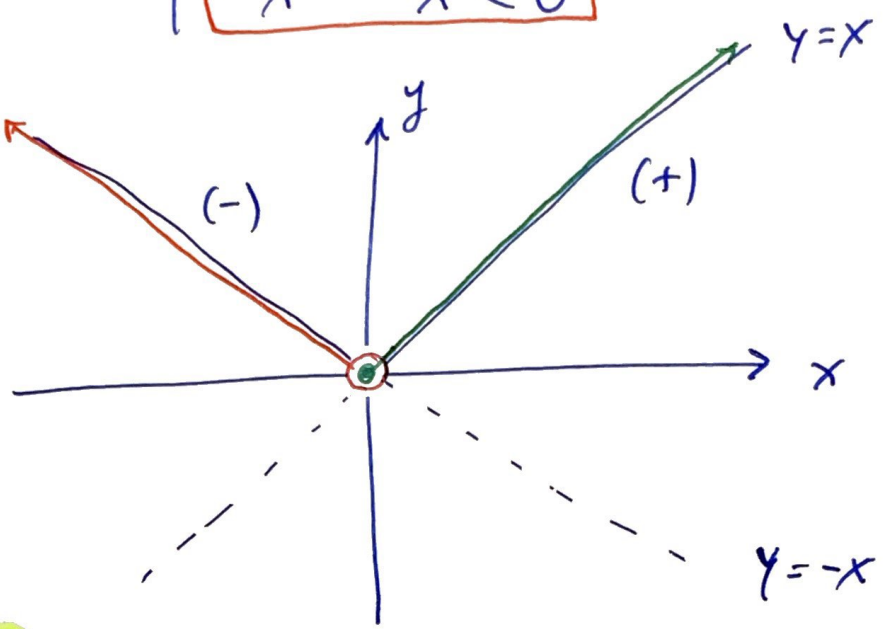
1.6 Absolute Value Functions

Def: $f(x) = |x|$ is really a piecewise functi

$$f(x) = \begin{cases} +x & x \geq 0 \\ -x & x < 0 \end{cases}$$

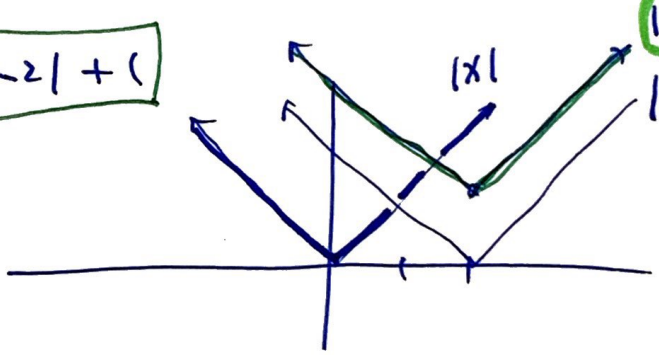


⊛ Graphing

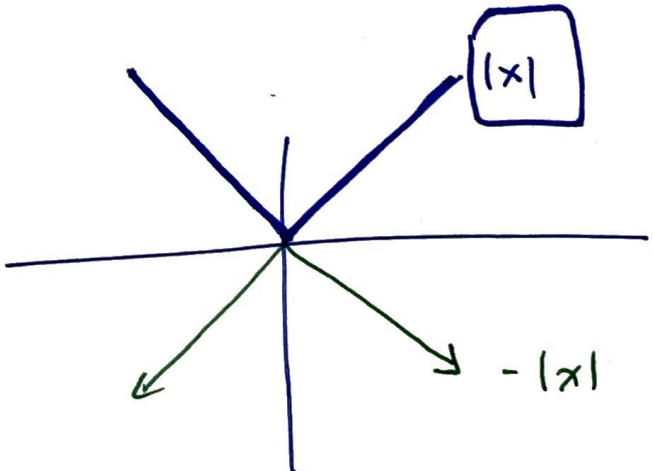
- shifts

$|x-2|+1$

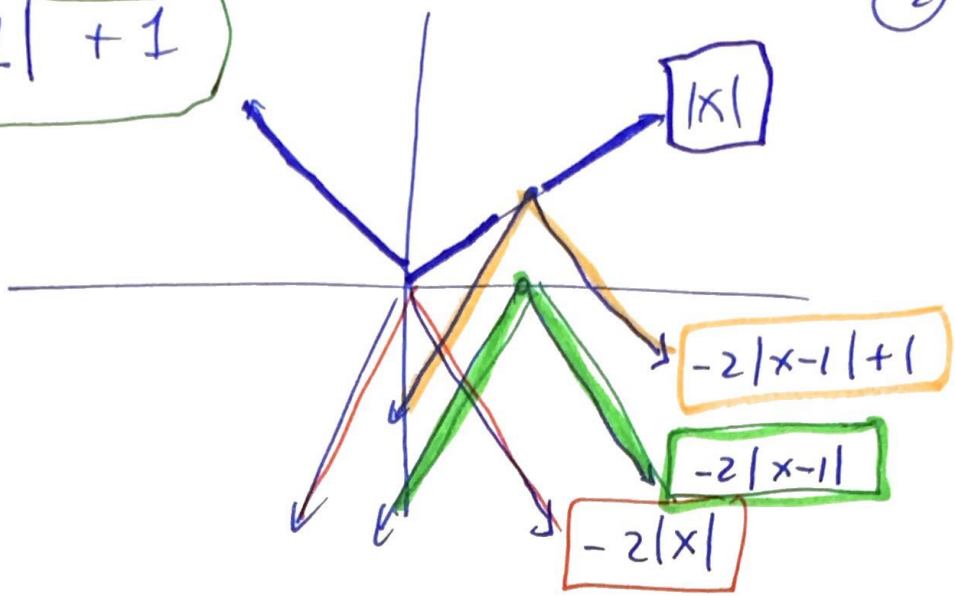
$|x-2|+1$



- flip $-|x|$



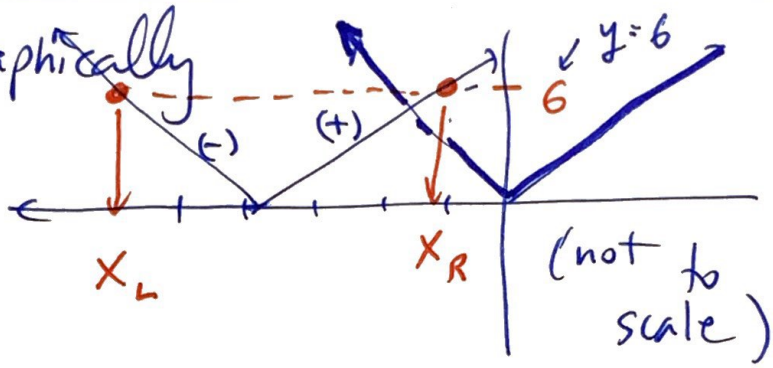
- $-2|x-1|+1$



✱ Solve eqns

EX Solve for x if $|x+4|=6$

- graphically



Find X_R & X_L (not to scale)

- analytical solution: Consider the piecewise definition

$|x+4|=6$

(−) Branch

$-(x+4)=6$

$x+4=-6$

$x=-6-4$

$\boxed{x=-10}$

−OR− $\boxed{X_L=-10}$

(+) Branch

$+(x+4)=6$

$x+4=6$

$x=6-4$

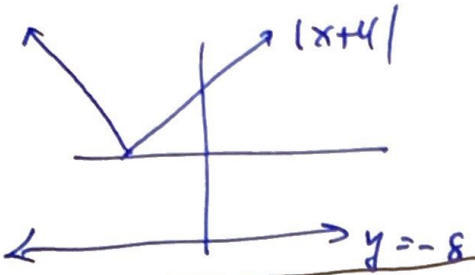
$\boxed{x=2}$

$\boxed{X_R=2}$

⊛ Trick eqn:

Ex Solve $|x+4| = -8$

always (+) — but this is (−) !!

