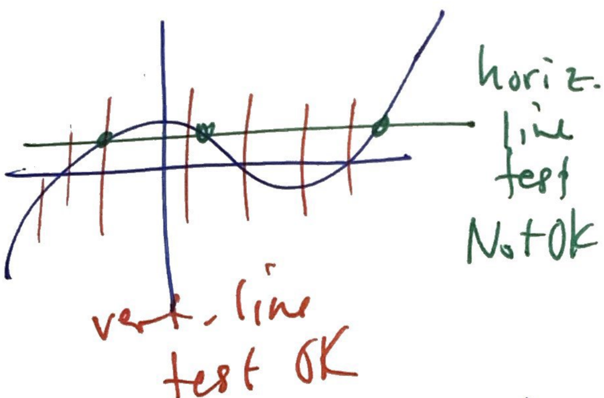
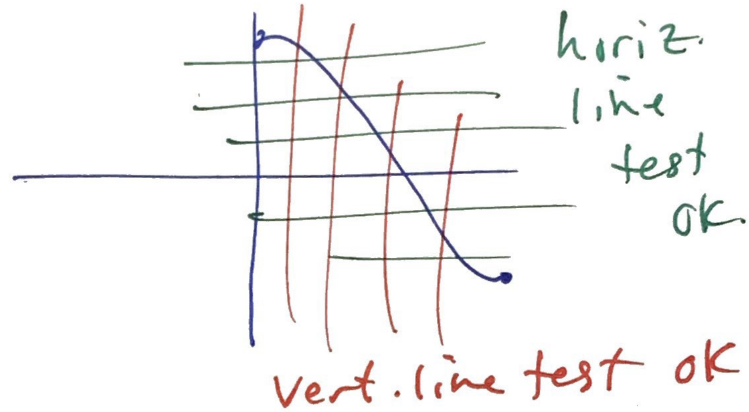
1.7 Inverse Functions

Recall

- A function is a relationship that has a single output for each valid input
- A 1-1 function is a function that, for each output, only one input works.



A graph of a function but not 1-1



A graph of a 1-1 funct.

Def: An inverse function of a function is such ~~that~~ the inv. function reverse the functi

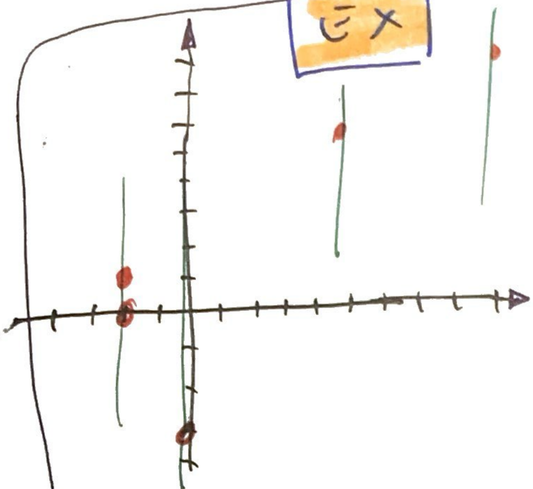
i.e. $y = f(x) \longrightarrow x = f^{-1}(y) \neq \frac{1}{f}$

For analytical relations

⊗ If a relation is a 1-1 function it has an inverse

Tables

Ex



input ↓ ↑ output

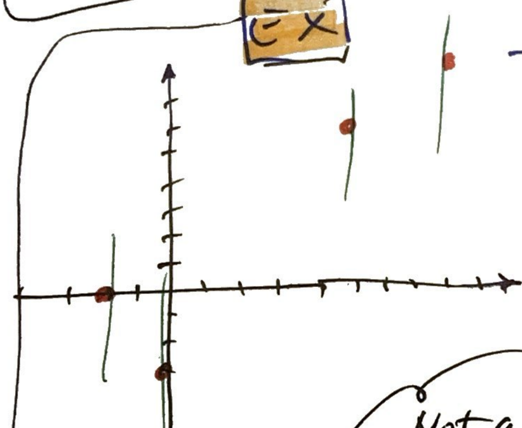
x	$y=f(x)$
-2	0
0	-3
5	6
10	8
-2	1

• Function?

