3.4 Graphs of Polynomials

((3.3 was skipped))

polynomial : $a_n x^n + a_{n-1} x^{n-1} + \cdots + a_2 x^2 + a_1 x^1 + a_0 = 0$

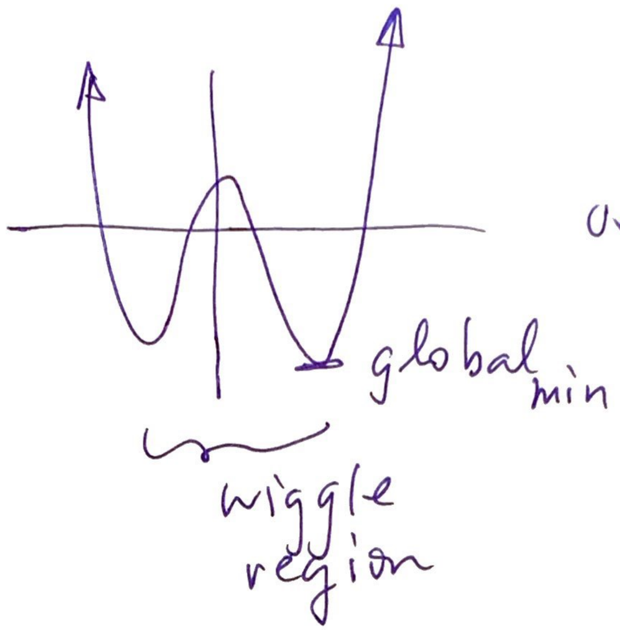
Ex $4x^6 - x^3 + 11x^2 - 14 = 0$

* no fractional or decimal powers.

* "degree" = highest degree of all terms in the polynomial.

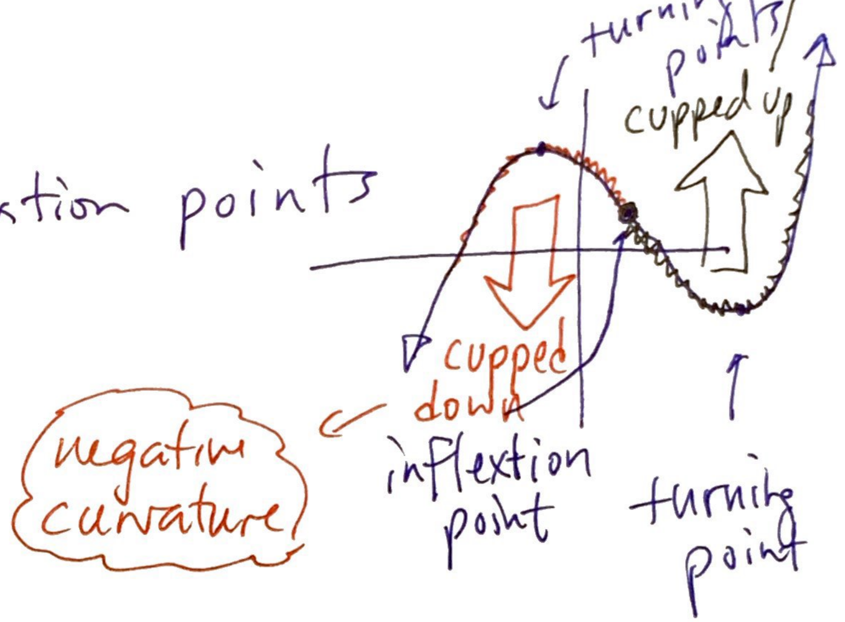
* decreasing power (please)

