10.2 Hyperbolas

①

Std Form:

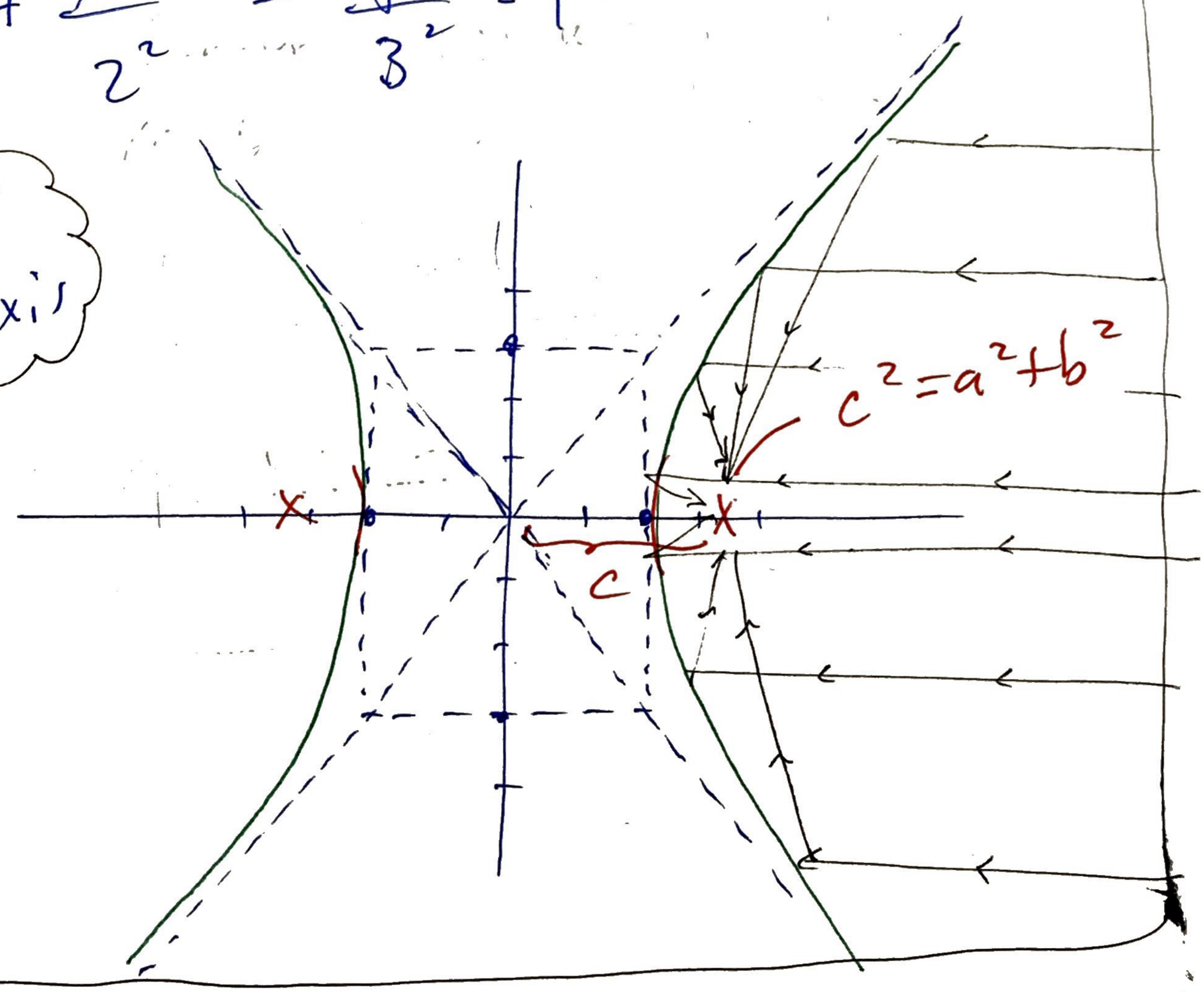
$$\boxed{\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1} \quad \text{or} \quad \boxed{\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1}$$

- graph is on the <u>outside</u> of the box.
- The hyperbola has <u>asymptotes</u> that connect the diagonals of the box.