Q: we get only one output for each input?

$\boxed{\text{No}}$ -2 yeilds 0 and 1

Ex



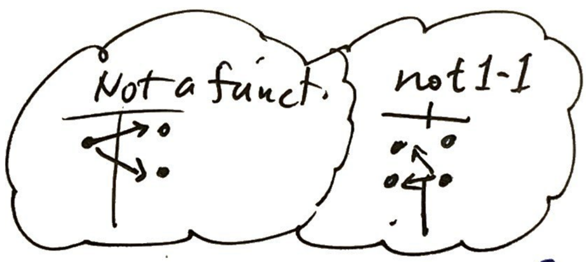
x	$y=f(x)$
-2	0
0	-3
5	6
10	8

• Function?

$\boxed{\text{yes}}$ ... all input has only one output.

• 1-1?

$\boxed{\text{yes}}$ each output has only one input



Now Find the inverse function

x	$y=f^{-1}(x)$
0	-2
-3	0
6	5
8	10

just swap x & y columns.

Ex Build the f^{-1} for this table

(i) Test to see if a funct.

x	$y=f(x)$
-2	0
0	-3
5	6
10	-3

• Function?

yes each input yeilds 1 output

(ii) Test to see if 1-to-1

• 1-1?

$\boxed{\text{No}}$ B/C -3 comes from two inputs

(iii) Find f^{-1} if i & ii are yes

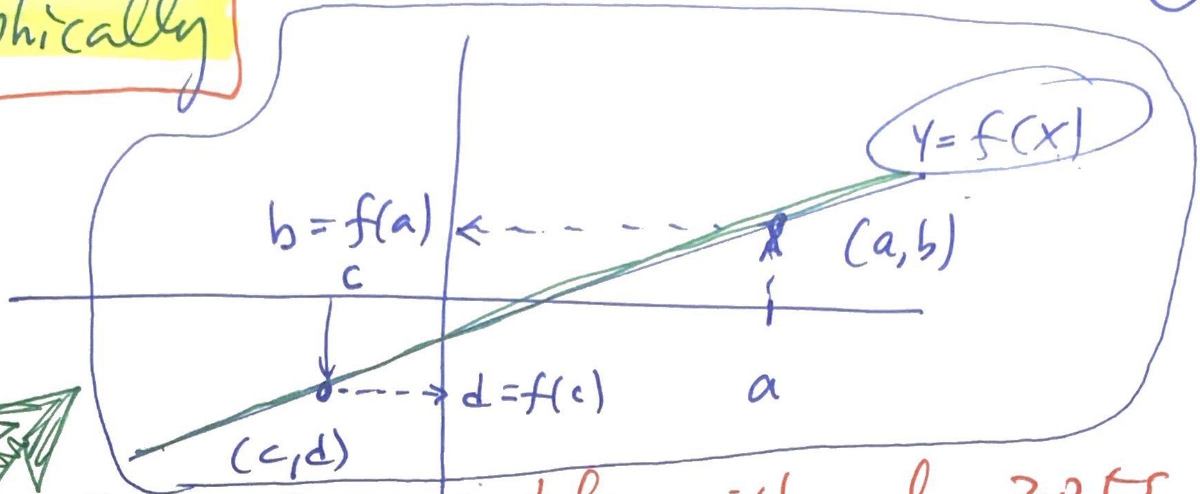
• Try to build f^{-1}: swapping x and y ...

x	$y=f^{-1}(x)$
0	-2
-3	0
6	5
-3	10

Contradition

cannot build an inv. function b/c f^{-1} is not a function.