* generally polynomials have a wooden roller coaster appearance:



or

* calculus " inflexion points



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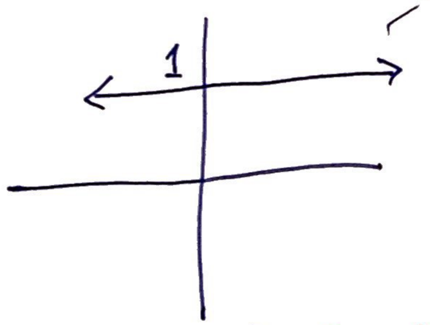
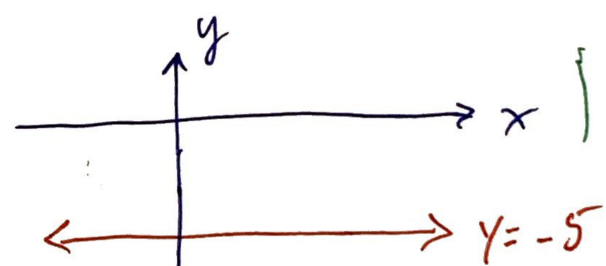
* generalities

n=0



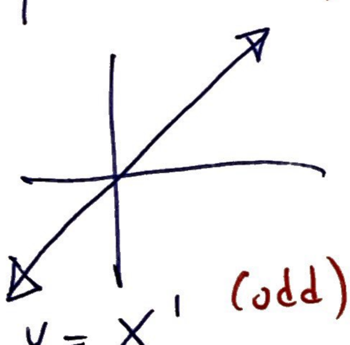
$y = x^0$ (even)

Constant



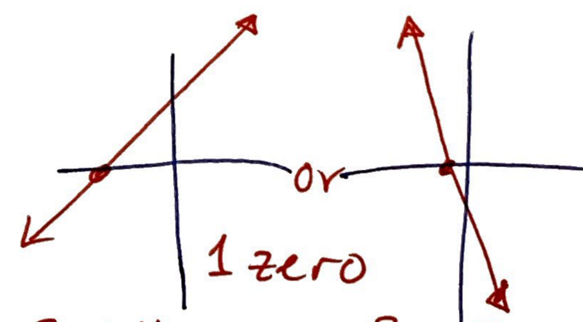
0 zeros

n=1



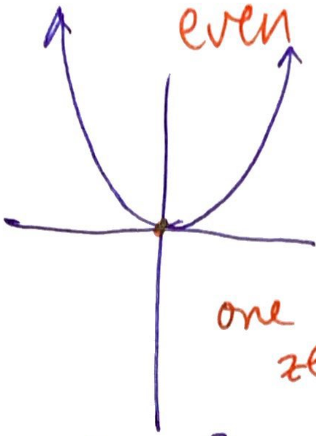
$y = x^1$ (odd)

