# 11.3 Geometric Sequences



In this section we will notice that the sequence terms vary by a ratio.

**ex** $\{2, 8, 32, 128, \ldots\}$

+6, +24, +96 ... ← Not Arithmetic

*4 *4 *4 ← Geometric series

Form ratio by ÷ consecutive terms:

$\frac{8}{2} = 4, \quad \frac{32}{8} = 4, \quad \frac{128}{32} = 4$ ratio

## The general term

Recursive form

$$a_n = r\,a_{n-1}$$

let "a" be the 1st term.

$n=1 \quad a_1 = a$

$n=2 \quad a_2 = r a_1 = r(a) = ra$

$n=3 \quad a_3 = r a_2 = r(ra) = r^2 a$

$$a_n = r^{n-1} a$$

direct general term.

↑ 1st term $a_1$

**EX** Find the first 5 terms in the sequence $\{3, 9, 27, 81, 243 \cdots\}$

$*3$ $*3$ $\leftarrow r$ common ratio

**EX** Find the general term in the above Example

$$a_n = r^{n-1} a$$

- $n=1$ : $a_1 = r^0 \cdot a = 1 \cdot 3$

So $\boxed{a = 3}$

we need the ratio "r":

$n=2$ $\quad a_2 = r^{2-1} \cdot 3$

$9 = r^1 \cdot 3$ $\quad \boxed{r = 3}$

together

$$\boxed{a_n = 3^{n-1} \cdot 3}$$

or $\boxed{a_n = 3^n}$

Test $\{3, 3^2, 3^3, \cdots, 3^n, \cdots\}$

$= \{3, 9, 27, \cdots, 3^n, \cdots\}$

$n=1$ 2 3 - - - n - - -

**EX** Find the general term for this sequence $\{-2, \frac{1}{2}, -\frac{1}{8}, \frac{1}{32}, -\frac{1}{128}, \dots\}$

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- Gen. Form: $a_n = r^{n-1} a$ — since it appears to be a geom. sequence

Q: what is "r" and what is "a"

↑ Common ratio

↑ starting term

- analytically: match terms
  - $a_2 = r^{2-1} a$
    $\frac{1}{2} = ra$ → $ra = \frac{1}{2}$
  - $a_3 = r^{3-1} a$
    $-\frac{1}{8} = r^2 a$ → $r^2 a = -\frac{1}{8}$

eqns in "r" & "a"

sub

- Solve:

$$\Rightarrow r^2 \left(\frac{1}{2r}\right) = -\frac{1}{8} \Rightarrow \frac{r}{2} = -\frac{1}{8}$$

$$\Rightarrow r = -\frac{1}{8} \cdot 2 = -\frac{1}{4} \qquad \boxed{r = -\frac{1}{4}}$$

Top eqn: $ra = \frac{1}{2}$ but $r = -\frac{1}{4}$

$$\Rightarrow -\frac{1}{4} a = \frac{1}{2} \Rightarrow \boxed{a = -2}$$

$$a_n = \left(-\frac{1}{4}\right)^{n-1} \cdot (-2)$$

(*) C.S.I.

Ex

Write the general term for a geometric sequence where $a_7 = 64$ and $a_{10} = 512$

(i) Start with the form $a_n = r^{n-1} a$

(ii) build two eqns

$a_7 = r^{7-1} a$

$64 = r^6 a$

$a_{10} = r^{10-1} a$

$512 = r^9 a$

(iii) Solve for "a" and "r"

Solve

(÷) $$\frac{64 = r^6 a}{512 = r^9 a} \Rightarrow \frac{64}{512} = r^{-3}$$

$$\Rightarrow r^3 = \frac{512}{64} = \frac{256}{32} = \frac{128}{16} = \frac{64}{8} = 8$$

$$\boxed{r = 2}$$

Use to eqn: $64 = 2^6 a \Rightarrow \boxed{a = 1}$

$$\boxed{a_n = 2^{n-1}}$$