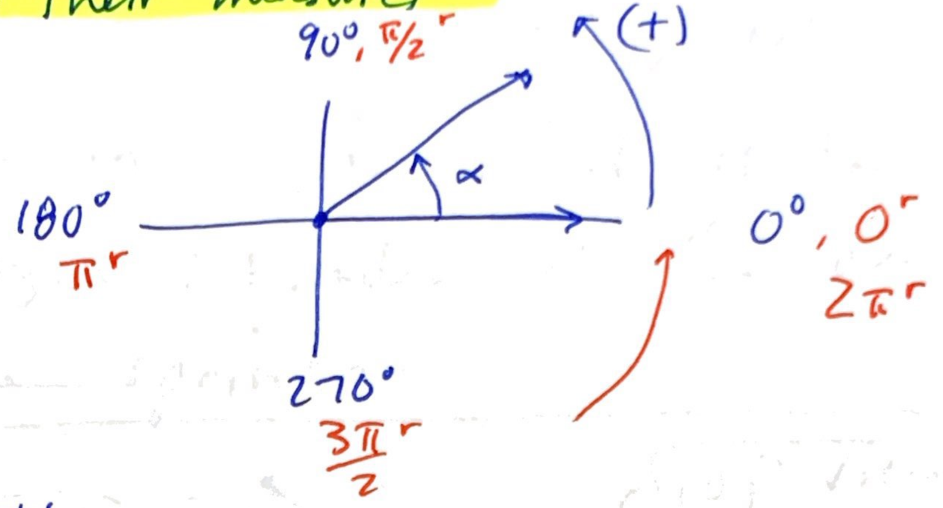
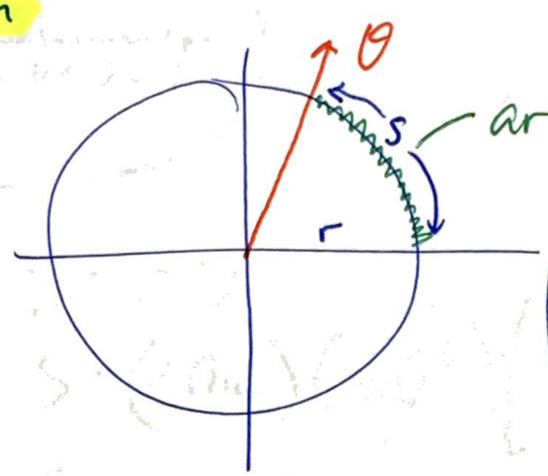
Chapter 5 | Trig Review

* Angles & their measures



* arc length



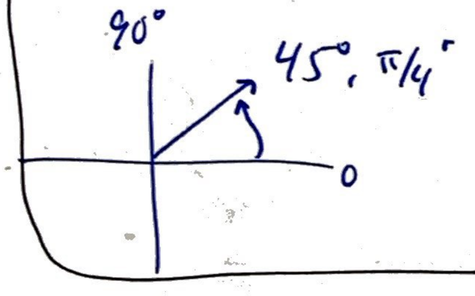
arc length covered by our angle θ

$$\boxed{s = r\theta}$$

but θ needs to be measured in radians: for the formula to work...

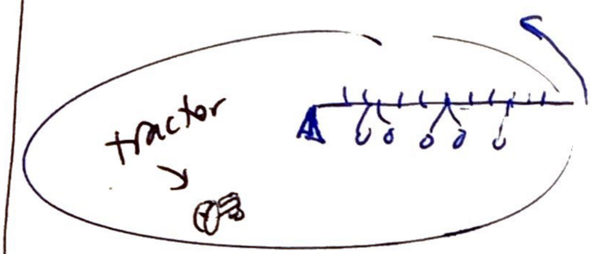
$$\boxed{2\pi^r = 360^\circ} \rightarrow \boxed{\pi = 180^\circ}$$

