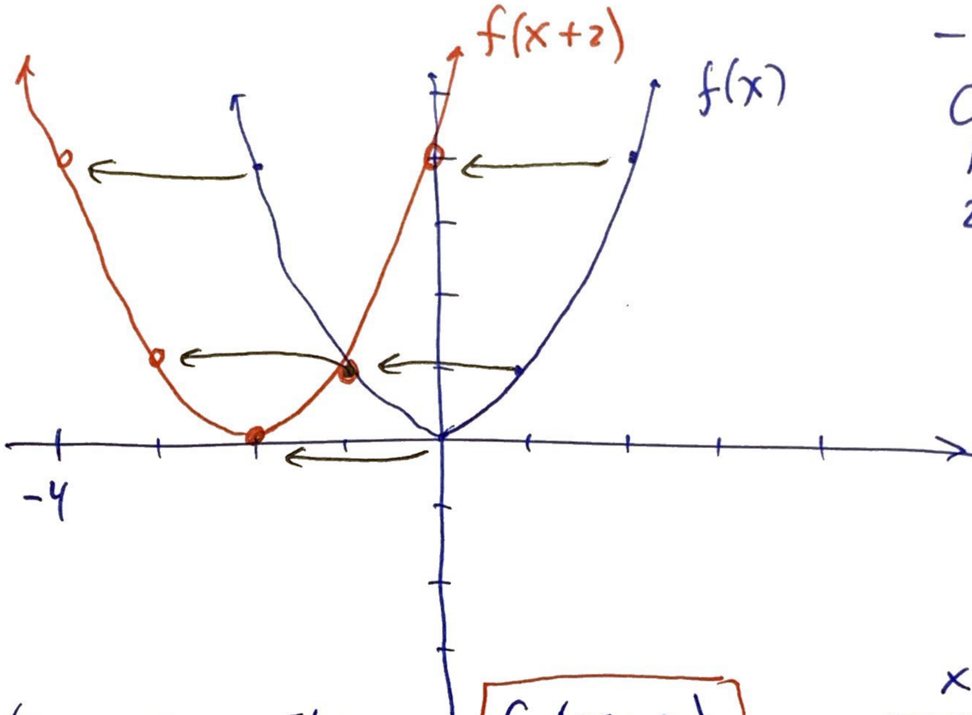
1.5 Function Transformations

In this section we start with a simple $f(x)$ and replace x with $x+a$ and study the effect on $f(x)$'s graph.

Horizontal Transformation

Consider $f(x) = x^2$ graph

x	$y = x^2$
-2	$(-2)^2 = 4$
-1	$(-1)^2 = 1$
0	$(0)^2 = 0$
1	$(1)^2 = 1$
2	$(2)^2 = 4$



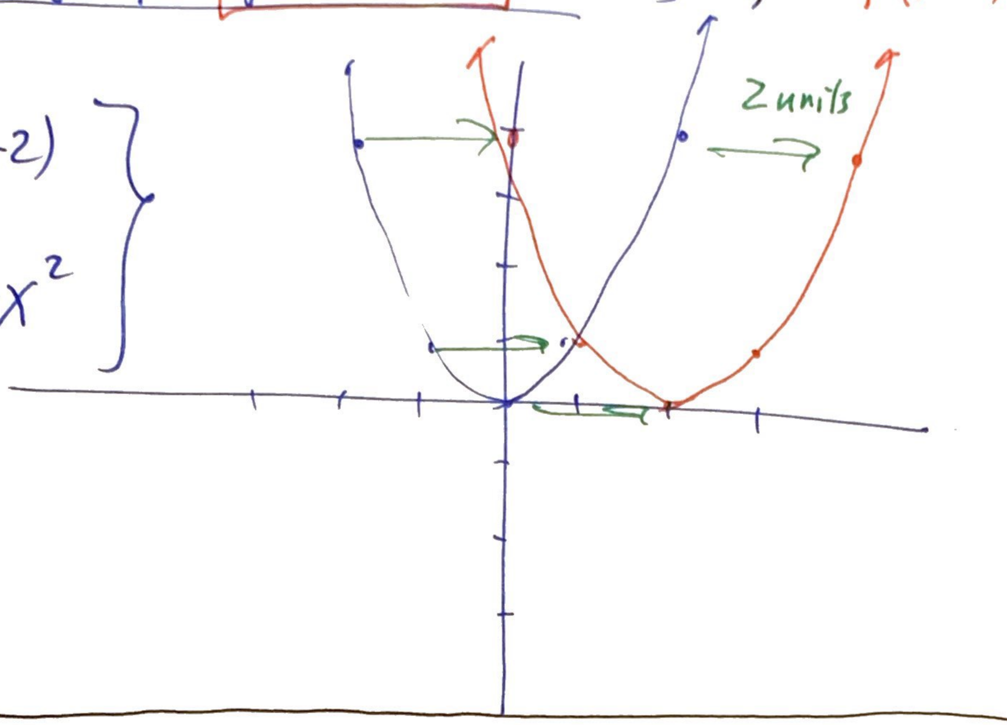
Now conside $\boxed{f(x+2)}$
$= (x+2)^2$