EX

$$+\frac{x^2}{2^2} - \frac{y^2}{3^2} = 1$$

open on x-axis



(*) Steps to graph

$$\pm \frac{(x-h)^2}{a^2} \mp \frac{(y-h)^2}{b^2} = 1$$

→ **Draw Box** ←

(i) mark the center with ☒ @ (h, k)

(ii) march over "a" units to the left and then "a" units to the right
⇒ left and right sides of the box

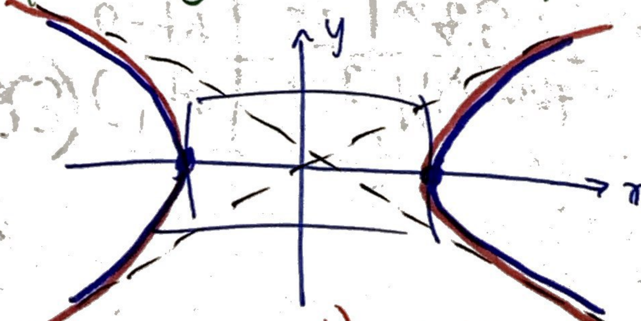
(iii) step up "b" units from the center ☒ and drops down "b" units to have
⇒ top and bottom of the box

(iv) Draw lines ~~through~~ the opposite corners of the box → these are the assymptotes.

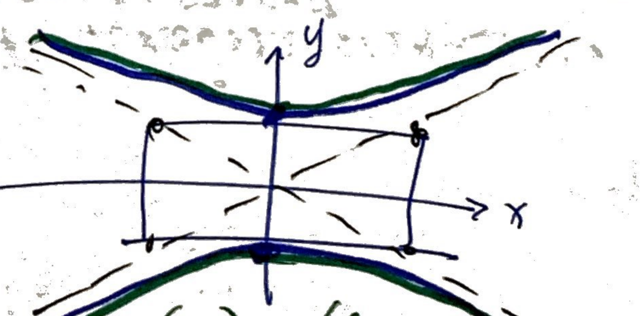
(v) → **draw hyperbola** ←

red case above — • if the sign on x-term is (+) the hyperbola opens to the left - right

green case above — • if the sign on the y-term is (+) the hyperbola open to up and down.