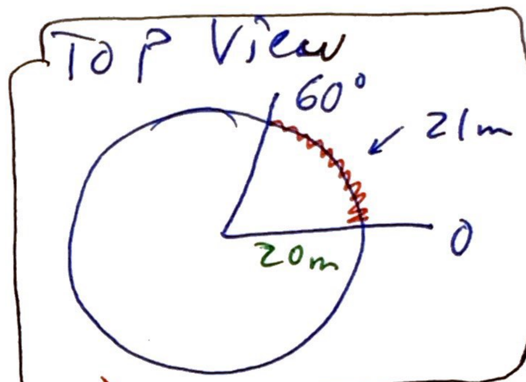
Ex Convert 45° into radians



$$45^\circ\left(\frac{2\pi^r}{360^\circ}\right) = \frac{90}{360}\pi = \boxed{\pi/4}$$

EX If a rotary sprinkler is sweeping a $60°$ arc, and if it is 20m in length what is the arc length subtended at the end of the sprinkler





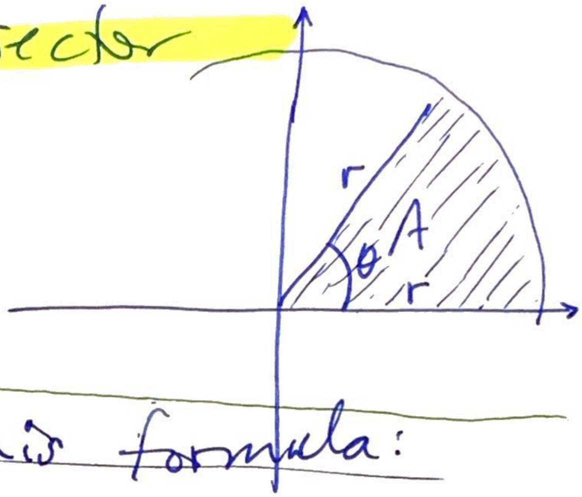
$$S = r\theta$$

$$S = (20m)(60°)\left(\frac{\pi}{180°}\right)$$

$$S = \frac{1200}{180}\pi$$

$$\boxed{S = \frac{20}{3}\pi \text{ meters}} \approx 20m$$

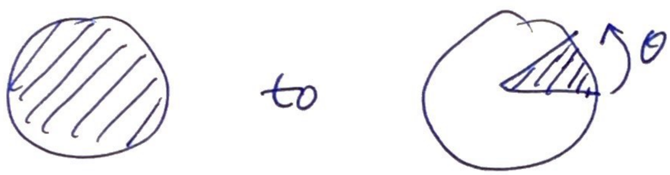
⊛ area of sector



$$\boxed{\text{Area} = \frac{1}{2} r^2 \theta}$$

θ = radians only

Derivation of this formula:



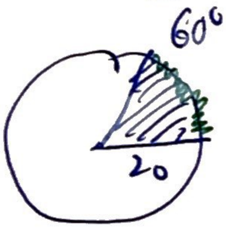
$\theta = 2\pi$ is to πr^2 as θ is to A_{sector}

$$\frac{2\pi}{\pi r^2} = \frac{\theta}{A_{sector}}$$

$$A_{sector} = \frac{\theta \pi r^2}{2\pi}$$

$$\boxed{A_{sector} = \frac{1}{2}\theta r^2}$$

Ex In the previous ex of the sprinkler, what is the area covered by 0° to 60°?



$$A_{sector} = \frac{1}{2}(20\text{m})^2\left[(60^\circ)\left(\frac{\pi}{180^\circ}\right)\right]$$

$$= \frac{\overset{200}{\cancel{400}} \cdot 60\pi}{\cancel{2} \cdot \underset{3}{\cancel{180}}}$$

$$\boxed{A = \frac{200\pi}{3}\text{m}^2} \approx 200\text{m}^2$$

⊛ angular speed (Application)

physics: $vt = \text{distance}$

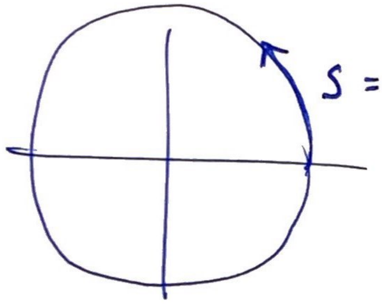
$$\boxed{v = \frac{x}{t}} \quad \text{linear speed}$$

rotational speed (a.k.a angular velocity)

