* Form: $\boxed{a_n = a_{n-1} + d}$ $\boxed{a_1 = a}$

difference — start value (seed)

- For any pair of a & d we get a unique sequence.

Ex let $a = 2$ and $d = 3$

$\Rightarrow a_n = a_{n-1} + 3, \quad a_1 = 2$

$a_1 = \text{seed} = 2$

$n=2 \quad a_2 = a_{2-1} + 3 = a_1 + 3 = 2 + 3 = 5$

$n=3 \quad a_3 = a_2 + 3 = 5 + 3 = 8$

$n=4 \quad a_4 = a_3 + 3 = 8 + 3 = 11$

$\vdots$

$$\boxed{\{a_n\} = \{2, \underbrace{5,}_{+3}\ \underbrace{8,}_{+3}\ \underbrace{11,}_{+3} \ldots\}}$$

$+3 \quad +3 \quad +3$ ← The behavior of an arithmetic sequence.

* general term, nth term:

$$\boxed{a_n = a + (n-1)d} \quad \begin{cases} a = a_1 = \text{seed} \\ d = \text{common difference} \end{cases}$$

* non-arithmetic seq. (by contrast)

EX Analyze

$\{4, 16, 64, 256, 1024, \ldots\}$

- How do I get from 4 to 16? $+12$ -or- $*4$
- How do I get from 16 to 64? $+48$ -or- $*4$ $\to +192$
- How do I get from 64 to 256? $\underline{\underline{*4}}$

Q: Is this arithmetic? No we are not adding a common difference.

{ B.T.W.: this sequence is a geometric sequence (11.3) since we are multiplying by a constant ratio, 4 }

EX $\{11.4, 9.3, 7.2, 5.1, 3.0, \ldots\}$ Find the gen. term

$11.4 \xrightarrow{-2.1} 9.3 \xrightarrow{-2.1} 7.2 \xrightarrow{-2.1} 5.1$

- recursive form

$a_n = a_{n-1} + d$

$\boxed{a_n = a_{n-1} - 2.1}$

- arithmetic
- $d = -2.1$
- general term

$a_n = a_1 + (n-1)d$

$a_n = 11.4 + (n-1)(-2.1)$

$\boxed{a_n = 11.4 - 2.1(n-1)}$

Ex Revisit 1st exaple, write gen. term

$$\boxed{\{2, 5, 8, 11, \dots, 2+(n-1)3, \dots\}}$$

Ex Find the 24th term of the sequence

$n = 1, 2, 3, 4$

$\{-3, 0, 3, 6, \dots\}$

Strategy: get the gen. term 1st

(i) determine a & d

$$a = \underline{-3} \quad \& \quad d = \underline{3}$$

(ii) general term:

$$a_n = a + (n-1)d = \boxed{-3+(n-1)3}$$

(iii) compute:
$$a_{24} = -3 + (24-1)3$$
$$= -3 + (23)3$$
$$= -3 + 69$$
$$a_{24} = \boxed{66}$$

(*) C.S.I. problems

Q: How do we build/determine a sequence given two members of the sequence and told that the sequence is arithmetic

Ex If $a_1 = 17$ and $a_7 = -31$ Find a_{20}

- general term $\boxed{a_n = a + (n-1)d}$ ⟸ arithmetic form
- ⟹ $a = a_1$ so $\boxed{a = 17}$ ⟸ start term

so then the form becomes $\boxed{a_n = 17 + (n-1)d}$ what is $d = ?$

- use the last piece of info: $a_7 = -31$

$$\Rightarrow a_7 = 17 + (7-1)d$$

$$-31 = 17 + 6d$$

solve for "d"

$$\frac{-31-17}{6} = d$$

$$d = \frac{-48}{6} = -8$$

- general term: $\boxed{a_n = 17 + (n-1)(-8)}$

answer: ~~the question~~ $a_{20} = 17 - 8(20-1) = 17 - 8 \cdot 19 = \boxed{-135}$

④

EX Write the 1st 3 terms of an arithmetic sequence if $a_{13} = -60$ and $a_{33} = -160$

Not a_1 like in the last example

general term : $\boxed{a_n = a + (n-1)d}$

2 eqns

- $n = 13$: $a_{13} = a + (13-1)d \rightarrow -60 = a + 12d$
- $n = 33$: $a_{33} = a + (33-1)d \rightarrow -160 = a + 32d$

2 unknowns

Solve

$$a + 12d = -60$$
$$+\ a + 32d = -160 \quad * -1$$
$$-20d = 100 \qquad \boxed{d = -5}$$

insert into top eqn

$$a + 12(-5) = -60 \Rightarrow \boxed{a = 0}$$

- General term:

$$\boxed{a_n = -5(n-1)}$$

- answer question:

$$a_1 = 0$$
$$a_2 = -5$$
$$a_3 = -10$$

$\{0, -5, -10, \ldots, -5(n-1), \ldots\}$

* Finally, recursive formula...

EX Use the given recursive formula to write the first four terms: $\begin{cases} a_1 = 39 \\ a_n = a_{n-1} - 3 \end{cases}$ ← "$d=-3$"

$\{39, 36, 33, 30, 27, \dots\}$ (−3, −3, −3)

In non-recursive form

$a_n = 39 - 3(n-1)$

EX Write a recursive relation if $\{-15, -7, 1, \dots\}$ (+8, +8) $d = 8$

$a_1 = -15$, $a_n = a_{n-1} + 8$

seed

Alternatively convert to general term ⟹ explicit general term $a_n = -15 + 8(n-1)$

11.2 is finnished