+) x term

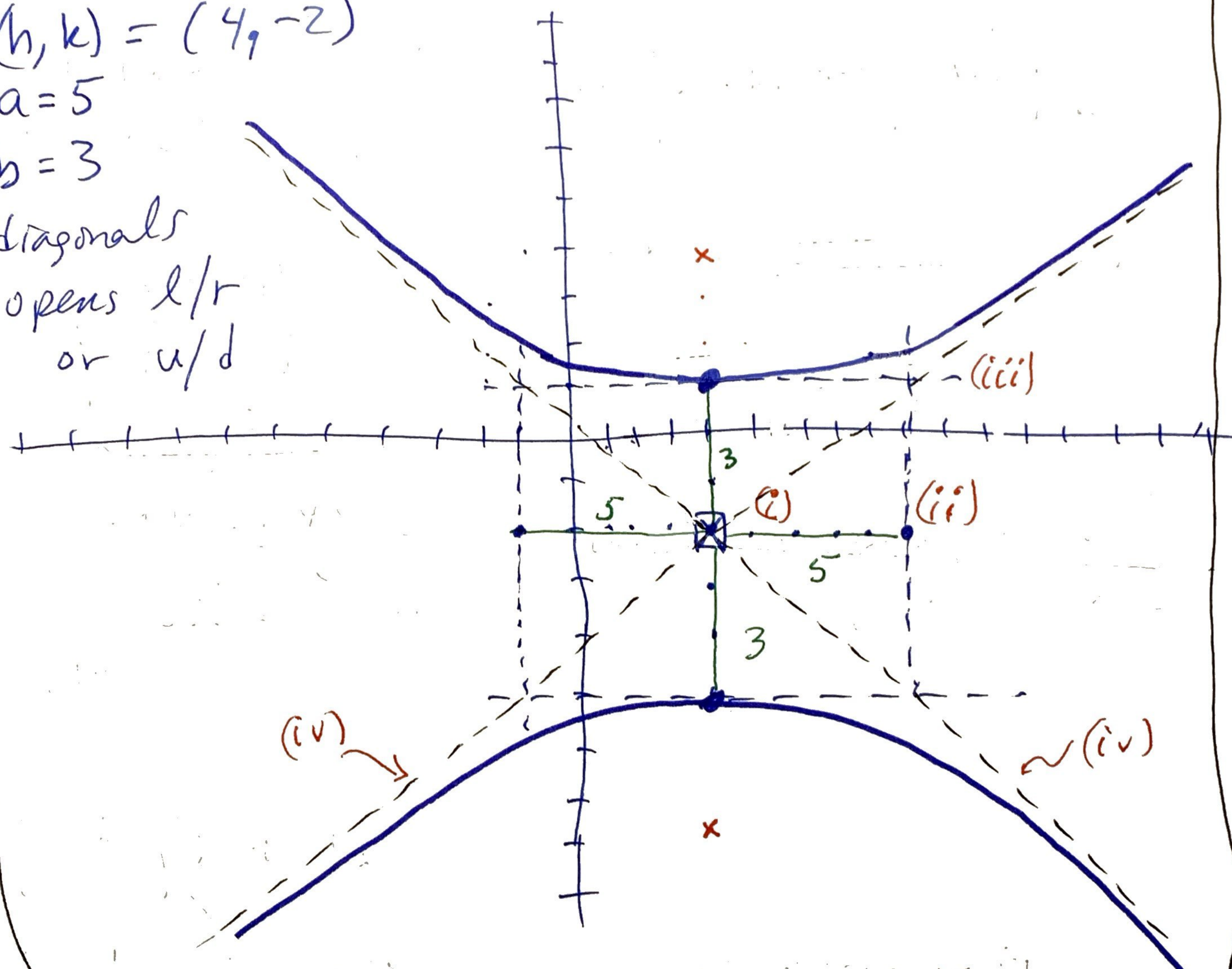


(+) term

Ex Sketch

$$\frac{(y+2)^2}{3^2} - \frac{(x-4)^2}{5^2} = 1$$

- $(h, k) = (4, -2)$
- $a = 5$
- $b = 3$
- diagonals
- opens l/r or u/d



focii : $a^2 + b^2 = c^2$

$3^2 + 5^2 = c^2$

$9 + 25 = c^2$

$34 = c^2$

$c = \pm\sqrt{34}$ from center

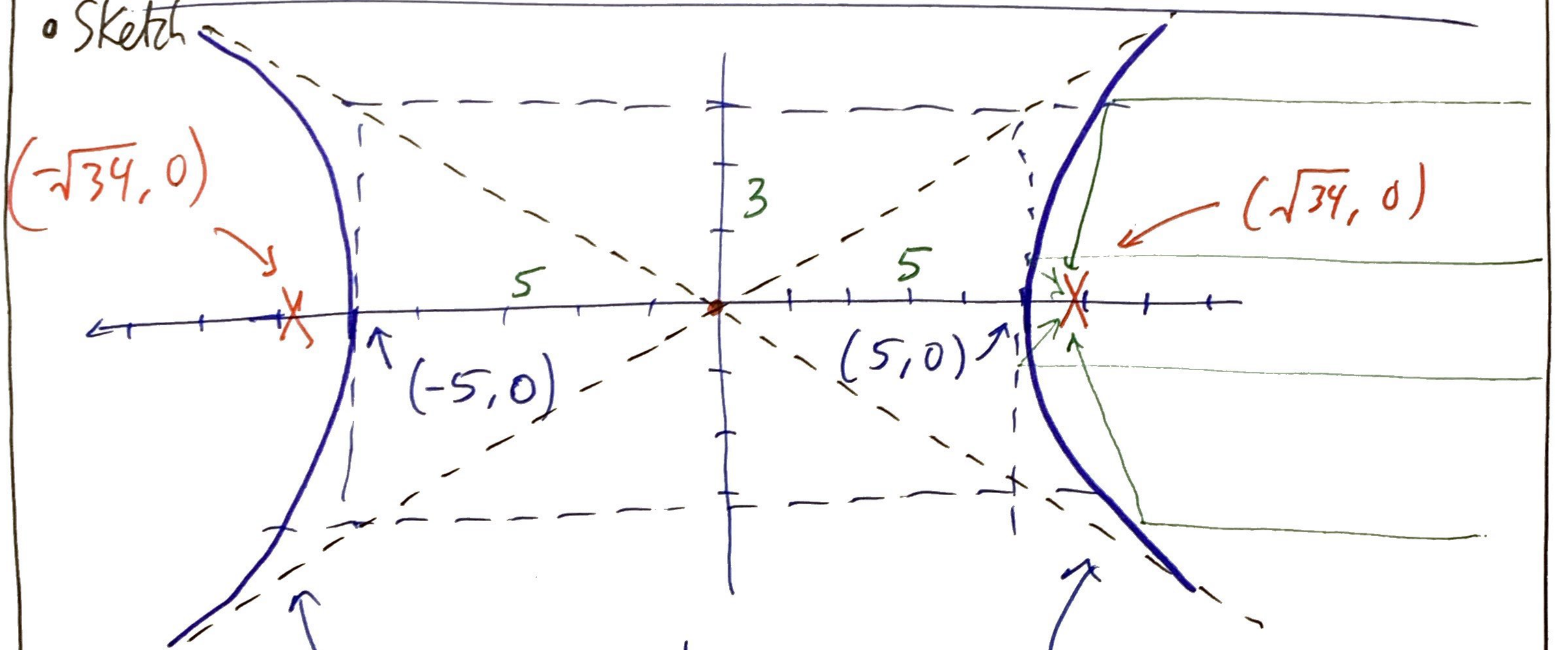
$\approx \underline{\underline{\pm 5.9}}$ $\{$since $\sqrt{36} = 6\}$