(*) graphically



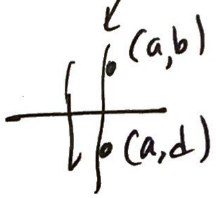
• Consider a table with only 2 pts

x	$y=f(x)$
a	b
c	d

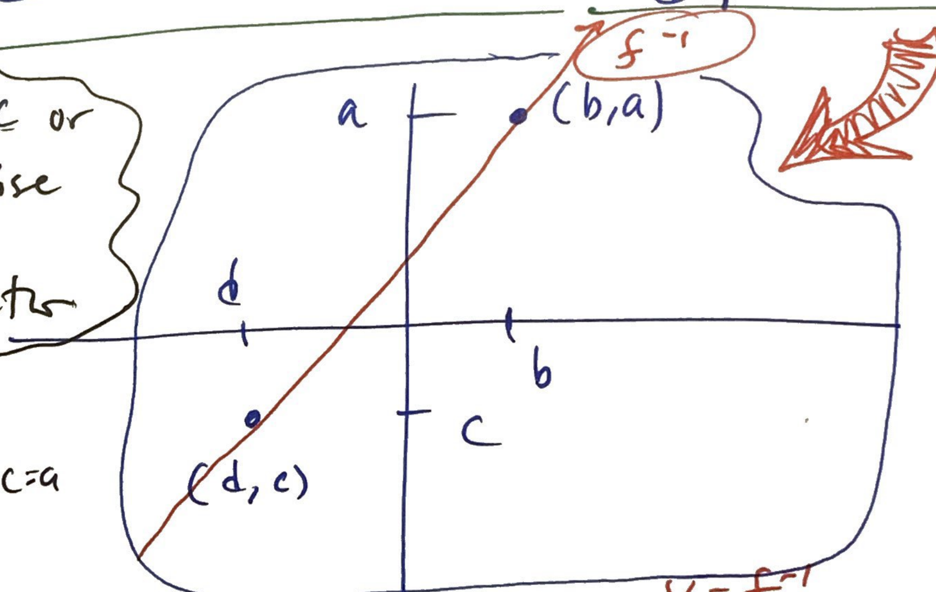
$\xrightarrow{f^{-1}}$

x	$y=f^{-1}(x)$
b	a
d	c

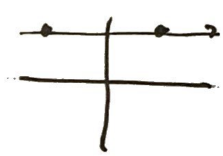
$a \neq c$ or otherwise not a function



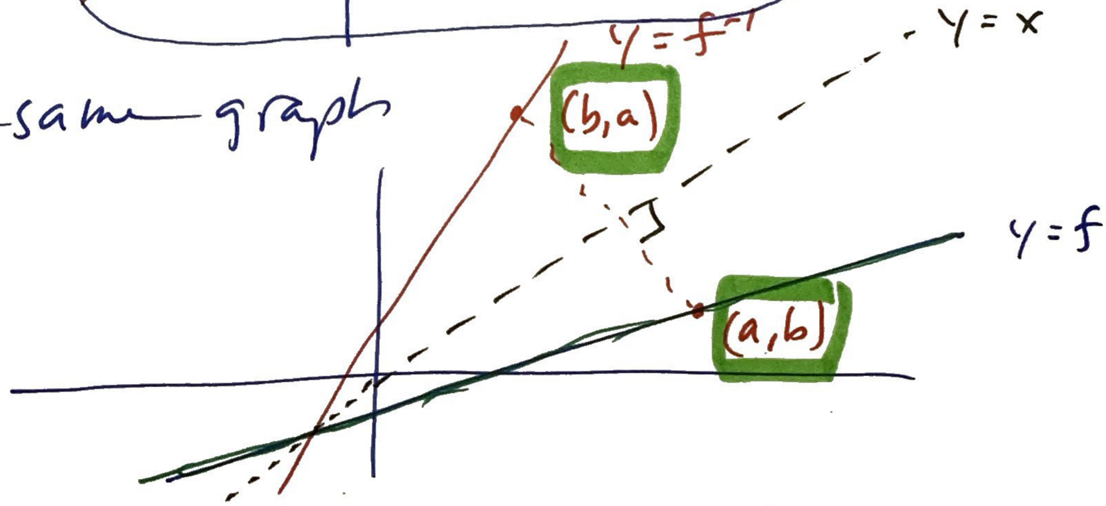
if $c=a$



$b \neq d$ OR the funct. is not 1-1



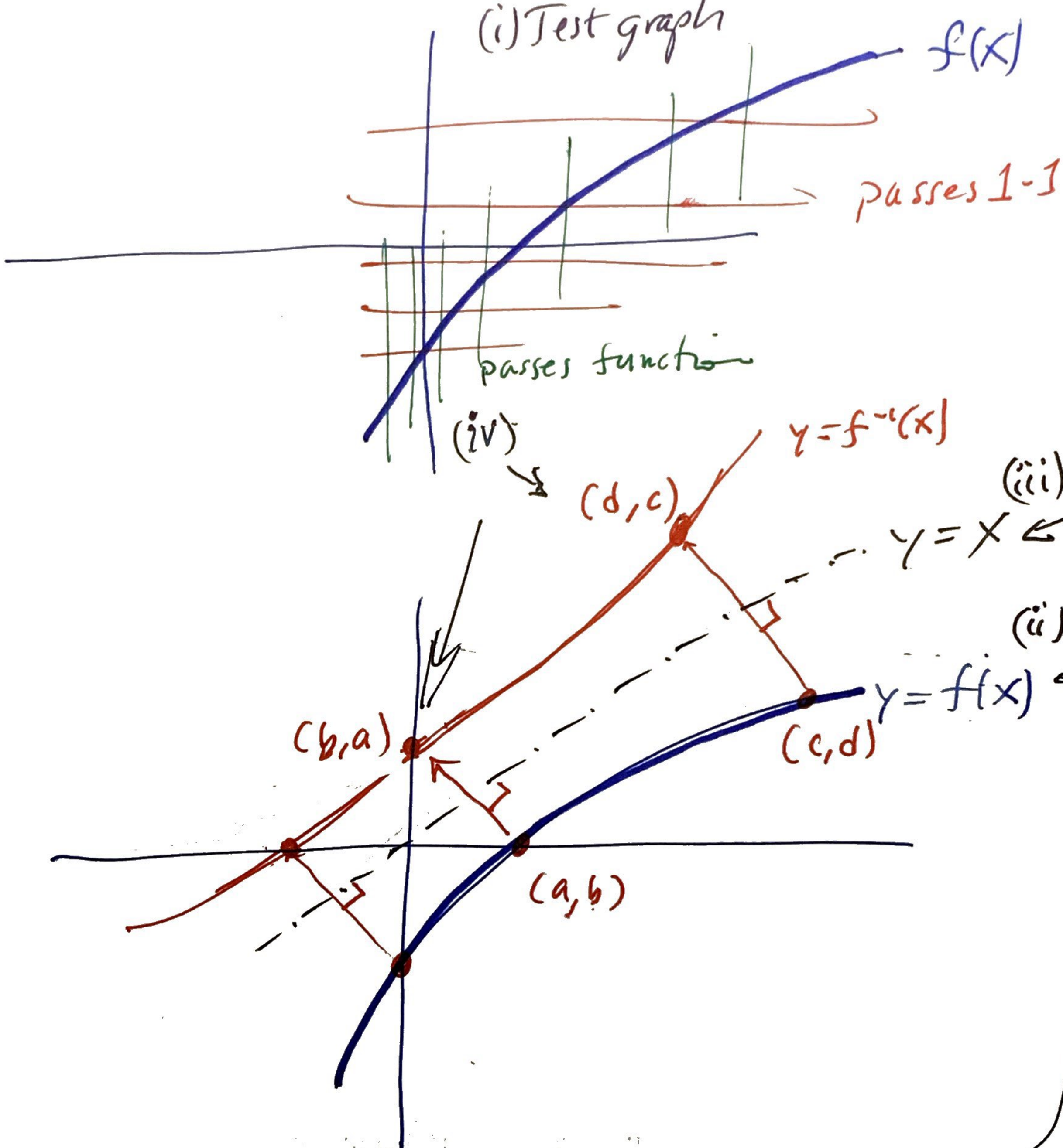
• on the same graph



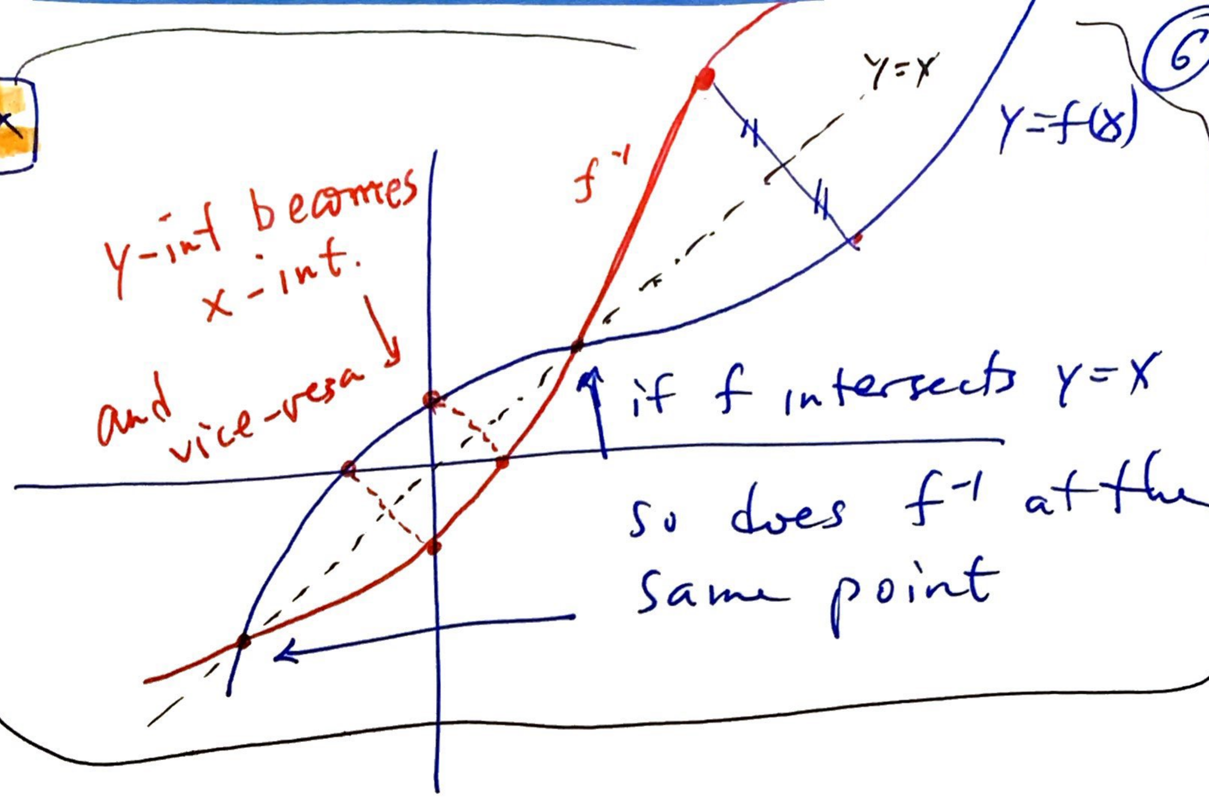