x	$\boxed{y = (x+2)^2}$
-2	$y = (-2+2)^2 = 0$
-1	$y = (-1+2)^2 = 1$
0	$y = (0+2)^2 = 4$
1	$y = (1+2)^2 = 9$
2	$y = (2+2)^2 = 16$
-3	$y = (-3+2)^2 = 1$
-4	$y = (-4+2)^2 = 4$

The results is that the graph of $f(x+a)$ moves the graph $f(x)$ back "a" units

EX graph $\boxed{g=(x-2)^2}$.

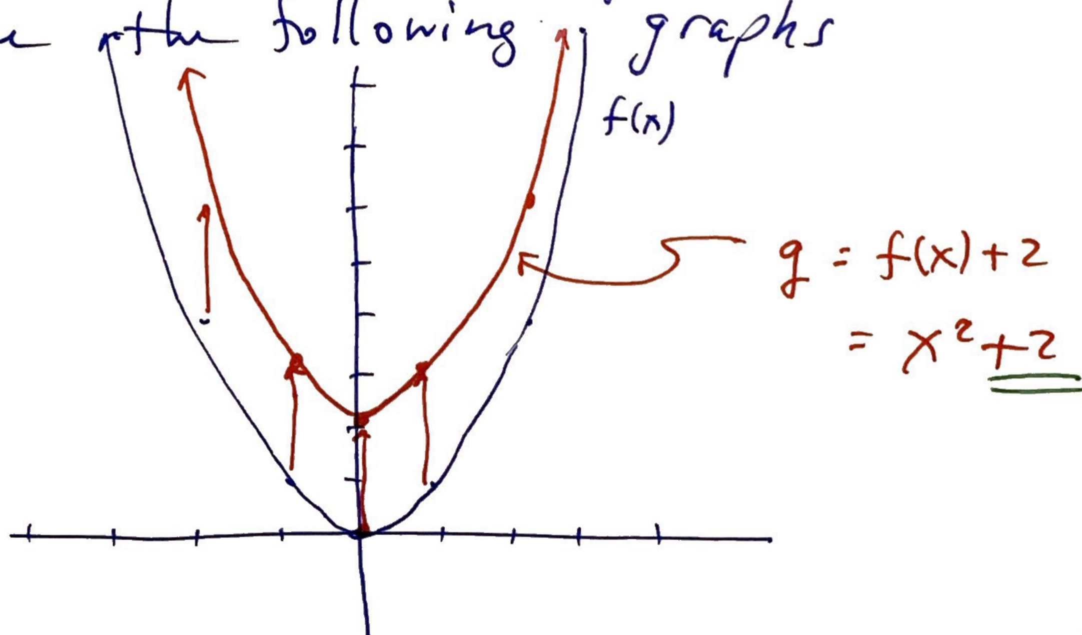
- Note $g=f(x-2)$ where $f(x)=x^2$



vertical transformation

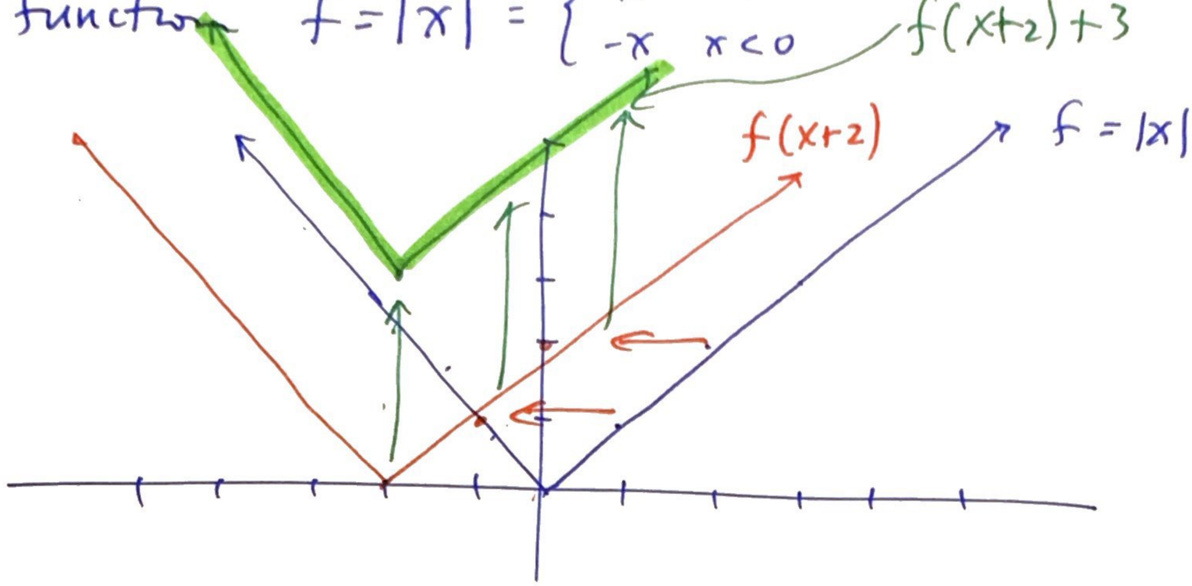
Here we not that the graph of $g=f(x)+c$ is the graph of $f(x)$ bosted up "c" units