EX (a) Sketch $\frac{x^2}{5^2} - \frac{y^2}{3^2} = 1$ and label vertices

(b) what are the eqns of the asymptotes

(c) what are the locations of the focii

• Sketch



• asymptotes: $y = mx + b$

$y = \left(\frac{\text{rise}}{\text{run}}\right)x + b$

$$\boxed{y = \left(\frac{3}{5}\right)x + \underline{0}}$$

$y = \left(\frac{3}{-5}\right)x + \underline{0}$

$y = -\frac{3}{5}x$

$$\boxed{y = \pm\frac{3}{5}x}$$

$y = \pm\frac{b}{a}x$

• Focii

$$\left.\begin{aligned} c^2 &= a^2 + b^2 \\ &= 5^2 + 3^2 \\ &= 25 + 9 \\ &= 34 \end{aligned}\right\} \quad \begin{aligned} c &= \pm\sqrt{34} \\ &\approx \pm\underline{5.9} \end{aligned}$$

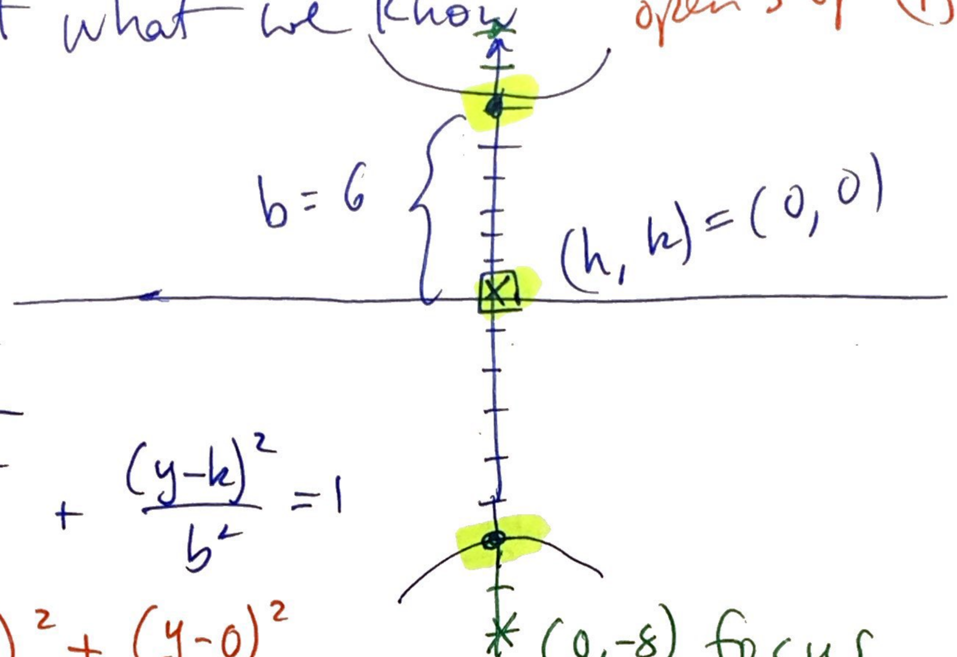
CSI (work backwards)

Ex If a hyperbola has vertices @ (0,6) and (0,−6) and has a focus @ (0,−8)