Def.

$$\boxed{\theta/t \equiv \omega} \quad \text{angular speed}$$

but for circular motion $x = r\theta$

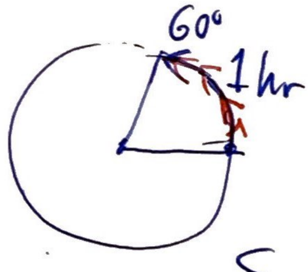


$$V = \frac{r\theta}{t}$$

$$\boxed{V = r\omega} \qquad [\omega] = \frac{\text{radians}}{\text{sec}}$$

linear ↔ angular conversion

EX The end of the sprinkler in the previous examples moves from 0° to 60° in 1 hr.



Q: what is the linear velocity of the end of the sprinkler?

A: $V = r\omega$ but $\omega = \theta/t$

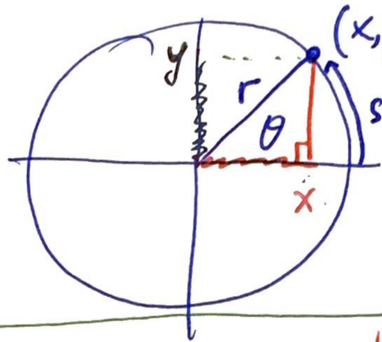
So then $V = r\left(\frac{\theta}{t}\right)$

$$= (20\,m)\left(\frac{\pi/3^{r}}{3600\,s}\right)$$

$$= \frac{\pi}{180 \cdot 3}\ m/s \rightarrow \approx 0.006\ m/s \quad \boxed{6\ mm/s}$$

$\boxed{1\ hr = 3600\ s}$

⊛ Unit Circle



$(x,y) = (r\cos\theta, r\sin\theta)$

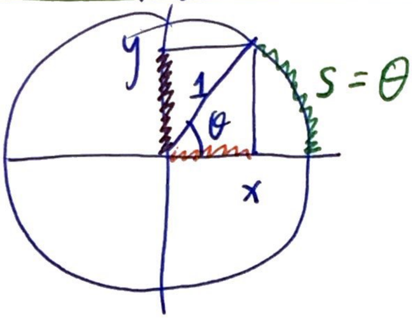
$s = r\theta$ but $r=1$ so $s=\theta$ (only for unit circle)

• arc length of unit circle

For $r = 1$ $\quad \boxed{\cos\theta \equiv x} \quad \boxed{\sin\theta \equiv y}$

projection of radius onto x-axis

projection onto y-axis



θ is in radians!

• In radians for the unit circle θ = arc length

Using this definition (unit circle) $r = 1$

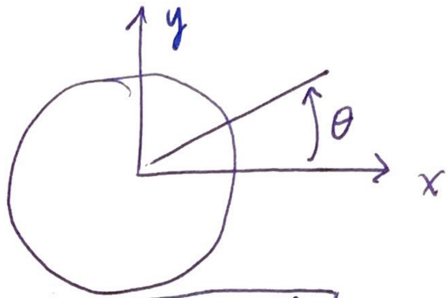
$$y = \sin\theta$$
$$x = \cos\theta$$
$$y/x = \tan\theta$$
$$\frac{1}{y} = \csc\theta$$
$$\frac{1}{x} = \sec\theta$$
$$x/y = \cot\theta$$