linear



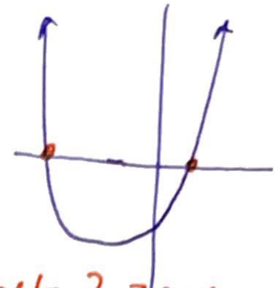
$y = 2x+4$ $\quad = -3 -2$

$n=2$



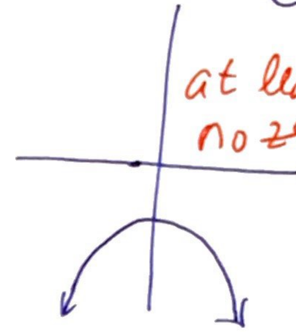
$y=x^2$

quadratic



at most 2 zeros

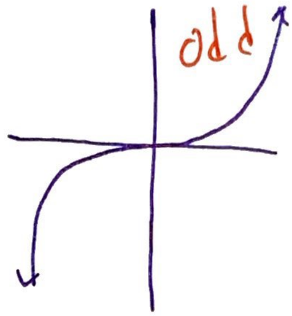
$y=2x^2-x-3$



at least no zeros

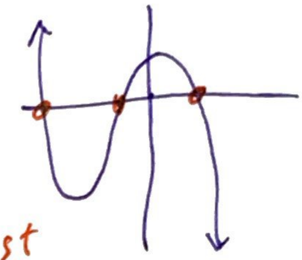
$y=-x^2-2$

$n=3$



$y=x^3$

cubic



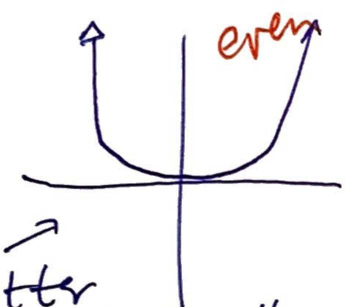
at most 3 zeros

$y=-x^3+x+2$

at least one zero

$y=x^3-4x-3$

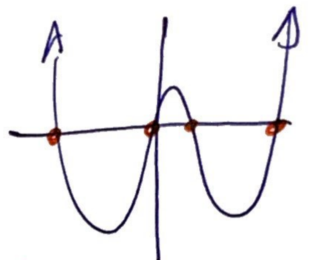
$n=4$



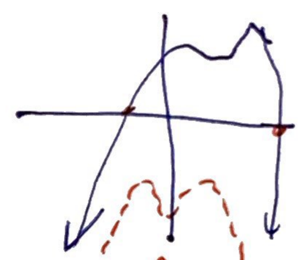
flatter bottoms

$y=x^4$

quartic



at most 4 zeros



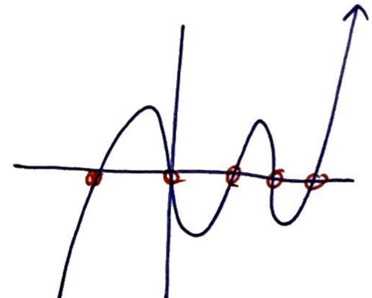
at least no zeros

$n=5$

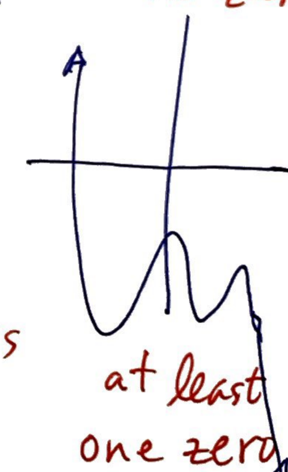
flatter

$y=x^5$

quintic



at most 5 zeros



at least one zero

⋮

factored form **Fundamental Thm of Algebra** 4

all polynomials of degree n, that is

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \cdots + a_2 x^2 + a_1 x^1 + a_0 x^0 = 0$$

can be factored into n = binomials

$$f(x) = a(x - r_1)(x - r_2)(x - r_3) \cdots (x - r_{n-1})(x - r_n)$$

where r_i can be real, zero or complex conjugate $3+2i,\ 3-2i$

or irrational conjugate $1+\sqrt{2},\ 1-\sqrt{2}$

EX $f(x) = x^6 - 2x^4 - 3x^2$ Q: what are the zeros and y-int

* factor

$$f(x) = x^2(x^4 - 2x^2 - 3)$$

$$= x^2(x^2+1)(x^2-3)$$

complex conjugates, irrational conjugates

$$f(x) = x^2(x+i)(x-i)(x+\sqrt{3})(x-\sqrt{3})$$

multiplicity 2, the rest are multiplicity one

- Real zeros [x-int] are $\begin{cases} x = 0 \text{ (multiplicity 2)} \\ x = \pm\sqrt{3} \end{cases}$
- Complex zeros are $x = \pm i$
- y-int : let $x=0 \Rightarrow f(0) = 0^6 - 2\cdot 0^4 - 3\cdot 0^2 = \boxed{0}$