EX let $f(x)=x^2$ then if $\boxed{g(x)=f(x)+2}$ we have the following graphs



Together: EX Consider and sketch $\boxed{y = |x+2| + 3}$

core function $f = |x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$



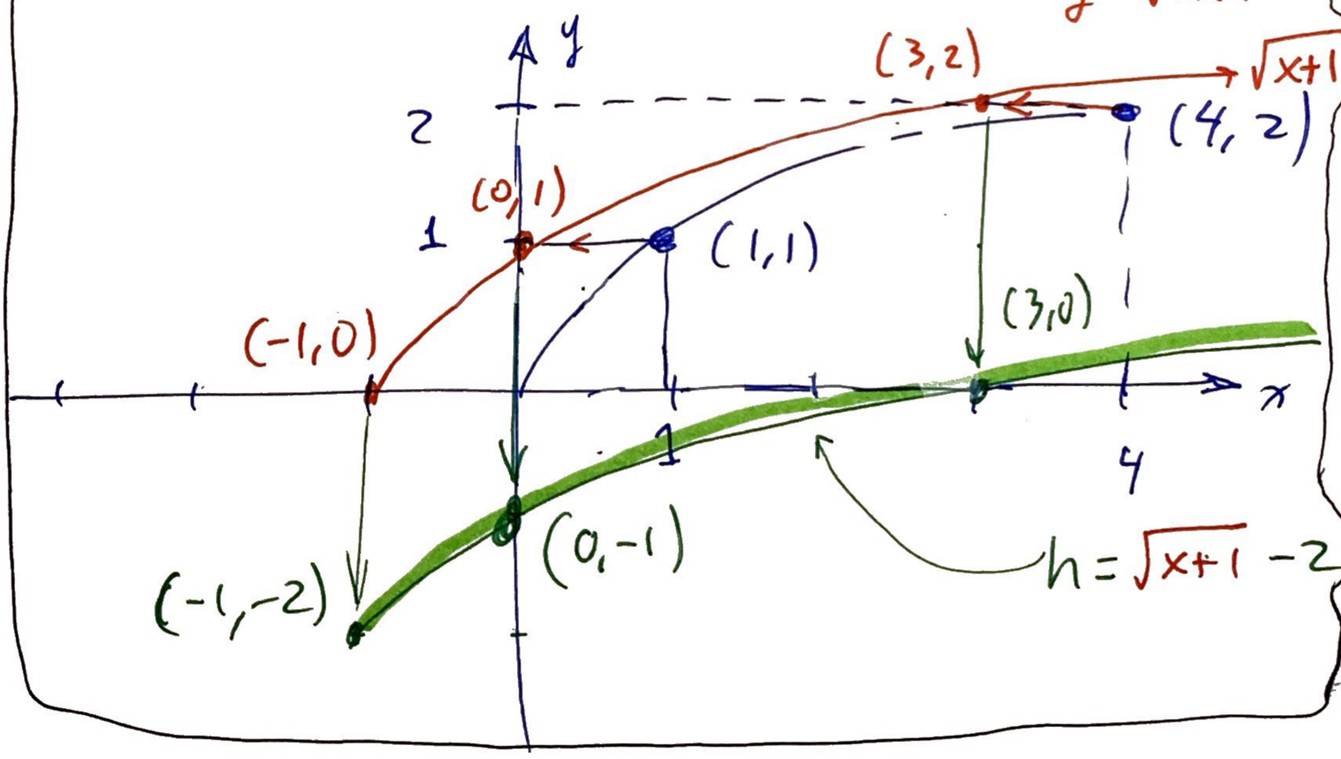
Next let $g = \boxed{f(x+2)}$

then let $h = \boxed{g(x+2) + 3}$

EX sketch $\boxed{y = \sqrt{x+1} - 2}$