non-unit circle

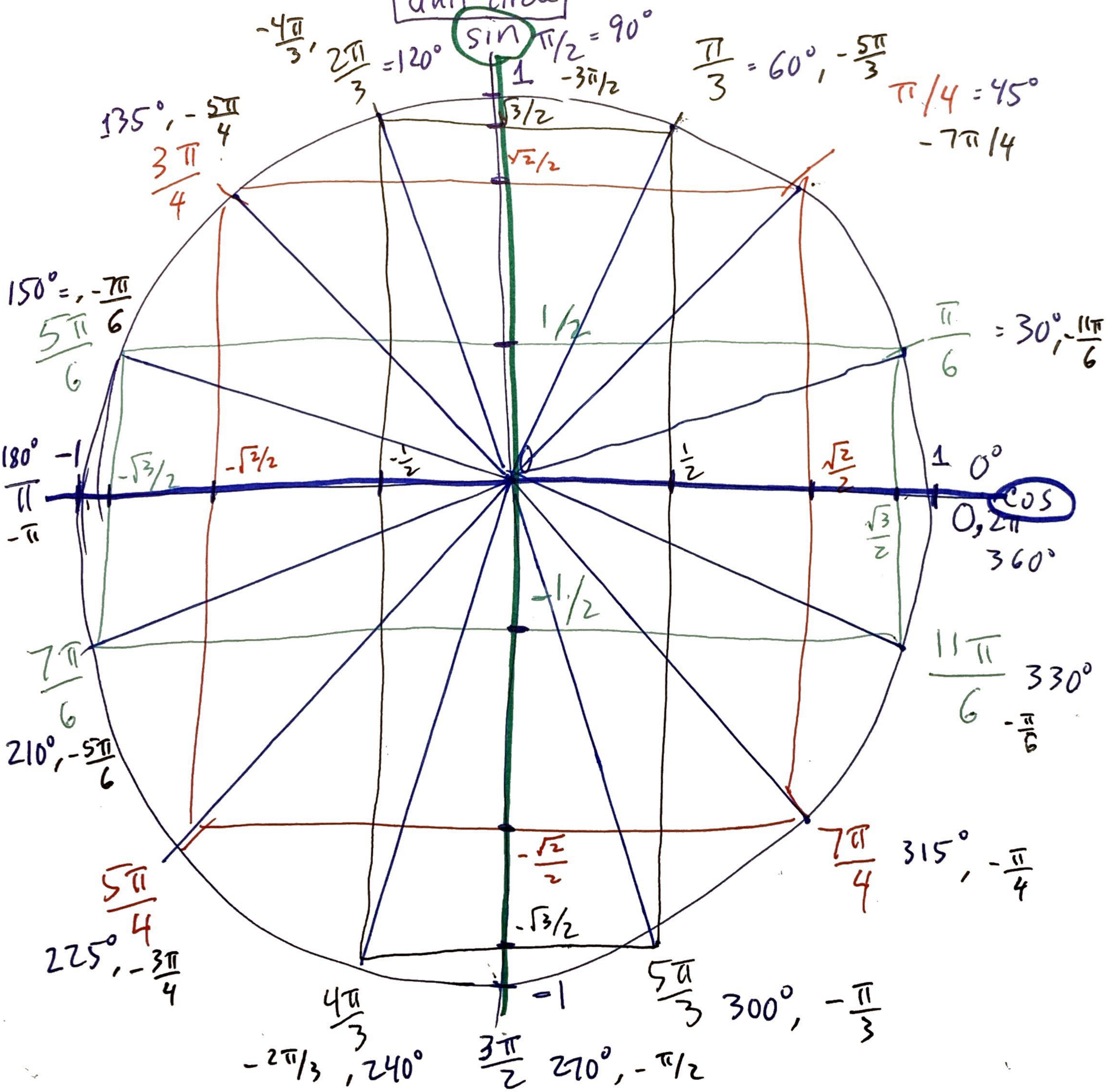
$$y = r\sin\theta$$
$$x = r\cos\theta$$
$$r/y = \csc\theta$$
$$r/x = \sec\theta$$

