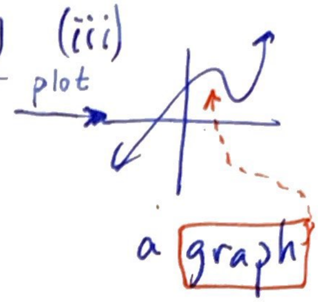
1.3 Rates of Change

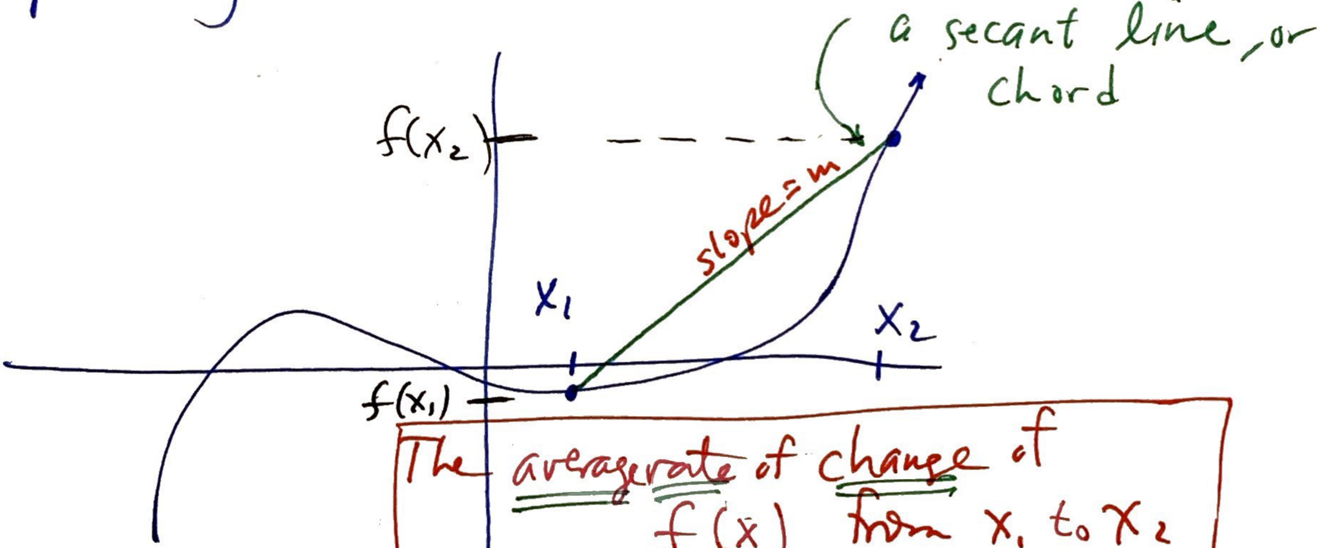
- When we study a function it helps to graph the function:
  - (i) set $f(x)$ to y
  - (ii) build table

x	$y=f(x)$
⋮	⋮

(iii) plot



- Frequently we desire to know how a function changes — particularly the rate at which a function changes.
- For Lines the rate of change is its slope, m
- For curves the rate of change is either the average rate of change or the instantaneous rate of change.
- Graphically



