

1.4

Function Composition

①

Frequently we place one function into another:

Ex

$$f(x) = 2x^2 + 1$$

$$g(x) = x + 3$$

Add'n: $f(x) + g(x)$

$$= (2x^2 + 1) + (x + 3)$$

$$= \boxed{2x^2 + x + 4}$$

Subt: $f(x) - g(x)$

$$= (2x^2 + 1) - (x + 3)$$

$$= \boxed{2x^2 - x - 2}$$

Mult: $f \cdot g$

$$= (2x^2 + 1)(x + 3)$$

$$= 2x^3 + x + 6x^2 + 3$$

$$= \boxed{2x^3 + 6x^2 + x + 3}$$

Div: f/g

$$= \frac{(2x^2 + 1)}{(x + 3)}$$

$$= \boxed{\frac{2x^2 + 1}{x + 3}}$$

Ex Cont.

(2)

$(f \circ g)(x)$ "f composed with g"

$$(f \circ g)(x) = f(g(x))$$

$$(f \circ g)(x) = 2[g(x)]^2 + 1$$

$$(f \circ g)(x) = 2[x+3]^2 + 1$$

$$= 2(x+3)^2 + 1$$

$$= 2[x^2+6x+9] + 1$$

$$f \circ g = [2x^2+12x+19]$$

$\boxed{\times} (g \circ f)(s)$

$$= g(f(s))$$

$$= [f(s)] + 3$$

$$= [2[s]^2+1] + 3$$

$$\begin{aligned} f[] &= 2[]^2 + 1 \\ g[] &= [] + 3 \end{aligned}$$

$$\begin{aligned} f[] &\xrightarrow{s} 2[]^2 + 1 \\ g[] &\xrightarrow{s} [] + 3 \end{aligned}$$

$$\Rightarrow (g \circ f)(s) = 2s^2 + 4$$

(3)

* Decomposition

Write a more complicated function as a composite of two simpler functions:

Ex

let $h(x) = \left(\frac{1}{2x-3}\right)^2$, decompose into simpler functions

let $\begin{cases} f[\] = \frac{1}{[\]^2} & \leftarrow \text{reciprocate} \\ g[\] = 2[\] - 3 \end{cases}$

then

$$h(x) = (f \circ g)(x)$$

$$= \frac{1}{[g(x)]^2}$$

$$= \frac{1}{(2x-3)^2}$$

$$(f \circ g)(x) = \frac{1}{4x^2 - 12x + 9}$$

Not unique

let $\begin{cases} f(\) = (\)^2 \\ g(x) = \frac{1}{2x-3} \end{cases}$

-OR-

$$f(\) = \frac{1}{(\)}$$

$$g(x) = (2x-3)^2$$

* Composite Tables of relationships ④

EX

let $g \circ f$ have the following tabular results

x	0	1	2	3	4	5	6	7	8	9
$f(x)$	7	6	5	8	4	0	2	1	9	3
$g(x)$	9	5	6	2	1	8	7	3	4	0

(a) Find $(f \circ g)(8)$

$$\begin{aligned}
 &= f(g(8)) \\
 &= f(4) \quad \text{start at the inside} \\
 &= \boxed{4}
 \end{aligned}$$

(b) Find $(g \circ f)(5)$

$$\begin{aligned}
 &= g(f(5)) \\
 &= g(0) \\
 &= \boxed{9}
 \end{aligned}$$

(c) $(f \circ f)(1)$

$$= f(f(1))$$

$$= f(6)$$

$$= \boxed{2}$$