The Trig Circle

* Angles



unit circle



EX

- $\cos\left(\frac{4\pi}{3}\right) = \boxed{-\frac{1}{2}}$
- $\sin\left(\frac{4\pi}{3}\right) = \boxed{-\frac{\sqrt{3}}{2}}$
- $\tan\left(\frac{4\pi}{3}\right) = \dfrac{\sin(4\pi/3)}{\cos(4\pi/3)}$

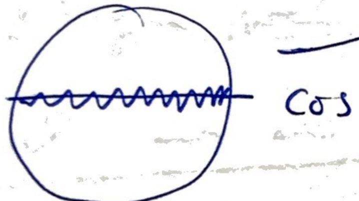
$$= \frac{-\sqrt{3}/2}{-1/2} = \boxed{\sqrt{3}}$$

EX

- $\csc\left(\frac{11\pi}{6}\right) = \dfrac{1}{\sin\left(\frac{11\pi}{6}\right)} = \dfrac{1}{-1/2} = \boxed{-2}$

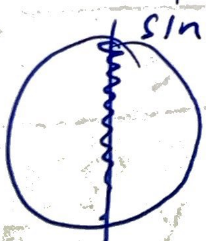
(A) Domains and Ranges

- $\cos\theta$:



$$\begin{cases} D_{\cos\theta} = \{\ \theta \in \mathbb{R}\ \} \\ R_{\cos} = \{-1 \leq \cos\theta \leq 1\} \end{cases}$$

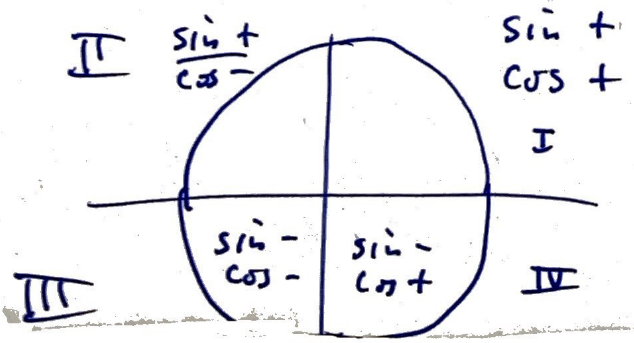
- $\sin\theta$:

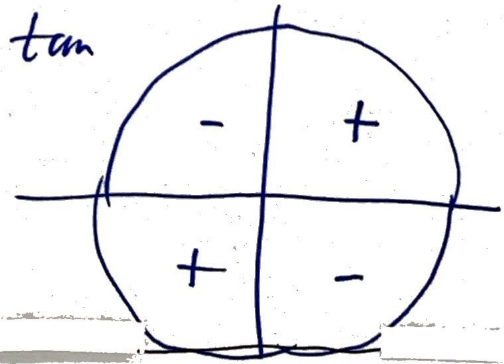


$$\begin{cases} D_{\sin\theta} = \{\theta \in \mathbb{R}\} \\ R_{\sin} = \{-1 \leq \sin\theta \leq 1\} \end{cases}$$

- $\boxed{\tan\theta = \dfrac{\sin\theta}{\cos\theta}}$ $\quad \cos\theta \neq 0 \Rightarrow \theta \neq \frac{\pi}{2}, \frac{3\pi}{2}, \ldots \left(\frac{2n-1}{2}\right)\pi$

odd multiples of $\pi/2$





- to avoid $\cos\theta = 0$

  we need $\theta \neq$ odd multiples of $\pi/2$

even multiples: $n\frac{\pi}{2}$ $2n$ always even

odd multiples: $(2n+1)\frac{\pi}{2}$ $2n+1$ always odd

n = integer $\ldots, -3, -2, -1, 0, 1, 2, 3 \ldots$

So $D_{\tan} : \left\{ \theta \,\middle|\, \theta \neq \frac{2n+1}{2}\pi \right\}$