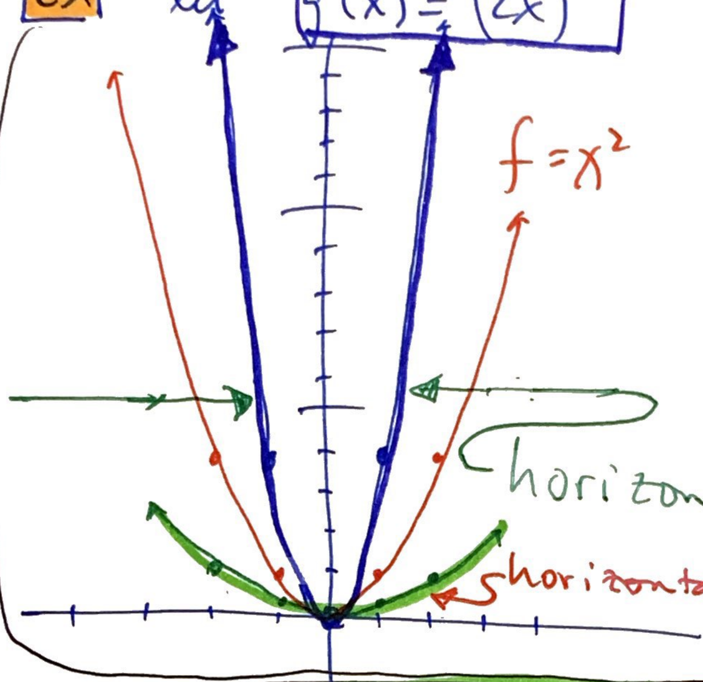
$f = \sqrt{x}$

$g = \sqrt{x+1}$



The graph of $f(ax)$ is the graph of $f(x)$ but stretched out horizontally if $a<1$ and squeezed in horizontally if $a>1$

Ex let $\boxed{g(x) = (2x)^2}$



x	$y=(2x)^2$	(x,y)
-2	$y=(2(-2))^2=16$	$(-2,16)$
-1	$y=(2(-1))^2=4$	$(-1,4)$
0	$y=(2\cdot 0)^2=0$	$(0,0)$
1	$y=(2\cdot 1)^2=4$	$(1,4)$
2	$y=(2\cdot 2)^2=16$	$(2,16)$