Find the eqn

(i) plot what we know

opens up (+) y term



(ii) Form

$$-\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$$

$$-\frac{(x-0)^2}{a^2}+\frac{(y-0)^2}{6^2}=1$$

$$\Rightarrow \boxed{\frac{y^2}{6^2}-\frac{x^2}{a^2}=1}$$ form to date

(iii) use remaining info to get a

we have not used the focus yet

- From the center $c = 8$

eqn $\boxed{a^2+b^2=c^2}$

$$a^2+6^2=8^2$$

$$a=\sqrt{64-36}$$

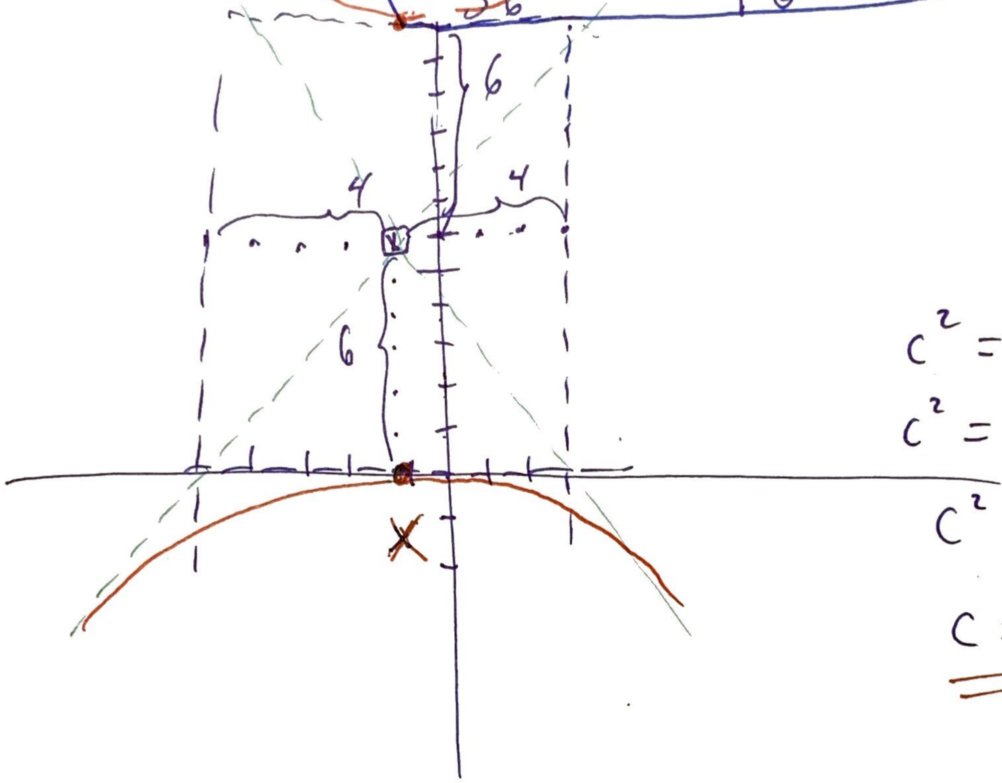
$$a=\sqrt{28}$$

(iv) Final form

$$\boxed{\frac{y^2}{36}-\frac{x^2}{28}=1}$$

Ex Sketch $\frac{(y-6)^2}{36} - \frac{(x+1)^2}{16} = 1$

(i) · centr $(-1, 6)$
(ii) · box
(iii) · vertices up/Dn
(v) · focii
(iv) · eqns of asymtotes



$c^2 = a^2 + b^2$

$c^2 = 16 + 36$

$c^2 = 52$

$c = \pm\sqrt{52} \approx \pm 7.5$

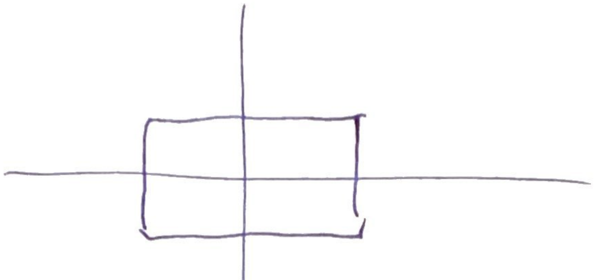
· vertices: $(-1, 6) \pm (0, 6) = (-1, 0)$ & $(-1, 12)$

· focii: $(-1, 6) \pm (0, \sqrt{52}) = (-1, 6-\sqrt{52})$ and $(-1, 6+\sqrt{52})$

