Chapter 3 Polynomial Functions

Polynomial functions have the form

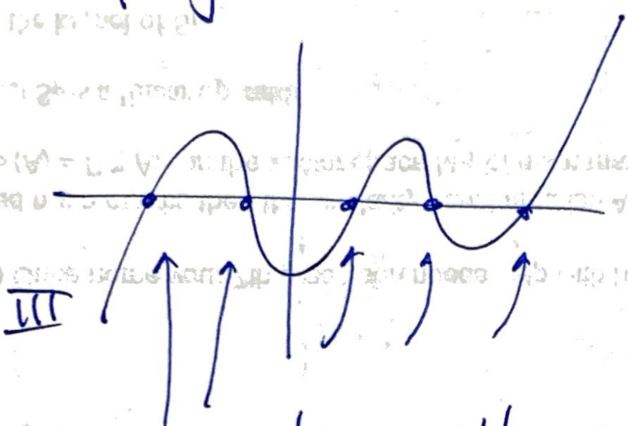
$$\boxed{f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0}$$

EX:

$$f(x) = 5x^3 + 2x + 1$$

$$g(x) = x^7 - x^5 + x^2 + 3x$$

- Roots of polynomial function:



5 roots so this 5^{th} degree or higher $\{7, 9, \cdots\}$

roots are the values of x such that $f(x) = 0$

{a.k.a. zeros of $f(x)$} Sec. 3.6

- Rational functions are ratios of polynomials

$$\boxed{h(x) = \frac{a_3 x^3 + a_2 x^2 + a_1 x + a_0}{b_2 x^2 + b_1 x + b_0}} \text{ etc.}$$

3.1 Complex Variables

Back in the days of pyrimid building a student asked his math teach "what number squared is -1?"

we call such an <u>imaginary number</u>, i

$$\boxed{i^2 \equiv -1}$$

some might use $\sqrt{-1} \equiv i$

we can mix <u>imaginary numbers</u> with <u>real numbers</u>

$$\boxed{c = a + \underbrace{bi}}$$

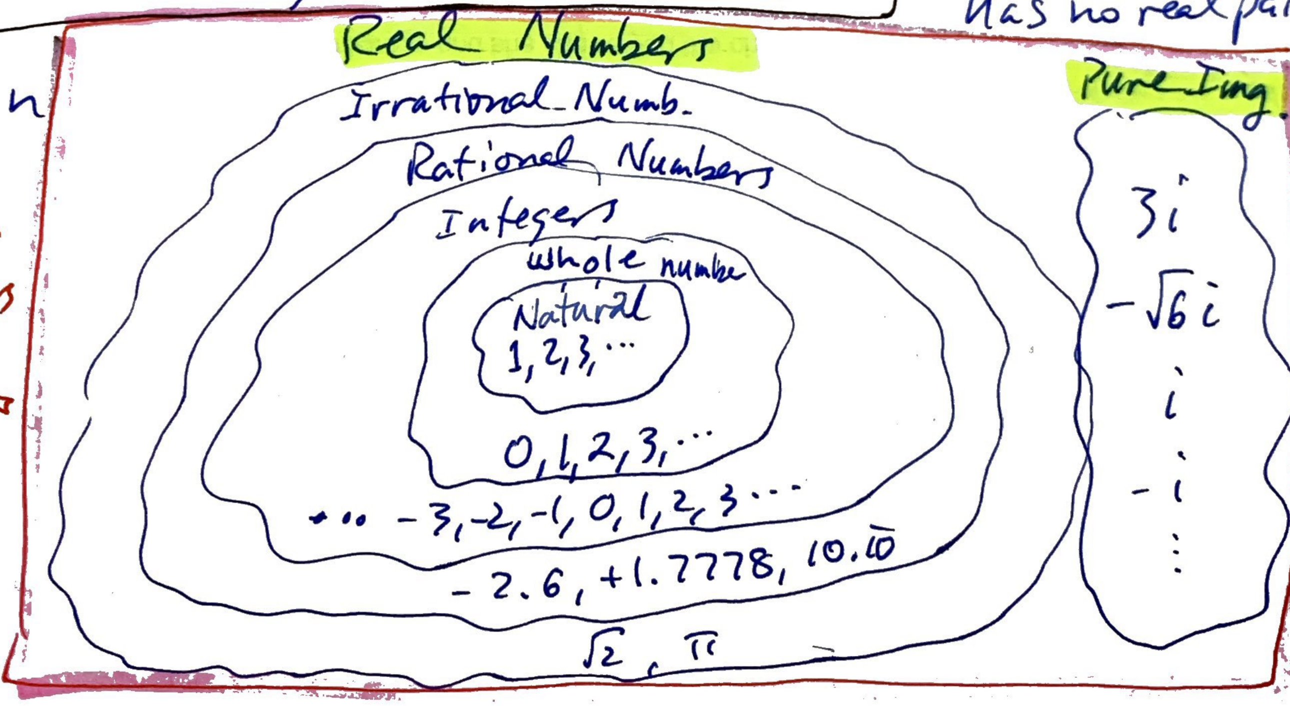
complex number — real part — imaginary part