$x \to 2x$

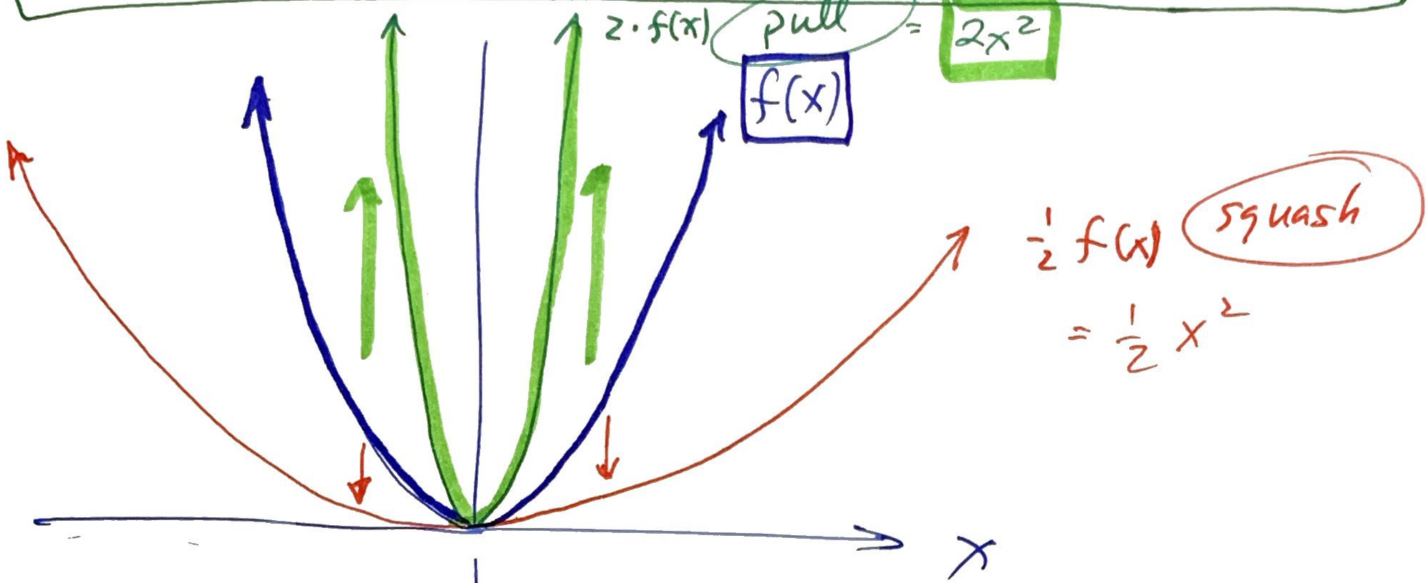
Ex let $\boxed{h(x) = \left(\frac{1}{2}x\right)^2}$

graph is above

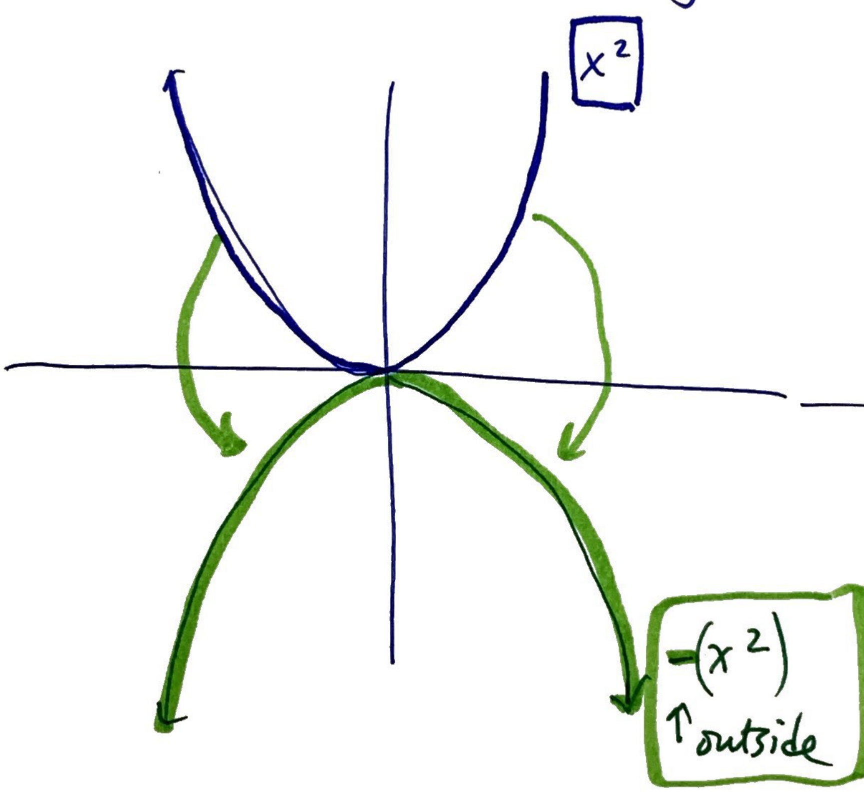
x	$y=\left(\frac{x}{2}\right)^2$	(x,y)
-2	$\left(-\frac{2}{2}\right)^2=1$	$(-2,1)$
-1	$\left(-\frac{1}{2}\right)^2=\frac{1}{4}$	$(-1,\frac{1}{4})$
0	$\left(\frac{0}{2}\right)^2=0$	$(0,0)$
1	$\left(\frac{1}{2}\right)^2=\frac{1}{4}$	$(1,\frac{1}{4})$
2	$\left(\frac{2}{2}\right)^2=1$	$(2,1)$