about the line y=

swap x

Ex Sketch the inv. function if the function has a graph shown



EX


y=x
y=f(x)
f^{-1}
y-int becomes x-int.
and vice-versa
if f intersects y=x
so does f^{-1} at the same point

⊛ Analytical functions

procedures for finding the inverse function given a 1-1 function $f(x)$

Steps

(a) Given $f(x)$ = relationship in "x" replace $f(x)$ with "y"

(b) Swap "x" $\leftrightarrow$ "y" {basically reflecting about the line $x=y$}

(c) Solve for "y"

(d) Replace "y" with $f^{-1}(x)$

Ex let $\boxed{f(x) = 2x}$

(a) $y = 2x$

(b) $x = 2y$

(c) $\frac{x}{2} = y \rightarrow y = x/2$

(d) $\boxed{f^{-1}(x) = \frac{1}{2}x}$

- Test : Use $\boxed{(f \circ f^{-1})(x) = x \text{ or } (f^{-1} \circ f)(x) = x}$

$f(f^{-1}(x)) \stackrel{?}{=} x$	$f^{-1}(f(x)) \stackrel{?}{=} x$
$f(\frac{1}{2}x) \stackrel{?}{=} x$	$f^{-1}(2x) \stackrel{?}{=} x$
$2(\frac{1}{2}x) \stackrel{?}{=} x$ ✓	$\frac{1}{2}(2x) \stackrel{?}{=} x$ ✓

Application!

