

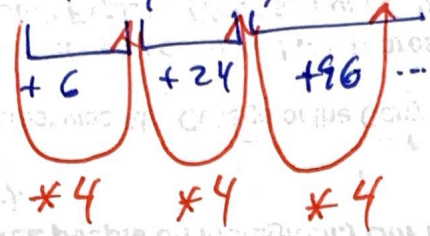
11.3 Geometric Sequences

①

In this section we will notice that the sequence terms vary by a ratio.

ex

$$\{ 2, 8, 32, 128, \dots \}$$



← Not Arithmetic

← Geometric Series

Form ratio by $\frac{8}{2} = 4$, $\frac{32}{8} = 4$, $\frac{128}{32} = 4$ ratio
 ÷ Consecutive terms.

The general term

$$a_n = r a_{n-1}$$

Recursive form

let "a" be the 1st term

$$n=1 \quad a_1 = a$$

$$n=2 \quad a_2 = r a_1 = r(a) = r a$$

$$n=3 \quad a_3 = r a_2 = r(r a) = r^2 a$$

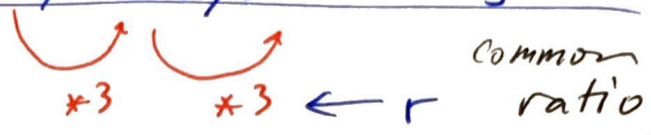
$$a_n = r^{n-1} a$$

direct general term.

1st term a.

EX

Find the first 5 terms in the sequence $\{3, 9, 27, 81, 243, \dots\}$



EX

Find the general term in the above Example

$$a_n = r^{n-1} a$$

• $n=1 : a_1 = r^0 \cdot a = 1 \cdot 3$

So $a=3$

we need the ratio "r":

$n=2$ $a_2 = r^{2-1} \cdot 3$

$9 = r^1 \cdot 3$

$r=3$

together

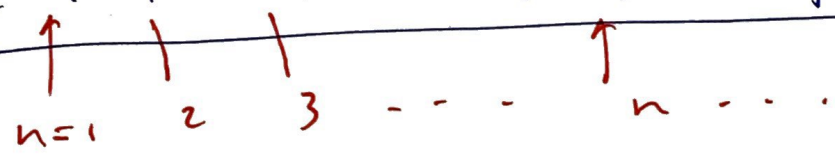
$$a_n = 3^{n-1} \cdot 3$$

or $a_n = 3^n$

Test

$$\{3, 3^2, 3^3, \dots, 3^n, \dots\}$$

$$= \{3, 9, 27, \dots, 3^n, \dots\}$$



EX

Find the general term for this sequence

$$\left\{ -2, \frac{1}{2}, -\frac{1}{8}, \frac{1}{32}, -\frac{1}{128}, \dots \right\}$$

• Gen. Form: $a_n = r^{n-1} a$

since it appears to be a geom. sequence

Q: what is "r" and what is "a"

↑
Common ratio

↑
starting term

• analytically: match terms

$$\begin{cases} a_2 = r^{2-1} a \\ \frac{1}{2} = r a \end{cases}$$

$$\begin{cases} a_3 = r^{3-1} a \\ -\frac{1}{8} = r^2 a \end{cases}$$

2 eqns in "r" & "a"

$$\begin{cases} r a = \frac{1}{2} \\ r^2 a = -\frac{1}{8} \end{cases}$$

sub

• Solve:

$$\Rightarrow r^2 \left(\frac{1}{2r} \right) = -\frac{1}{8} \Rightarrow \frac{r}{2} = -\frac{1}{8}$$

$$\Rightarrow r = -\frac{1}{8} \cdot 2 = -\frac{1}{4} \quad \boxed{r = -\frac{1}{4}}$$

Top eqn: $r a = \frac{1}{2}$ but $r = -\frac{1}{4}$

$$\Rightarrow -\frac{1}{4} a = \frac{1}{2} \Rightarrow \boxed{a = -2}$$

$$a_n = \left(-\frac{1}{4}\right)^{n-1} \cdot (-2)$$

Ex

write the general term for a geometric sequence where $a_7 = 64$ and $a_{10} = 512$

(i) Start with the form $a_n = r^{n-1} a$

(ii) build two eqns

$$\begin{cases} a_7 = r^{7-1} a \\ 64 = r^6 a \\ a_{10} = r^{10-1} a \\ 512 = r^9 a \end{cases}$$

(iii) Solve for "a" and "r"

Solve

$$\begin{array}{l} \div \\ \frac{64 = r^6 a}{512 = r^9 a} \Rightarrow \frac{64}{512} = r^{-3} \end{array}$$

$$\Rightarrow r^3 = \frac{512}{64} = \frac{256}{32} = \frac{128}{16} = \frac{64}{8} = 8$$

$$\boxed{r = 2}$$

use to eqn: $64 = 2^6 a \Rightarrow \boxed{a = 1}$

$$\boxed{a_n = 2^{n-1}}$$