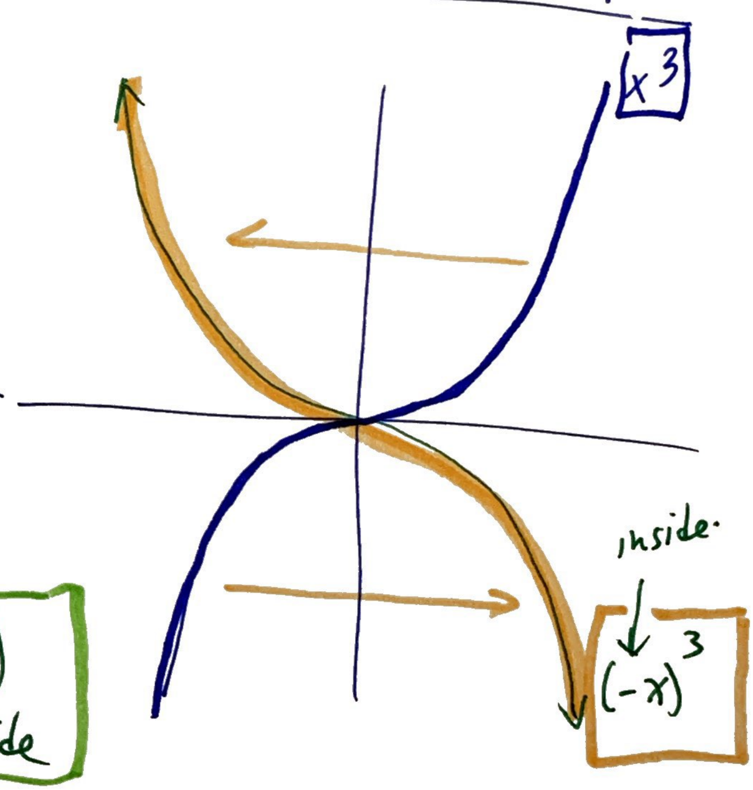
$x \to x/2$

If the graph of the function $f(x)$ is multiplied by "a" than the graph of $a \cdot f(x)$ is pulled vertically if $a > 1$ and is squashed vertically if $a < 1$