- For the range we see that as $\cos\theta \to 0^+$

  So tangent $\to \infty$

  and as $\cos\theta \to 0^-$

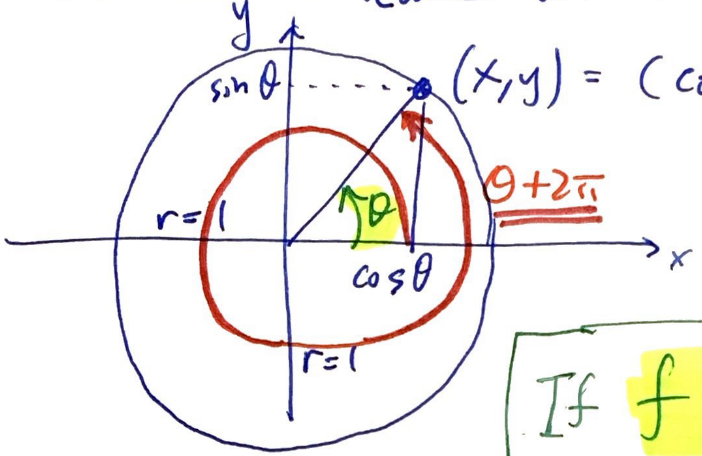
  we see tanent $\to -\infty$

- Thus $R_{\tan} = \{ y \mid y \in \mathbb{R} \}$

  or $\{ y \mid -\infty < y < \infty \}$

*periodicity of trig functions

- sin and cos land on the same location after 2π radians



$(x,y) = (\cos\theta, \sin\theta) = (\cos(\theta+2\pi), \sin(\theta+2\pi))$

- we call this a periodic function
2π is periodicity

If $f(x+p) = f(x)$

then $f(x)$ has a period of "p" units

$\sin(\theta + 2\pi k) = \sin(\theta) \longrightarrow$ period $= 2\pi$

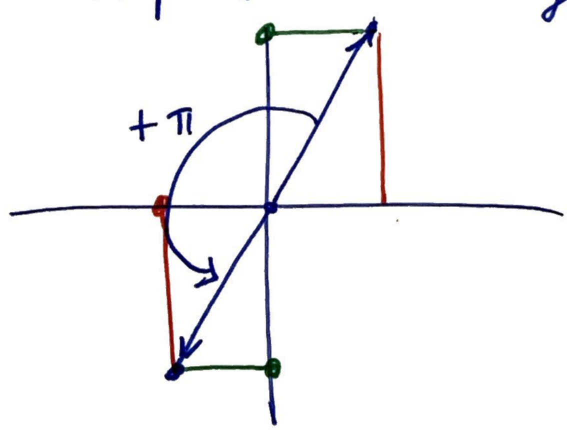
$\cos(\theta + 2\pi k) = \cos(\theta) \longrightarrow$ period $= 2\pi$