Solve for x: $\quad 2x = 17$

let $f(x) = 2x$

$$f(x) = 17$$

- apply the inverse function $f^{-1}(x) = \frac{x}{2}$

take the inverse of both sides...

$$\Rightarrow f^{-1}(f(x)) = f^{-1}(17)$$

$$\boxed{x = \frac{(17)}{2}}$$

Ex Find $f^{-1}(x)$ if $f(x) = x^2 + 4$,

(a) $y = x^2 + 4$

(b) $x = y^2 + 4$

(c) $x - 4 = y^2$

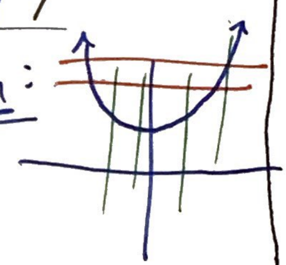
$y = \pm\sqrt{x-4}$

(d) $f^{-1}(x) = \pm\sqrt{x-4}$

* not a function...

There are two outputs for each input

- graph:



- Not a 1-1 function
- So an inverse function is not defined

restrict domain

Ex Find $f^{-1}(x)$ if $f(x)=x^2+4$ for $x \geq 0$

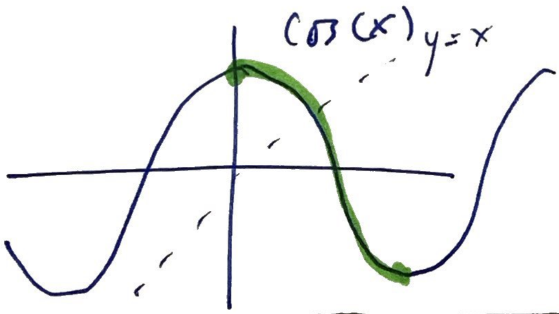
(a) $y = x^2+4$, $x \geq 0$

(b) $x = y^2+4$

(c) $y = \pm\sqrt{x-4}$

(d) $\boxed{f^{-1}(x) = \sqrt{x-4}}$ $x \geq 4$

* Trig functions are periodic so we need to apply domain constraints



Principle Cosine function

$\cos(x)$ $x \in [0, \pi]$

is 1-1

Ex Find $f^{-1}(x)$ if $f(x) = \dfrac{x-2}{x+4}$ $x \neq -4$

(a) $y = \dfrac{x-2}{x+4}$

(b) $x = \dfrac{y-2}{y+4}$

(c) $x(y+4) = y-2$

$xy+4x-y=-2$

$y(x-1) = -2-4x$

$y = -\dfrac{4x+2}{x-1}$

$$\boxed{f^{-1}(x) = -\frac{4x+2}{x-1}}$$

EX if $f(x) = \dfrac{1}{x-1}$ find $f^{-1}(x)$

(a) $y = \dfrac{1}{x-1}$

(b) $x = \dfrac{1}{y-1}$

(c) $x(y-1) = 1$