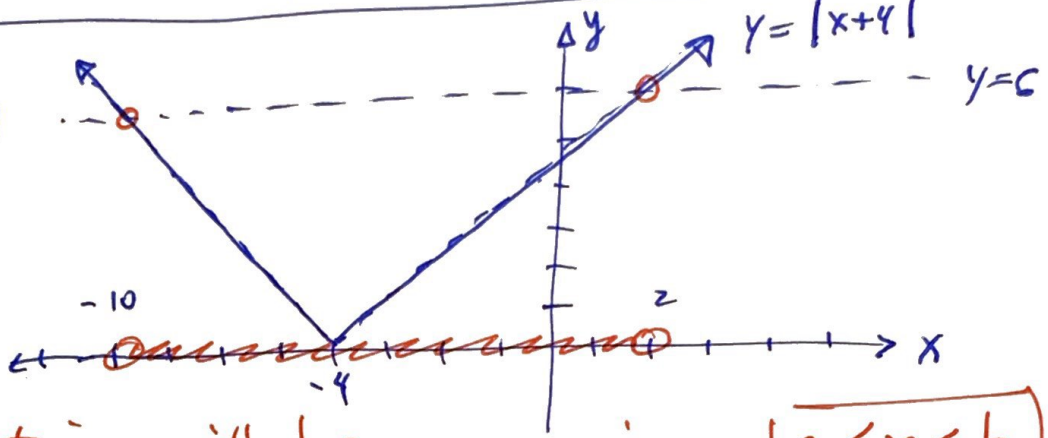


No Solution

⊛ Solving inequalities

Ex Solve for x $|x+4| \leq 6$

- graphically



The solution will be a region : $a < x < b$

- analytically

$|x+4| \leq 6$

(−) $-(x+4) \leq 6$

$x+4 \geqslant -6$

$x \geqslant -6-4$

$\boxed{x \geqslant -10}$

(+) $+(x+4) \leq 6$

$x+4 \leqslant 6$

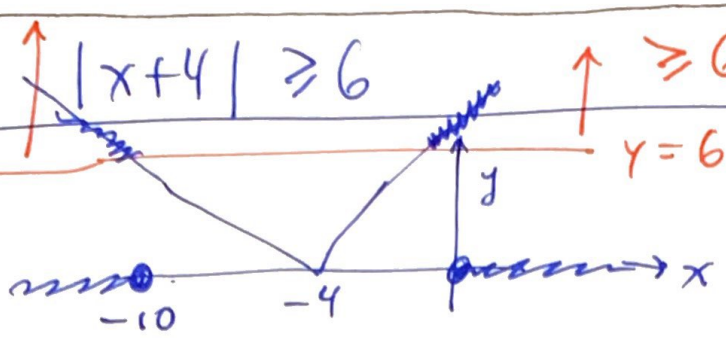
$\boxed{x \leqslant 2}$

ans: $\{x \mid x \in [-10, 2]\}$ ← set builder form

intvl: $-10 \leq x \leq 2$

Ex Solve $|x+4| \geq 6$

- graphically speaking



- analytically

$|x+4| \geq 6$

(−) $-(x+4) \geq 6$

$x+4 \leq -6$

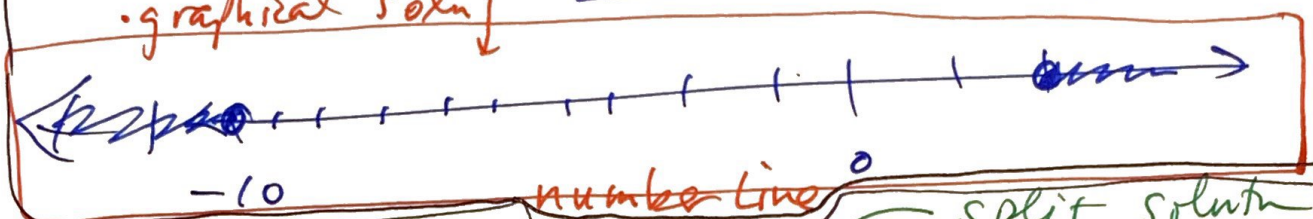
$\boxed{x \leq -10}$

(+) $+(x+4) \geq 6$

$x+4 \geq 6$

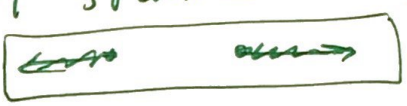
$\boxed{x \geq 2}$

- graphical soln



number line

Ex Solving $|3x-5| \geq 13$ — split soln



(−) $-(3x-5) \geq 13$

$3x-5 \leq -13$

$3x \leq -8$

$\boxed{x \leq -8/3}$

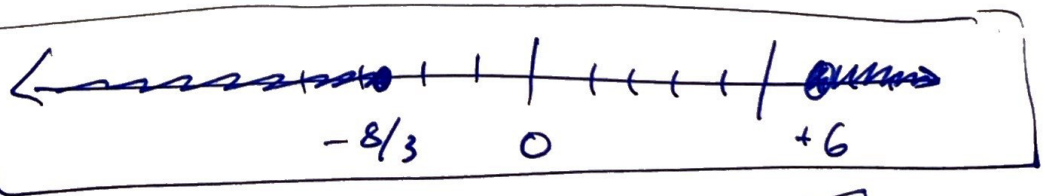
(+) $+(3x-5) \geq 13$

$3x \geq 18$

$\boxed{x \geq 6}$

- algeb.

- graph



- int'vl

$\boxed{(-\infty, -8/3) \cup (6, \infty)}$

Ex Solve $2|v-7|-4 \geq 42$

(i) isolate the abs. value 1st.

$$2|v-7| \geq 46$$
$$|v-7| \geq 23$$

(ii)

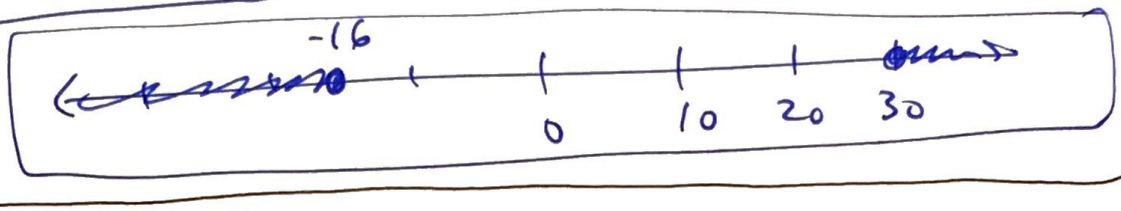
(−) $-(v-7) \geq 23$

$v-7 \leq -23$

$\boxed{v \leq -16}$

(+) $+(v-7) \geq 23$

$\boxed{v \geq 30}$



⊗ Finding Intercepts

Ex Find intercepts for $f = 2|x+3|+1$

- y-int: $x=0 \Rightarrow f(0) = 2|0+3|+1$

$f(0) = 7$ → y-int. $\boxed{(0,7)}$

- x-int: $y=0 \Rightarrow 0 = 2|x+3|+1$

(i) isolate the abs. value

$|x+3| = -1/2$

(ii) Solve this... No Solution → $\boxed{\text{No x-int.}}$

sketch