what are the zeros of

$$f(x) = 2x^3 - x^2 - 8x + 4$$

$$= x^2(2x-1) - 4(2x-1)$$

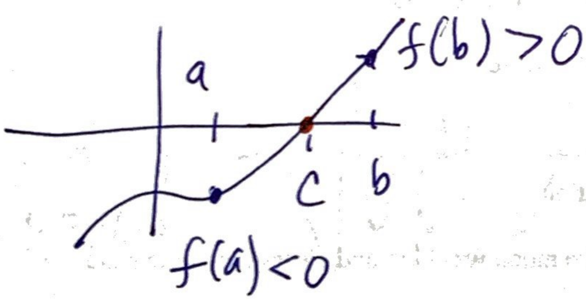
$$= (2x-1)[x^2-4]$$

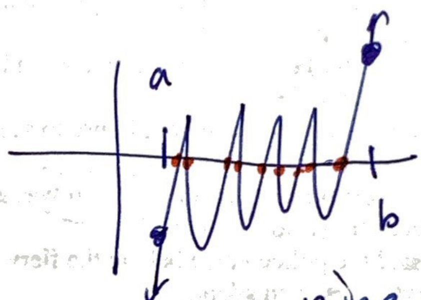
$$\boxed{f(x) = (2x-1)(x+2)(x-2)}$$

$\overbrace{2x-1=0}$	$\overbrace{x+2=0}$	$\overbrace{x-2=0}$
$\boxed{x = 1/2}$	$\boxed{x=-2}$	$\boxed{x=2}$

* Intermediate Value Theorem

Let $f(x)$ be a polynomial function. The I.V.T. states that if $f(a)$ and $f(b)$ have opposite signs then there is at least one value "c" between a and b such that $f(c)=0$





nine crossings

EX Confirm there exists a zero between $x=2$ & $x=4$ for $f(x) = x^3 - 9x$

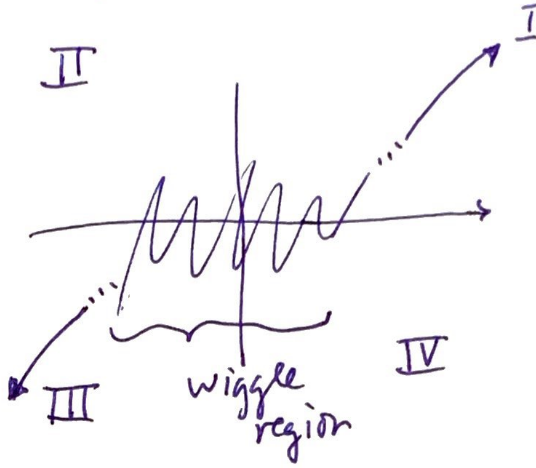
- $f(2) = 2^3 - 9 \cdot 2 = 8 - 18 = -10$ (−)
- $f(4) = 4^3 - 9 \cdot 4 = 64 - 36 = 28$ (+)

"there exists"

an element of

so, yes, $\exists$ at least one $x \in [2,4]$ where $f(x)=0$.

Far away behavior: for large x (+) or large x (−)