⊛ Flipping vertically, ⊛ horizontal flip



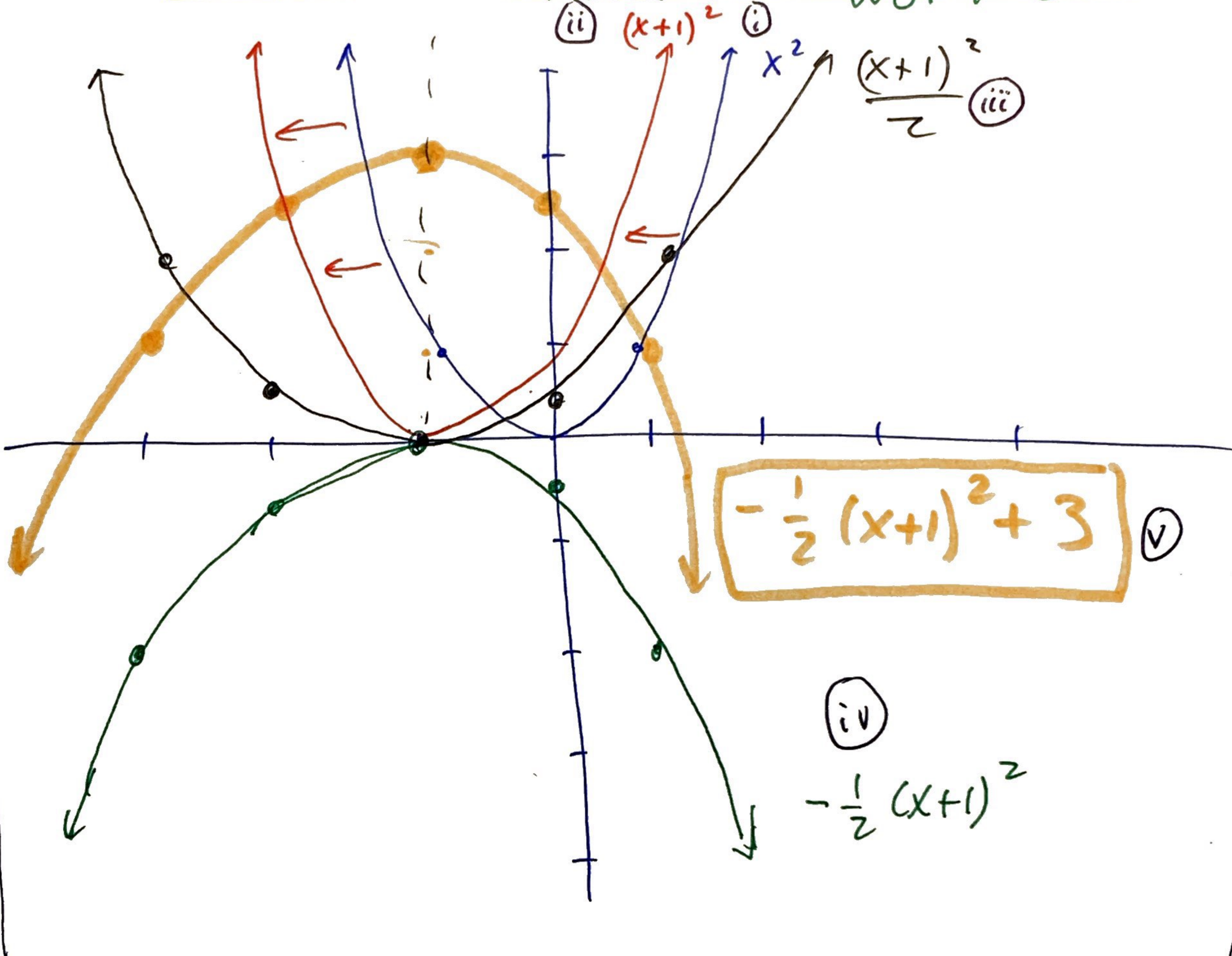


⊛ Perform multiple transformations

EX Sketch $f(x) = -\frac{1}{2}(x+1)^2 + 3$

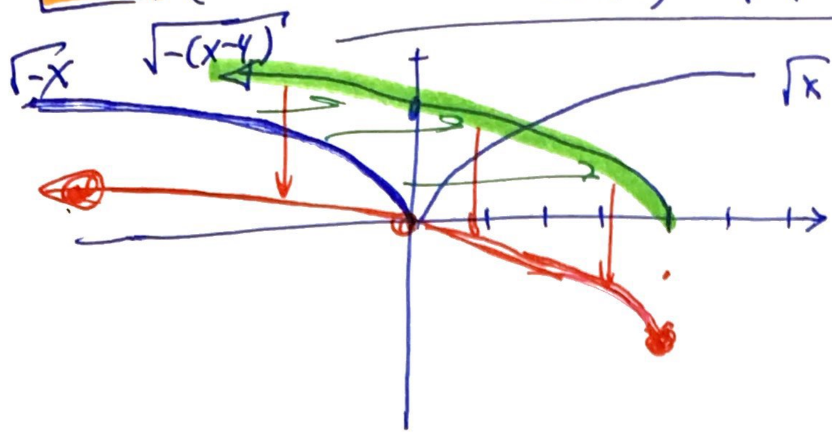
- Boost up 3 units
- shift left one unit
- squash ½ way
- flip vertically

* Start on the inside and work out.



Ex (cont.)