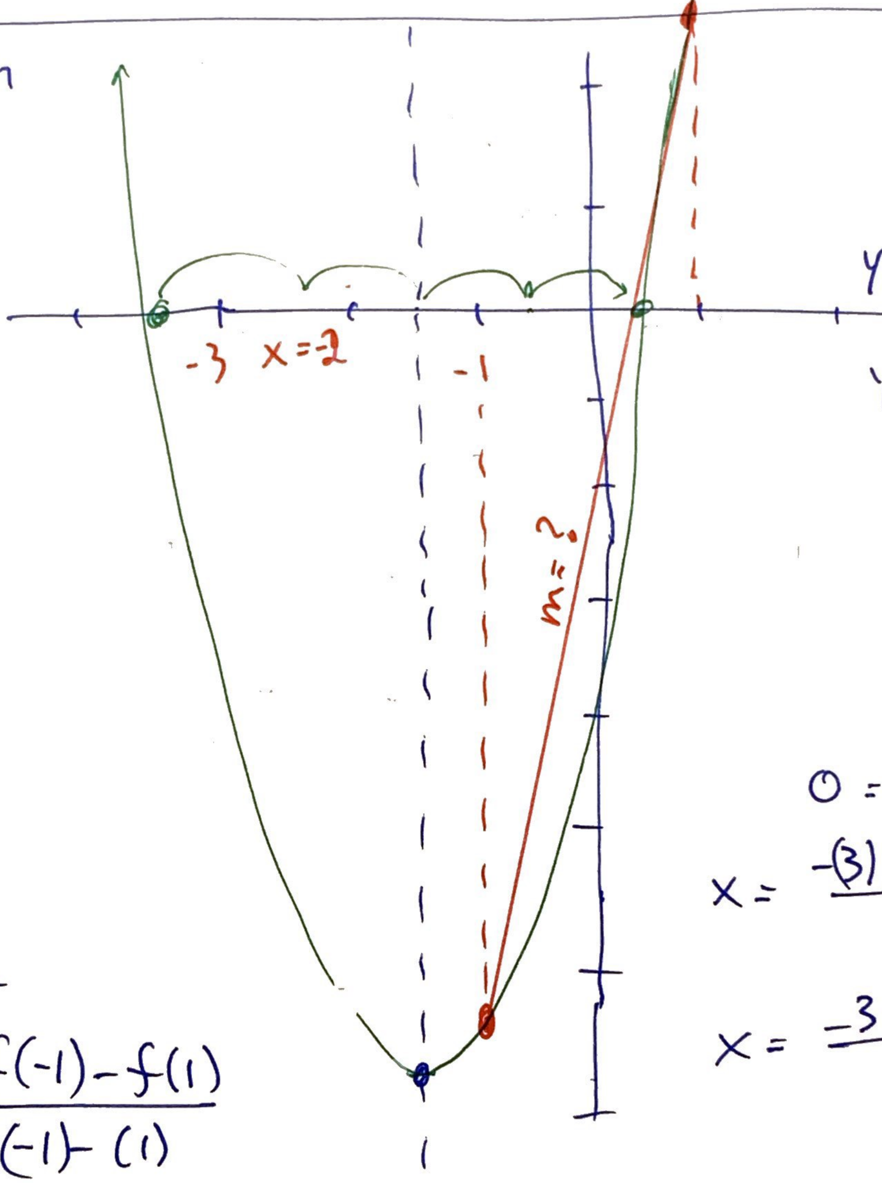
The average rate of change of $f(x)$ from x_1 to x_2

$$\boxed{m = \frac{f(x_2)-f(x_1)}{x_2-x_1}} \rightarrow \frac{\Delta y}{\Delta x}$$

EX let $f(x) = x^2 + 3x - 2$

Find the average rate of change of $f(x)$ from $x = -1$ to $x = 1$

• graph



• LoS $x = \frac{-3}{2(1)} = \frac{-3}{2}$ $\quad \frac{-b}{2a}$

• vertex

$y = \left(-\frac{3}{2}\right)^2 + 3\left(-\frac{3}{2}\right) - 2$

$y = \frac{9}{4} - \frac{9}{2} - 2$

$= \frac{9}{4} - \frac{18}{4} - \frac{8}{4}$

$= -\frac{27}{4} \approx -6\frac{3}{4}$

• roots:

$0 = x^2 + 3x - 2$

$x = \frac{-(3) \pm \sqrt{(3)^2 - 4(1)(-2)}}{2(1)}$

$x = \frac{-3 \pm \sqrt{17}}{2} = -\frac{3}{2} \pm \frac{\sqrt{17}}{2}$

• formula

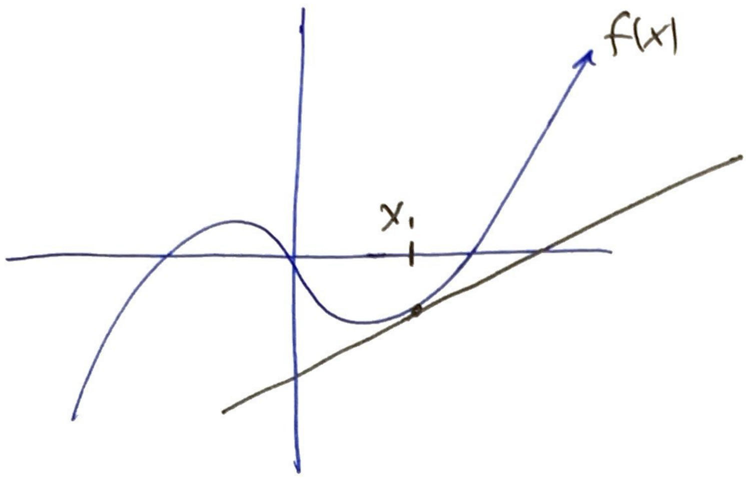
$\frac{\Delta f}{\Delta x} = \frac{f(-1) - f(1)}{(-1) - (1)}$