pure imaginary since this complex number has no real part

EX $\quad 2+3i, \quad -7+\sqrt{2}\,i, \quad \sqrt{3}\,i$

• Venn

Complex Numbers

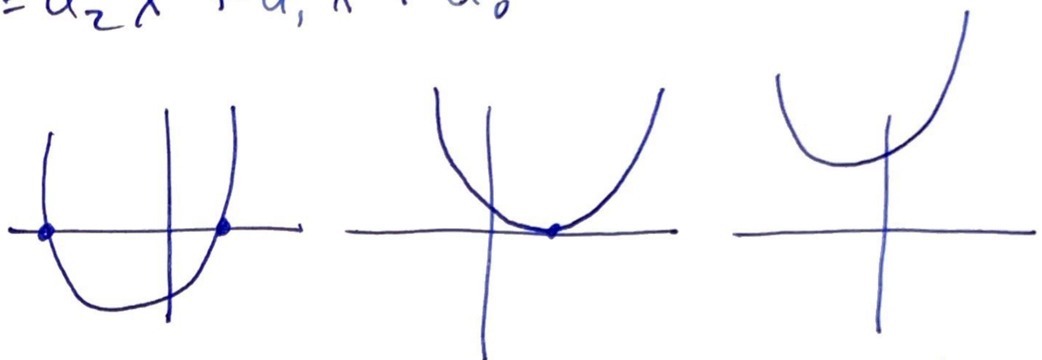


⊛ Focus on the quadratic function

$$f(x) = ax^2 + bx + c \quad \text{Degree } 2$$

$$g(x) = a_2 x^2 + a_1 x + a_0$$

• graph



2 zeros

1 zero {double}

2 complex conjugate zeros.

• Quad. Equation

$$ax^2 + bx + c = 0$$

• zeros are found by the Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Nomenclature

$ax^2 + bx + c$	quadratic expression
$ax^2 + bx + c = 0$	quadratic equation
$ax^2 + bx + c \geq 0$	quadratic inequality
$f(x) = ax^2 + bx + c$	quadratic function
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	quadratic formula

⊛ Deriving the quadratic formula

$$ax^2+bx+c=0$$

$$a\left[x^2+\frac{b}{a}x\right]+c=0$$

$$a\left[\underbrace{x^2+\frac{b}{a}x+\left(\frac{b}{2a}\right)^2}-\left(\frac{b}{2a}\right)^2\right]+c=0$$

$$a\left[\left(x+\frac{b}{2a}\right)^2-\left(\frac{b}{2a}\right)^2\right]+c=0$$

$$a\left(x+\frac{b}{2a}\right)^2-a\left(\frac{b}{2a}\right)^2+c=0$$

$$a\left(x+\frac{b}{2a}\right)^2=-c+\frac{b^2}{4a}$$

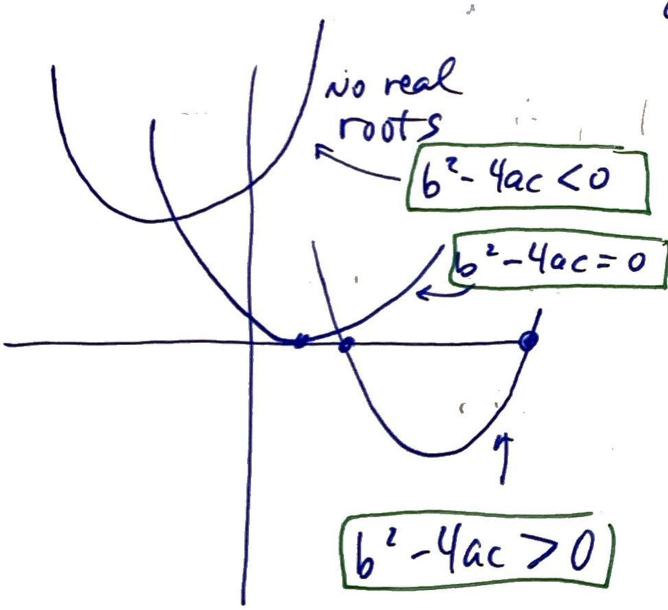
$$\left(x+\frac{b}{2a}\right)^2=-\frac{c}{a}+\frac{b^2}{4a^2}$$

$$\left(x+\frac{b}{2a}\right)=\pm\sqrt{-\frac{c}{a}\left(\frac{4a}{4a}\right)+\frac{b^2}{4a^2}}$$

