11.6 Binomial Thm

* A binomial is an expression with two terms in it.

$(a+b)$, or $(3+x)$, (y^2-y), ...

* We consider now a binomial raised to the n^{th} power. These terms arise freq'tly in STEM fields

Ex

$(a+b)^0 = 1$

$(a+b)^1 = a+b$

$(a+b)^2 = a^2+2ab+b^2$

$(a+b)^3 = (a+b)(a^2+2ab+b^2)$

$= a^3+\underline{2a^2b}+ab^2+\underline{ba^2}+2ab^2+b^3$

$(a+b)^3 = a^3+3a^2b+3ab^2+b^3$

$(a+b)^4 = (a+b)(a^3+3a^2b+3ab^2+b^3)$

$= a^4+\underline{3a^3b}+\underline{3a^2b^2}+\underline{ab^3}$

$+\underline{ba^3}+\underline{3a^2b^2}+\underline{3ab^3}+b^4$

$n=4$ $\quad (a+b)^4 = a^4+4a^3b+6a^2b^2+4ab^3+b^4$

getting tired yet? There is a better way ...

*pascal's triangle

1

1 1

n=2 → 1 2 1

n=3 → 1 3 3 1

n=4 → 1 4 6 4 1

n=5 → 1 5 10 10 5 1

n=6 1 6 15 20 15 6 1

* compare these numbers to the numerical coefficients for $(a+b)^n$ – they match!

$\boxed{\text{EX}}$ State the expansion of $(a+b)^6$:

$$(a+b)^6 = a^6 + 6a^5b + 15a^4b^2 + 20a^3b^3 + 15a^2b^4 + 6ab^5 + b^6$$

* For the $(a+b)^N$ general expansion we have:

$$\boxed{(a+b)^N = \sum_{k=0}^{N} \frac{N!}{(N-k)!\,k!}\, a^{N-k} b^k} = \sum_{k=0}^{N} \binom{N}{k} a^{N-k} b^k$$

N factorial

$$\boxed{N! \equiv N\cdot(N-1)\cdot(N-2)\cdots\cdot(3)(2)(1)}$$

$\boxed{\text{EX}}$ $6! = 6\cdot5\cdot4\cdot3\cdot2\cdot1 = 720$

- The term $\boxed{\dfrac{N!}{(N-k)!\,k!}\text{ is abbreviated as }\begin{pmatrix}N\\k\end{pmatrix}}$

"$\begin{pmatrix}N\\k\end{pmatrix}$" is called different names: "Combination", "binomial coefficient" to name two.

* Learning how to count:

In counting, $\begin{pmatrix}N\\k\end{pmatrix}$ is written as ${}_NC_k$ and is

the number of ways to extract k items from a collection of N items, ordering is not important.

$\boxed{\text{Ex}}$ We need to send to the office 3 ppl from our collection of 19 students.

The number of ways to do this is ${}_{19}C_3$

or $\begin{pmatrix}19\\3\end{pmatrix}$

- This button is on your calculator.

19 $\boxed{2^{nd}}$ $\boxed{nC_r}$ 3 $\boxed{=}$

don't ask your calculator to perform larger factorials!

use this trick instead.

- Manually

$${}_{19}C_3 = \begin{pmatrix}19\\3\end{pmatrix} = \frac{19!}{(19-3)!\,3!} = \frac{19!}{16!\,3!} = \frac{19\cdot18\cdot17\cdot\cancel{16!}}{\cancel{16!}\,3\cdot2\cdot1} = \frac{19\cdot\overset{3}{\cancel{18}}\cdot17}{\cancel{6}}$$

$= \boxed{969 \text{ ways to choose 3 ppl from a group of 19}}$ To be cont.

$\boxed{III.6}$ (Cont.) Binomial expansion ①

$$\boxed{(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k}$$

$\boxed{EX}$

$n=3$ $$(x+y)^3 = \sum_{k=0}^{3} \binom{3}{k} x^{3-k} y^k$$

$$= \underbrace{\binom{3}{0} x^{3-0} y^0}_{k=0} + \underbrace{\binom{3}{1} x^{3-1} y^1}_{k=1} + \underbrace{\binom{3}{2} x^{3-2} y^2}_{k=2} + \underbrace{\binom{3}{3} x^{3-3} y^3}_{k=3}$$

$$\boxed{\binom{n}{0} = 1, \quad \binom{n}{n} = 1, \quad \binom{n}{n-1} = n, \quad \binom{n}{1} = n}$$

$$= 1x^3y^0 + 3x^2y^1 + 3x^1y^2 + 1\cdot x^0y^3$$

$$\boxed{(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3}$$

If needed $$\binom{n}{k} = \frac{n!}{(n-k)!\,k!}$$

⊛ the r+1 term of the binomial expansion $(x+y)^n = \ldots\ldots$

$$a_{r+1} = \binom{n}{r} x^{n-r} y^{r}$$

r^{th} term

the 1^{st} term has $r = 0$

$\boxed{EX}$ the 11^{th} term, starting counting at 1 of $(x+y)^{17}$ is what?

$$\binom{17}{10} x^{17-10} y^{10}$$

$r = 10$

$$= \frac{17!}{(17-10)!\,10!} x^7 y^{10}$$

$$= \frac{17 \cdot \cancel{16}^{4} \cdot \cancel{15}^{5} \cdot \cancel{14}^{2} \cdot 13 \cdot \cancel{12}^{6} \cdot 11 \cdot \cancel{10!}}{(\cancel{7} \cdot \cancel{6}_{3} \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot 1)\,\cancel{10!}} x^7 y^{10}$$

$$= \frac{17 \cdot 4 \cdot 13 \cdot \cancel{6}^{2} \cdot 11}{\cancel{3}} x^7 y^{10}$$

$$= \boxed{19{,}448\, x^7 y^{10}}$$ 11^{th} term start counting @ 1

$$\begin{array}{r} 22 \\ \times 52 \\ \hline \\ \times 17 \\ \hline 1 \end{array}$$

~~TI-30IIa~~ TI-30Xa ✓

end of 11.6