$\tan(\theta + \pi k) = \tan(\theta) \longrightarrow$ period $= \pi$

*repeats itself every π radians

$$= \frac{\sin(\theta+\pi)}{\cos(\theta+\pi)}$$

$$= \frac{-\sin(\theta)}{-\cos(\theta)}$$



$= \tan(\theta)$

$\csc(\theta + 2\pi k) = \csc(\theta)$

$\sec(\theta + 2\pi k) = \sec(\theta)$

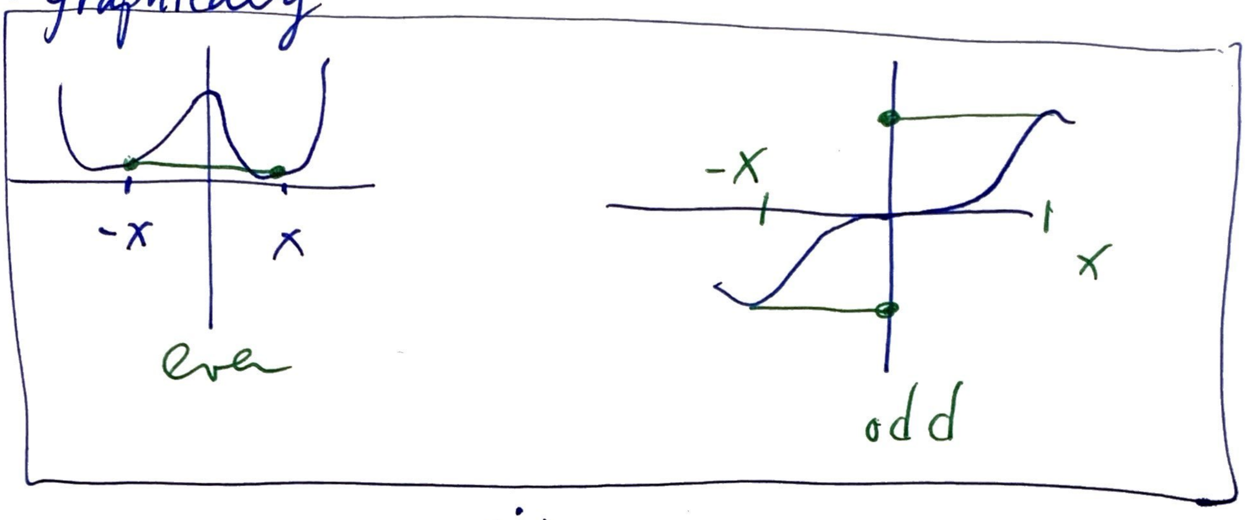
$\cot(\theta + \pi k) = \cot(\pi)$

* even and odd trig function

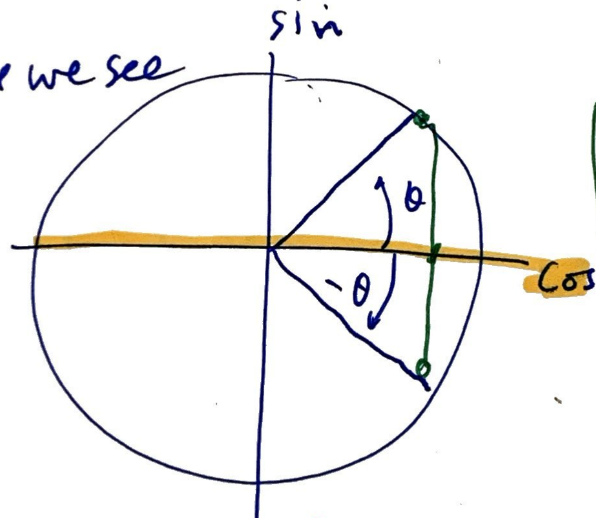
If $f(-x) = f(x)$ $\quad$ f is an even funct

$\quad$ $f(-x) = -f(x)$ $\quad$ f is an odd funct.

• graphically

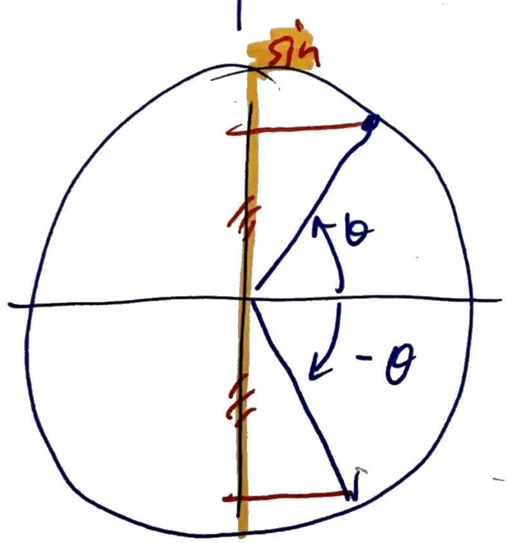


On the Trig circle we see



$\cos(-\theta) = \cos(\theta)$

even



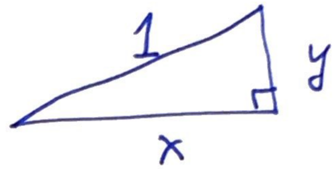
$\sin(-\theta) = -\sin(\theta)$

odd

* Fundamental Identities

pythagorean identities:

$x^2 + y^2 = 1$ if



but $x = \cos(\theta)$ and $y = \sin(\theta)$ for the unit circle

then the thm becomes

$$\boxed{\cos^2(\theta) + \sin^2(\theta) = 1}$$

$\div \cos^2\theta$:
$$\frac{\cos^2(\theta)}{\cos^2(\theta)} + \frac{\sin^2(\theta)}{\cos^2(\theta)} = \frac{1}{\cos^2(\theta)}$$

$$\rightarrow \boxed{1 + \tan^2(\theta) = \sec^2(\theta)}$$

$\div \sin^2\theta$:
$$\frac{\cos^2(\theta)}{\sin^2(\theta)} + \frac{\sin^2(\theta)}{\sin^2(\theta)} = \frac{1}{\sin^2(\theta)}$$

$$\boxed{\cot^2(\theta) + 1 = \csc^2(\theta)}$$

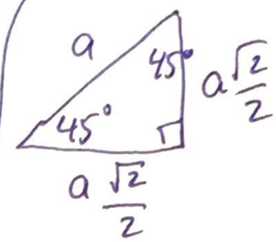
These three are the pythagorean identities of trig. functions

Chpt 5 Review (Cont.)

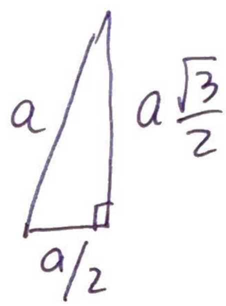
* Right triangles (Δ)

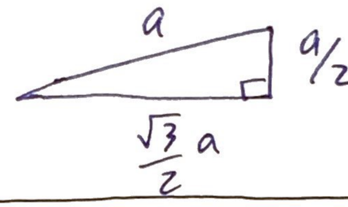
• Recall the popular Δ's

• 45°-45°-90° Δ



• 30°-60°-90° Δ





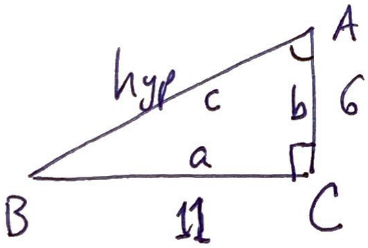
Recall also

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenues}}$$

$$\sin\theta = \frac{\text{opposite}}{\text{hypoteneus}}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}}$$

EX Complete the Δ values:

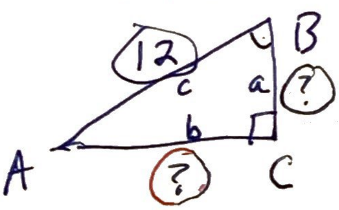


(a) $\sin A = \frac{\text{opp of } A}{\text{hyp}} = \boxed{\frac{11}{\sqrt{157}}}$

(b) $\tan B = \frac{\text{opp of } B}{\text{adj of } B} = \boxed{\frac{6}{11}}$

$c = \text{hyp} = \sqrt{11^2 + 6^2} = \sqrt{157}$

EX Find missing sides of ΔABC if $\sin B = \frac{3}{4}$ & $c = 12$



given

$\sin B = \frac{3}{4}$

but

$\sin B = \frac{\text{opp}}{\text{hyp}} = \frac{b}{c} = \frac{b}{12}$

match these to see

$\boxed{b = \frac{3}{4} \cdot 12 = 9}$

For a: use pyth. Ident. $a^2 + b^2 = c^2$

$a = \sqrt{c^2 - b^2} = \sqrt{144 - 81} = \boxed{\sqrt{63} = a}$