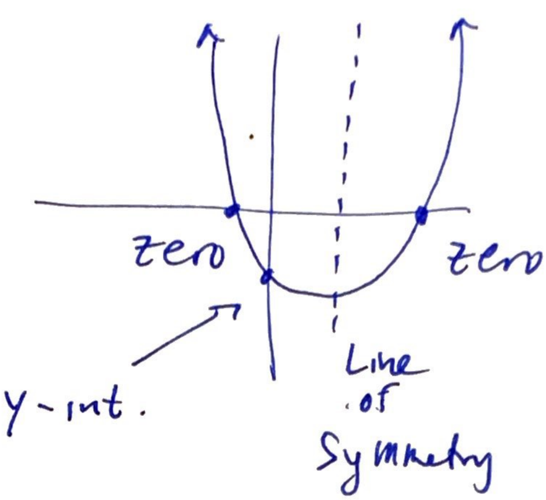
$$x=-\frac{b}{2a}\pm\sqrt{\frac{-4ac+b^2}{4a^2}}$$

$$x=-\frac{b}{2a}\pm\frac{\sqrt{b^2-4ac}}{2a}$$

$$\boxed{x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}}$$



EX Graph $f = 2x^2 - 4x - 10$



- $\boxed{y = 2x^2 - 4x - 10}$
- **zeros** : x-int $y=0$

$2x^2 - 4x - 10 = 0$

$\div 2$

$x^2 - 2x - 5 = 0$

$a=1 \quad b=-2 \quad c=-5$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-5)}}{2(1)}$$

$$x = \frac{2 \pm \sqrt{4+20}}{2}$$

$$x = 1 \pm \frac{\sqrt{24}}{2}$$

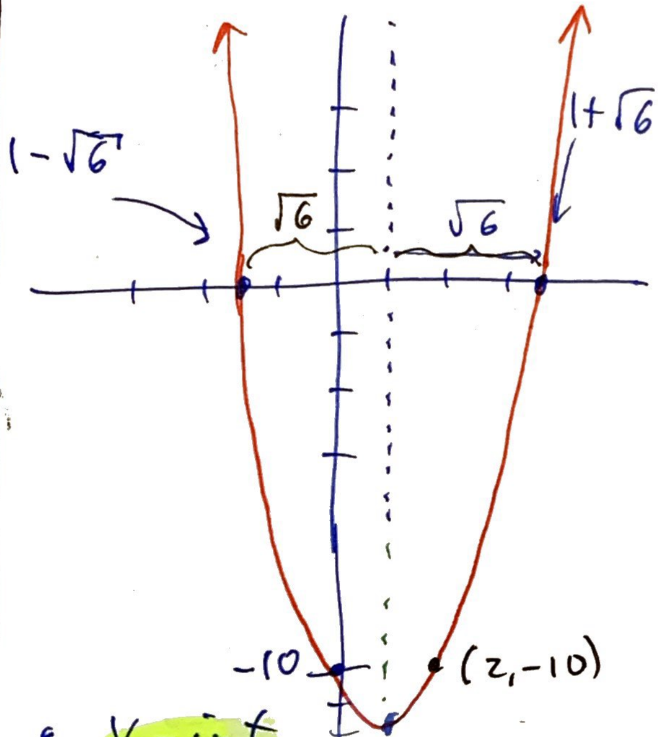
4·6

$$x = 1 \pm \frac{2\sqrt{6}}{2}$$

$$\boxed{x = 1 \pm \sqrt{6}}$$

$\sqrt{6} \approx 2.5$

L.of.Symmetry



- **y-int**

$y = 2 \cdot 0^2 - 4 \cdot 0 - 10$

$\boxed{(0,-10)}$

- **vertex**

$f(1) = 2(1)^2 - 4(1) - 10$

$= 2 - 14$

$= -12$

$\boxed{(1,-12)}$

$$\boxed{x_{l.o.s.} = \frac{-b}{2a}}$$

$$\boxed{f(x_{l.o.s.}) = y \text{ value of vertex}}$$

EX Find the zeros of the function

$f(x) = 2x^2 - 4x + 10$

$$0 = 2x^2 - 4x + 10 \quad \div 2$$

$$0 = x^2 - 2x + 5$$

$a = 1 \quad b = -2 \quad c = 5$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(5)}}{2\cdot(1)}$$