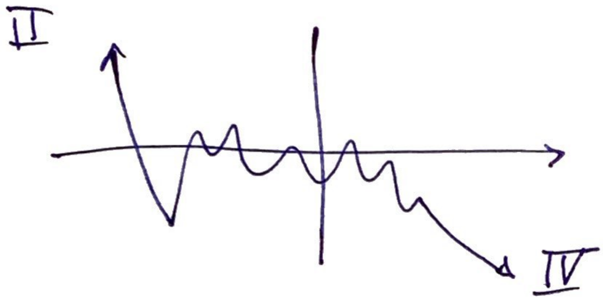


$f(x) = a_n x^n + \cdots + a_0$

a_0: y-int. BTW

odd degree
$n = 1, 3, 5, \ldots$

$a_n > 0$ (+)
starts in III ends in I



$a_n < 0$ (−)
starts in II ends in IV

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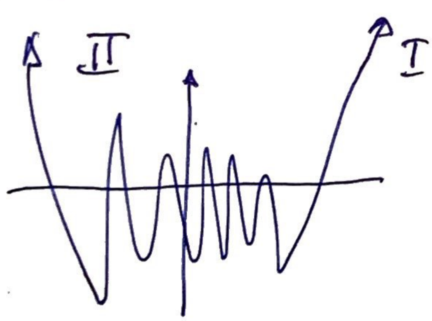
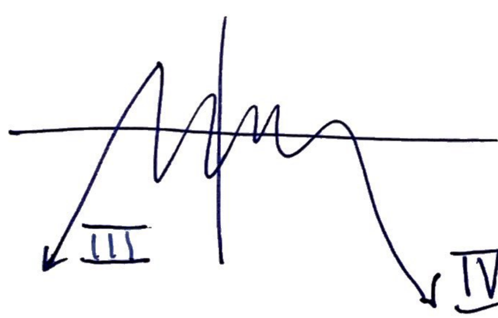
even degree
$n = 0, 2, 4, \ldots$



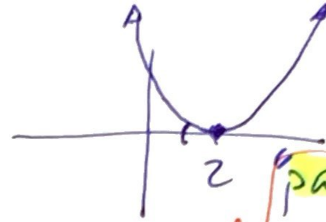
if $a_n > 0$ (+)
starts in II & leaves in I



if $a_n < 0$ (−)
starts in III & ends IV

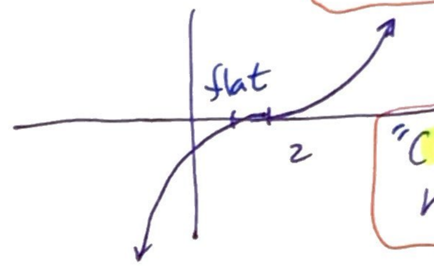
Local crossing behavior

$f(x) = (x-2)^2$



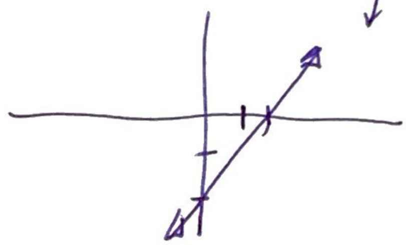
"parabolic" near $x=2$
"touch-n-go"

$f(x) = (x-2)^3$



"cubic" like near $x=2$

$f(x) = (x-2)^1$



"line-like" near $x=2$

EX Discuss the crossing mechanisms for

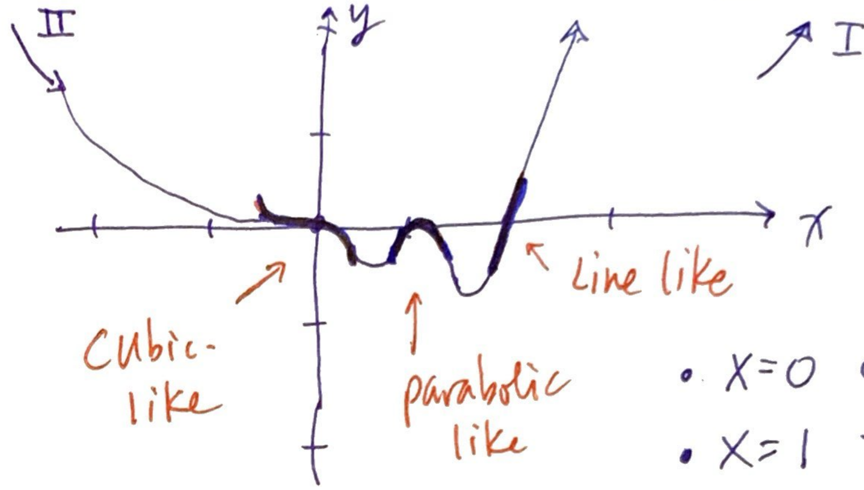
$f(x) = (x-2)(x-1)^2(x)^3$

- $x=0$ multiplicity 3 "cubic-like" crossing
- $x=1$ multiplicity 2 "parabolic-like" crossing
- $x=2$ multiplicity 1 "line-like" crossing

6th degreee

EX cont. Sketch $f(x)=(x-2)(x-1)^2x^3 = x^6+\dots+0$

• end behavior



• $x=0$ cubic-like

• $x=1$ touch-n-go

• $x=2$ linear-like

Genform:

$$x^3(x-2)(x-1)^2 = x^3(x-2)(x^2-2x+1)$$

$$= x^3\left[x^3-2x^2+x-2x^2+4x-2\right]$$

$$\boxed{f(x)=x^6-4x^5+5x^4-2x^3}$$ Gen.