$= \frac{[(-1)^2 + 3(-1) - 2] - [(1)^2 + 3(1) - 2]}{-2}$

$= \frac{[-4] - [2]}{-2}$

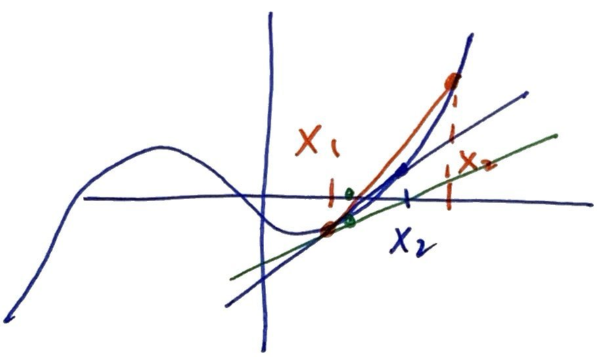
$= \boxed{3}$ average rate of change of f between $x = -1$ and 1

Instantaneous Rate of change



slope of the tangent line at x_1 is the instantaneous rate of change of $f(x)$

procedure: take a second point (to the right usually) and compute the ave range and then move that second point closer and closer to the 1st point



$\frac{\Delta f}{\Delta x}$ between x_2 & x_1

let x_2 approach x_1

the "limit" of $\frac{\Delta f}{\Delta x}$ as x_2 approaches x_1 is the instantaneous slope of $f(x)$ @ x_1

<u>Note:</u> $\Delta f = f(x_2) - f(x_1) \xrightarrow[\text{to}]{\text{goes}} 0$ as $x_2 \xrightarrow{\text{approaches}} x_1$

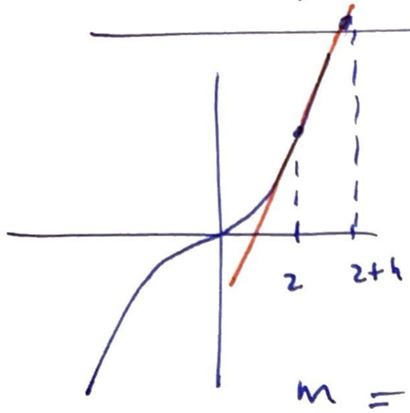
but $\Delta x = x_2 - x_1$ also $\longrightarrow 0$ as $x_2 \rightarrow x$

So $m = \text{"}\frac{0}{0}\text{"}$ yet the ratio remains a valid number

This process is called differentiation (Calc. I chpt 1)

Ex what is the ave. rate of change of

$r(t) = 4t^3$ from 2 to $2+h$?



$$m = \frac{f(2+h) - f(2)}{(2+h) - (2)} \leftarrow \frac{\Delta f}{\Delta x}$$

$$m = \frac{[4(2+h)^3] - [4(2)^3]}{h}$$

$$m = \frac{4(2+h)(2+h)(2+h) - 4 \cdot 8}{h}$$

or via Pascal's Δ

1

1 1

1 2 1

1 3 3 1

1 4 6 4

$$(2+h)^3 = 2^3 h^0 + 3 \cdot 2^2 \cdot h^1 + 3 \cdot 2^1 \cdot h^2 + 2^0 \cdot h^3$$

$$\boxed{(2+h)^3 = 2^3 + 3 \cdot 2^2 h + 3 \cdot 2h^2 + h^3}$$

$$= 8 + 12h + 6h^2 + h^3$$

$$m = \frac{4[8 + 12h + 6h^2 + h^3] - 32}{h}$$

$$m = \frac{\cancel{32} + 48h + 24h^2 + 4h^3 - \cancel{32}}{h}$$

$$\boxed{m = 48 + 24h + 4h^2}$$

if $h = 0.1$

$$m = 48 + 24(0.1) + 4(0.1)^2$$

$$= 48 + 2.4 + 0.04$$

$$= \underline{\underline{50.44}}$$

* pascals △

$$
\begin{array}{ccccccccccc}
 & & & & & 1 & & & & & \\
 & & & & 1 & & 1 & & & & \\
 & & & 1 & & 2 & & 1 & & & \\
 & & 1 & & 3 & & 3 & & 1 & & \\
 & 1 & & 4 & & 6 & & 4 & & 1 & \\
1 & & 5 & & 10 & & 10 & & 5 & & 1
\end{array}
$$

EX

So $(a+b)^5$

$= a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$