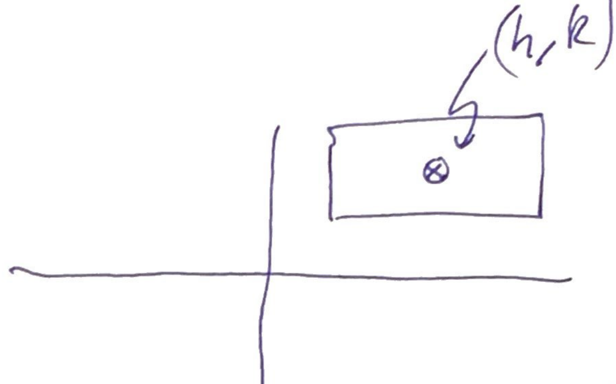
· asymptotes: $y = \pm \frac{6}{4} x$ if @ $(0,0)$

$$\boxed{(y-6) = \pm \frac{6}{4}(x+1)}$$

* off - center

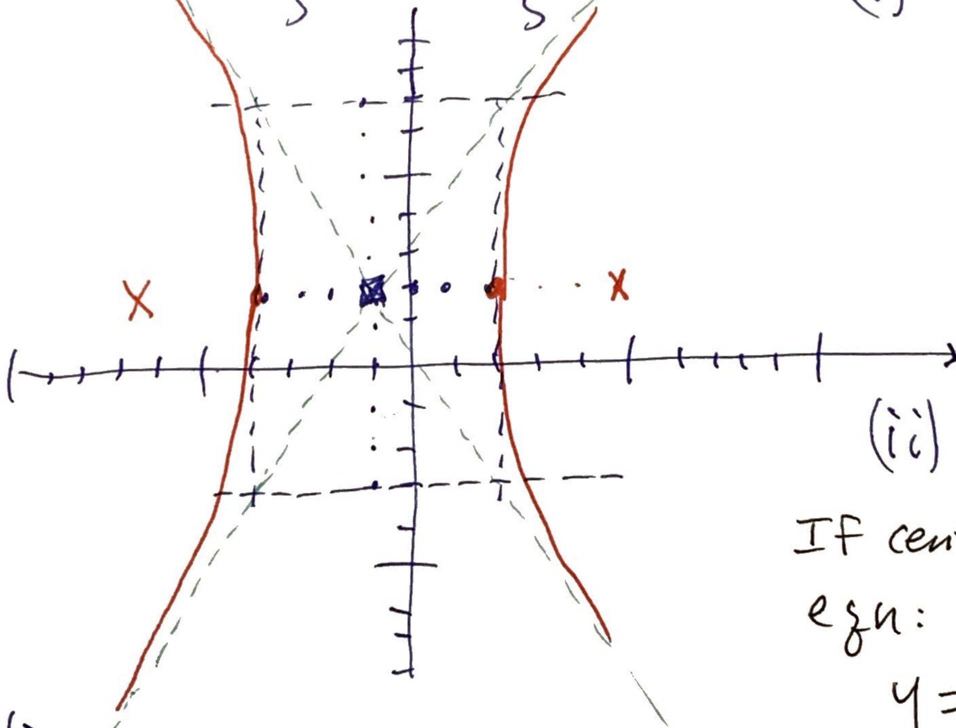


$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$$



$$\boxed{\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1}$$

EX Graph $\frac{(x+1)^2}{3^2}-\frac{(y-2)^2}{5^2}=1$



(i) BOX
- center $(-1,2)$
- sides @ ± 3
- top/bottom @ ± 5

(ii) Asymptotes

If center @ $(0,0)$

eqn:

$$y=\pm\frac{5}{3}x$$

Now shift the line:

$$\boxed{(y-2)=\pm\frac{5}{3}(x+1)}$$

(iii) vertices

opens L/R b/c $\oplus\frac{(x+1)^2}{3^2}$

(iv) focii

$$c^2=a^2+b^2$$

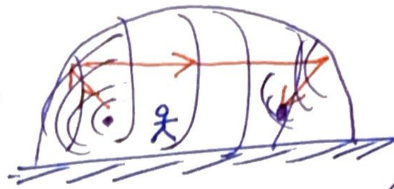
$$c^2=3^2+5^2$$

So $c=\pm\sqrt{34}\approx\pm 5.9$ from the center (not from the origin)