EX Sketch $m(x)=-2x(x-1)^2(x+3)^1 = -2x^4+\dots x$

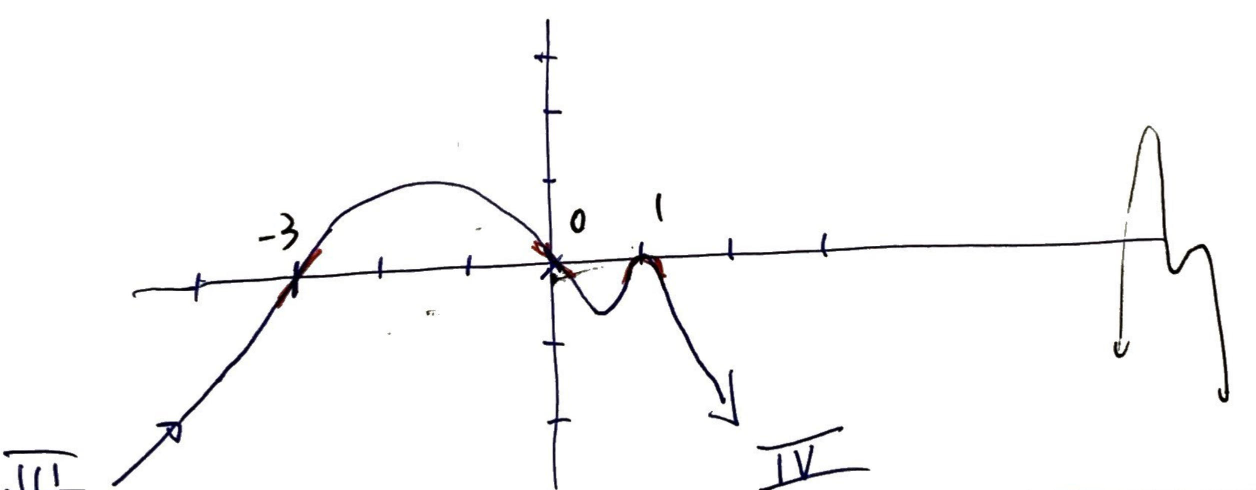
• $x=0$ linear, mult is 1

• $x=1$ parabolic, mult is 2

• $x=-3$ linear, mult. is 1

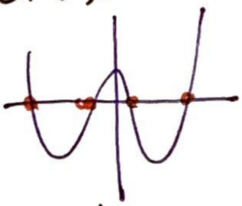
• far away end behavior

III $\to$ IV



EX what is the eqn of a degree 4 polynomial that has a multiplicity 2 zero @ $x=4$ and multiplicity 1 zeros @ $x=1$ and $x=-2$. It also has a y-intercept @ 3. (0,3) ← pt.

• zeros: we have 4 zeros



or

• Factored Form

$$\boxed{f(x) = a(x-4)^2(x-1)(x+2)}$$ ← Form

$$\underbrace{f(0)}_{3} = a(0-4)^2(0-1)(0+2)$$ ← Point

$$3 = a \cdot 16 \cdot (-1) \cdot 2)$$

$$3 = -32a \Rightarrow \boxed{a = -\frac{3}{32}}$$ Solve

So $$\boxed{f(x) = -\frac{3}{32}(x-4)^2(x-1)(x+2)}$$ ← Final

3.4 is done