$a(x) = \sqrt{4-x} - 2$



$a(x) = \sqrt{-(x-4)} - 2$

$\sqrt{x}$

Flip

$\boxed{\sqrt{-x}}$

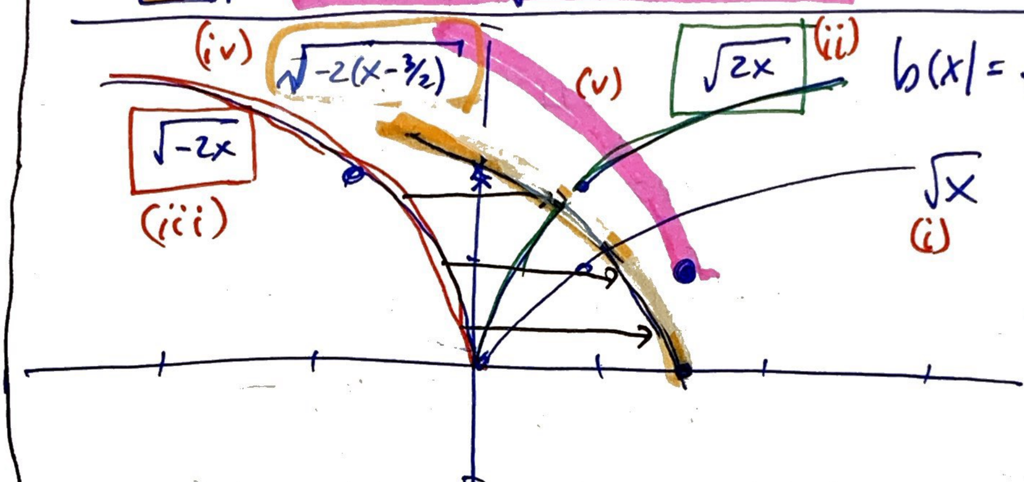
shift

$\boxed{\sqrt{-(x-4)}}$

$\boxed{a(x) = \sqrt{-(x-4)} - 2}$

Ex

sketch $\boxed{b(x) = \sqrt{3-2x} + 1}$



$b(x) = \sqrt{-2(x - 3/2)} + 1$

x is (+)

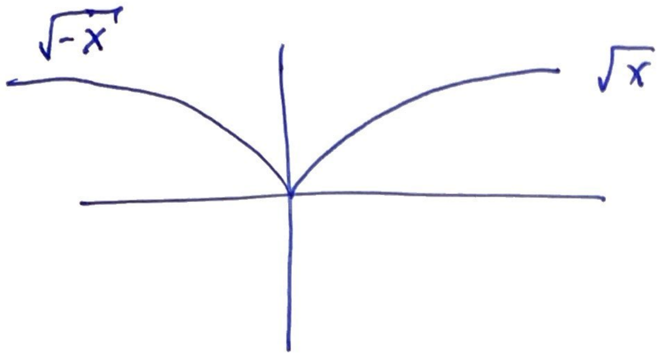
$\sqrt{}$

EX Sketch $a(x) = \sqrt{4-x} - 2$

(i) Isolate x: Choices...

$$\boxed{a(x) = \sqrt{-(x-4)} - 2}$$

↑ Isolate x so that is does not have neg. or a scaling factor



Rules: (0) flip vertically

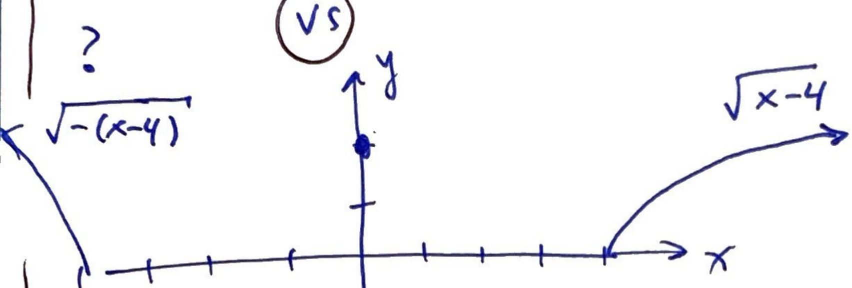
(i) squash or pull vertically

(ii) shift

(iii) stretch/~~squeez~~ horiz.

(iv) drop down or boost up.

vs



Resolve ⇒ Test points:

$x = 0$

$y = \sqrt{-(0-4)}$

$= \sqrt{4}$

$= 2$

$(0, 2)$

But this is not on the sketch

LHS Reflection

Not the Right reflection