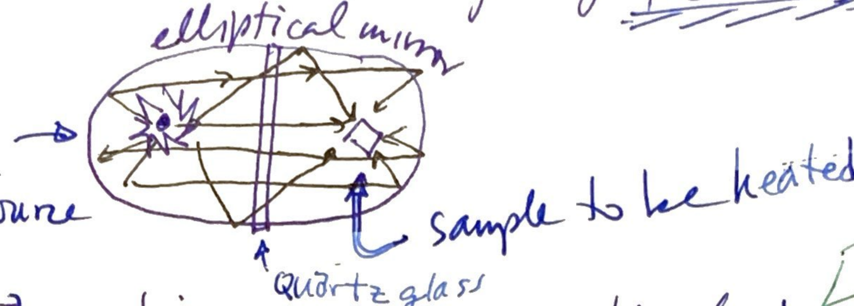
Application — "Conic Cuts"

* Ellipses

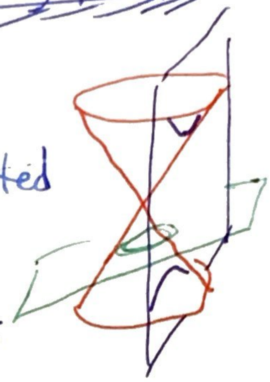
applications: whisper gallery



- Isolation oven:

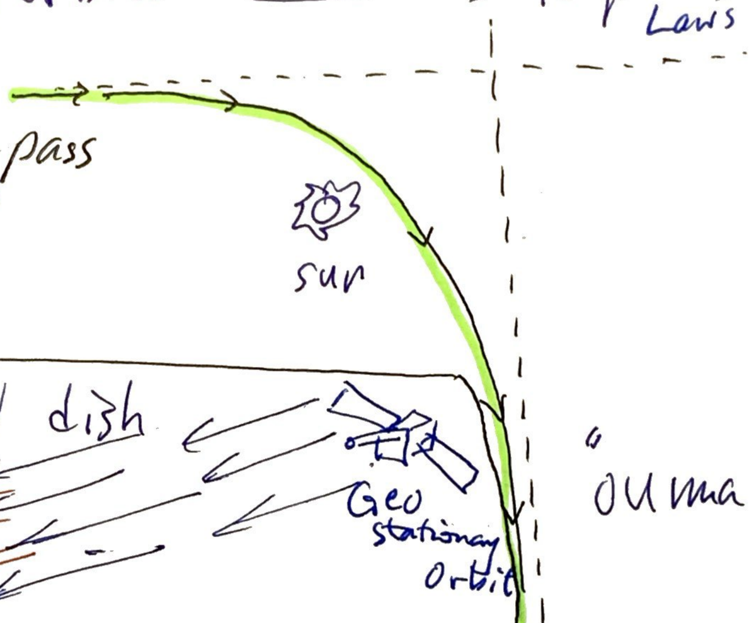


- planetary orbits → Kepler's Laws

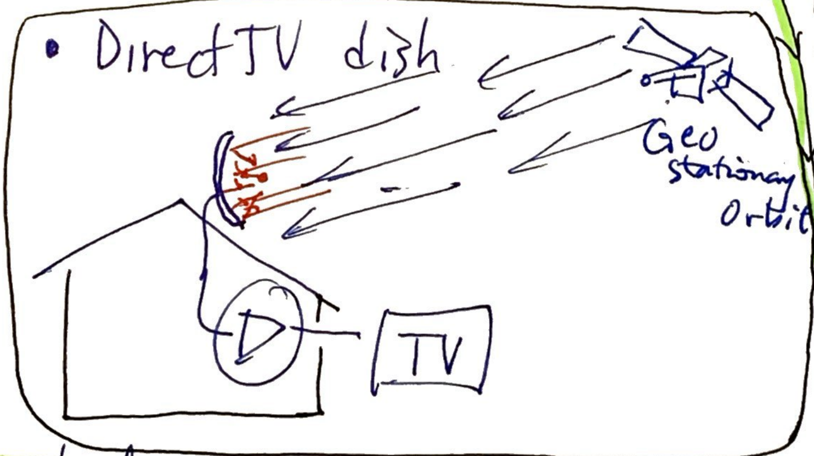


* Hyperbolas

- One time pass orbits

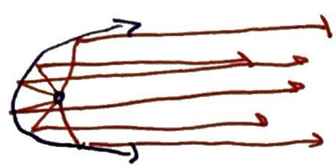


- DirectTV dish



* parabolas

- special orbits (not very frequently)
- Head lamps



- "sideline" mics (ditto)