$xy - x = 1$

$xy = 1 + x$

$y = \dfrac{1+x}{x}$

(d) $\boxed{f^{-1}(x) = \dfrac{x+1}{x}}$

The reciprocal function

EX Find f^{-1} if $\boxed{f(x) = \dfrac{1}{x}}$, $x \neq 0$

(a) $y = \dfrac{1}{x}$

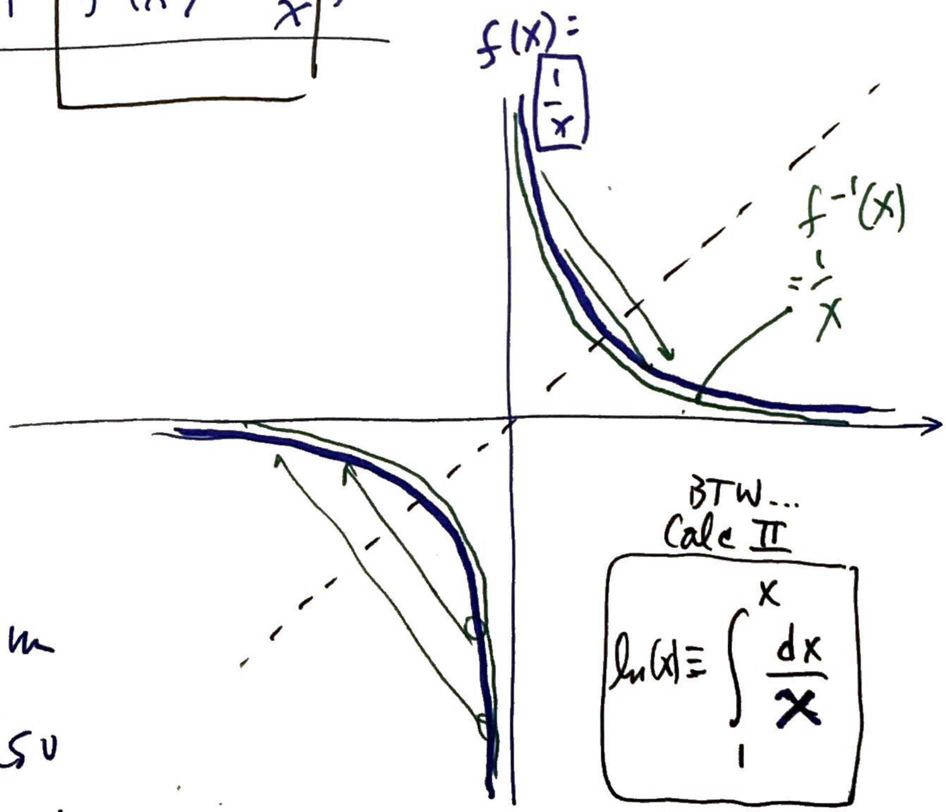
(b) $x = \dfrac{1}{y}$

(c) $xy = 1$

$y = \dfrac{1}{x}$

(d) $\boxed{f^{-1}(x) = \dfrac{1}{x}}$ Hm

also $f(x)$



BTW... Calc II

$$\boxed{\ln(x) \equiv \int_1^x \frac{dx}{x}}$$

⊛ A table of functions and their inverses (11)

• Function / Inv. function pair → The inv. of an inv. function is the funct

function	Inverse function
$f(x) = x$	$f^{-1}(x) = x$
$f(x) = \frac{1}{x}$	$f^{-1}(x) = \frac{1}{x}$
$f(x) = \frac{x}{2}$ A function inv-funct. pair	$f^{-1}(x) = 2x$
$f(x) = \sqrt{x}$	$f^{-1}(x) = x^2$
$f(x) = x^2$ "	$f^{-1}(x) = \sqrt{x}$
$f(x) = \cos(x)$	$f^{-1}(x) = \cos^{-1}(x)$
$f(x) = e^x$	$f(x) = \ln(x)$
$f(x) = \ln(x)$	$f^{-1}(x) = e^x$
$f(x) = \frac{2x}{x-4}$	$f^{-1}(x) = \frac{4x}{x-2}$
⋮	⋮

EX Solve for x : { jumping ahead to chpt 4 }

$$e^{3x+1} = 10$$

apply the inv. funct. of $e^x \to \ln(x)$

$$\underbrace{\ln\left(e^{3x+1}\right)} = \ln(10)$$

why $f^{-1}(f(x)) = x$

$$3x+1 = \ln(10)$$

Solve for x

$$\boxed{x = \frac{\ln(10)-1}{3}}$$

10 $\boxed{\ln}$ $\boxed{-}$ 1 $\boxed{=}$ $\boxed{\div}$ 3 $\boxed{=}$