126 \\
 & \qquad -(25x - 125) \\
\cline{2-2}
 & \qquad\qquad -1
\end{array}
$$

Ans: $\boxed{\dfrac{x^2+5x+25}{x-5} = (x^2+5x+25) + \dfrac{-1}{x-5}}$

Ex $\dfrac{x^4-3x^2+5x-11}{x^2-1}$

$$
\begin{array}{r l}
 & x^2 - 2 \\
\cline{2-2}
x^2+0x-1 \,\big) & x^4 + 0x^3 - 3x^2 + 5x - 11 \\
 & -(x^4 + 0x^3 - x^2) \\
\cline{2-2}
 & \qquad -2x^2 + 5x - 11 \\
 & \qquad -(-2x^2\ 0x + 2) \\
\cline{2-2}
 & \qquad\qquad 5x - 13
\end{array}
$$

r

Ans $\boxed{\dfrac{x^4-3x^2+5x-11}{x^2-1} = x^2 - 2 + \dfrac{5x-13}{x^2-1}}$

If the divisor is a linear binomial with a leading coefficient on the x term to be 1 then we can use synthetic division

Ex

$$\frac{3x^3-2x^2+x-4}{x+3}$$

$$\begin{array}{r} 3x^2-11x+34 \\ x+3\,\overline{)\,3x^3-2x^2+x-4} \\ -(3x^3+9x^2) \\ \hline -11x^2+x \\ -(-11x^2-33x) \\ \hline 34x-4 \\ -(34x+102) \\ \hline -106 \end{array}$$

$$\Rightarrow \boxed{\frac{3x^2-11x+34}{x+3} = 3x^2-11x+34-\frac{106}{x+3}}$$

- Synthetic Division looses all the baggage of "+", "x"

change sign insert "0"

$$\begin{array}{r|rrrr} -3 & 3 & -2 & 1 & -4 \\ & & -9 & 33 & -102 \\ \hline & 3 & -11 & 34 & -106 \end{array}$$

remainder

$$\boxed{\frac{3x^3-2x^2+x-4}{x+3} = 3x^2-11x+34-\frac{106}{x-3}}$$

- Consider $f(x) = \underbrace{3x^3 - 2x^2 + x}_{} - 4$

$$f(x) = x\,(3x^2 - 2x + 1) - 4$$

$$f(x) = x\,[x(3x-2)+1] - 4$$

Next let $x = -3$ {comes from $x + 3 = 0$}

then $f(-3) = -3\,[-3\,(\underbrace{3\cdot(-3)}_{-9} - 2) + 1] - 4$

$$= -3\,[-3\,(\underbrace{-9-2}_{})+1] - 4$$

$$= -3\,[\underbrace{-3\,(-11)}_{}+1] - 4$$

$$= -3\,[\underbrace{33+1}_{}] - 4$$

$$= \underbrace{-3\,[34]}_{} - 4$$

$$= \underbrace{-102-4}_{}$$

$$\boxed{f(-3) = -106}$$

$f(c)$ = Remainder left over after dividing f by $x - c$

The remainder in synthetic division is the original polynomial evaluated by the constant term in the divisor {change of sign}.

ie, by the root of the divisor