$$x = \frac{2 \pm \sqrt{4-20}}{2}$$

$$x = 1 \pm \frac{\sqrt{-16}}{2} \qquad \sqrt{-1} = i$$

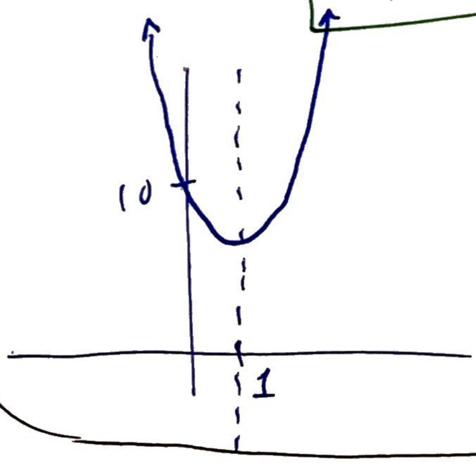
$$x = 1 \pm \frac{4i}{2}$$

$$\boxed{x = 1 \pm 2i}$$

No real roots

complex conjugates

• Roots $\boxed{x_1 = 1 + 2i \qquad x_2 = 1 - 2i}$



• $x_{los} \quad \frac{-b}{2a}$

$$= \frac{-(-2)}{2(1)}$$

$$\underline{x_{los} = 1}$$

* Adding complex numbers

$$\boxed{(a+bi)+(c+di) = (a+c)+(b+d)i}$$

Ex $(2+3i)+(-1+4i)$

$= (2-1)+(3+4)i$

$= \boxed{1+7i}$

* Multiplying complex numbers (FOIL)

$(a+bi)(c+di)$

Ex $(2+3i)(-1+4i)$

$= 2(-1)+2(4i)+(3i)(-1)+(3i)(4i)$

$= -2+8i-3i+12i^2$

$= -2+5i+12i^2$ but $i^2=-1$

$= -2+5i-12$

$= \boxed{-14+5i}$

- Formulas

$(a+b)(a+b) = a^2+2ab+b^2$

$\boxed{(a+b)(a-b) = a^2-b^2}$

$(a+bi)(a-bi)$

$= a^2-(bi)^2$

$= a^2+b^2$

$\boxed{(a+bi)(a-bi) = a^2+b^2}$

Dividing Complex Numbers

This is just a way to keep the denominator free of complex numbers thus the name "rationalizing the denominator"

- form: $\dfrac{a+bi}{c+di}$

- To rationalize the denominator we multiply top and bottom by the conjugate of the denominator:

$\boxed{\text{conjugate of } a+bi \text{ is } a-bi}$

Ex $\dfrac{-1+2i}{-2+3i}$

$$\frac{(-1+2i)}{(-2+3i)}\left(\frac{-2-3i}{-2-3i}\right)$$

$$= \frac{(-1)(-2)+(2i)(-2)+(-1)(-3i)+(2i)(-3i)}{(-2)^2+(3)^2}$$

$$= \frac{2-4i+3i-6i^2}{13}$$

$$= \frac{2-i+6}{13}$$

$$= \frac{8-i}{13}$$

$$= \boxed{\frac{8}{13}-\frac{1}{13}i}$$

* powers of i

$\boxed{i^n}$

$\boxed{i}$

$i^2 = \boxed{-1}$

$i^3 = i\, i^2 = \boxed{-i}$

$i^4 = i^2\, i^2 = (-1)(-1) = \boxed{1}$

$i^5 = i\, i^4 = \boxed{i}$

$i^6 = i^2 i^4 = \boxed{-1}$

$i^7 = i\, i^6 = i(-1) = \boxed{-i}$

$i^8 = i^4 i^4 = 1\cdot 1 = \boxed{1}$

$\vdots$

Q: $i^{63} = ?$

use modulus function $\mathrm{mod}(n,4)$

$\Rightarrow$ $4\overline{)n}$ we use the remainder to select one of the four outcomes

$$\boxed{\begin{aligned} r=0 &\to 1 \leftarrow i^0 \\ r=1 &\to i \leftarrow i^1 \\ r=2 &\to -1 \leftarrow i^2 \\ r=3 &\to -i \leftarrow i^3 \end{aligned}}$$

Ex

$$\begin{array}{r} 15 \text{ r. } 3 \\ 4\overline{)63} \\ \underline{4} \\ 23 \\ \underline{20} \\ 3 \end{array}$$

$(i^4)^{15}$ → $1^{15} = 1$

So $i^{63} = i^{60} \cdot i^3 = 1(-i)$

$\boxed{i^{63} = -i}$