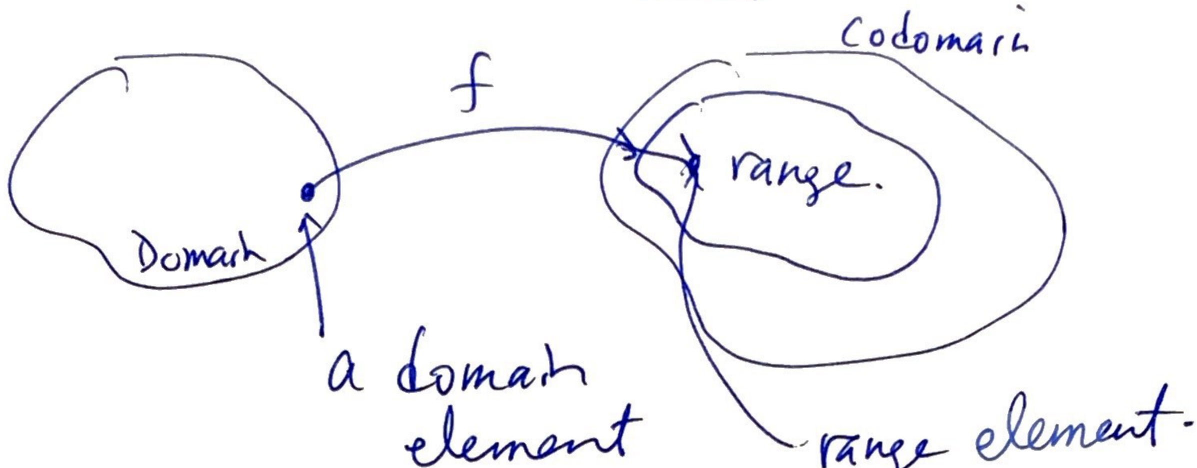
1.2 Domain and Range

For <u>analytical expressions</u> we call the set of all possible inputs the "Domain"

the set of all possible outputs from domain inputs is called the Range.



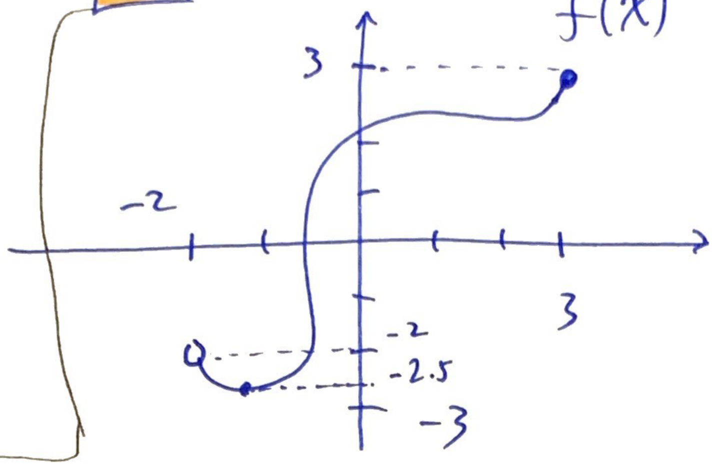
Ex — Tabular Relation

x	$R(x)$
-5	11
-4	7
-3	-6
-2	6
0	0
1	-7
2	11

- $D_{R(x)}: \{-5,-4,-3,-2,0,1,2\}$ inputs
- $R_{R(x)}: \{-7,-6,0,6,7,11\}$ output

Ex

⊛ Graph



all ex

open boundary

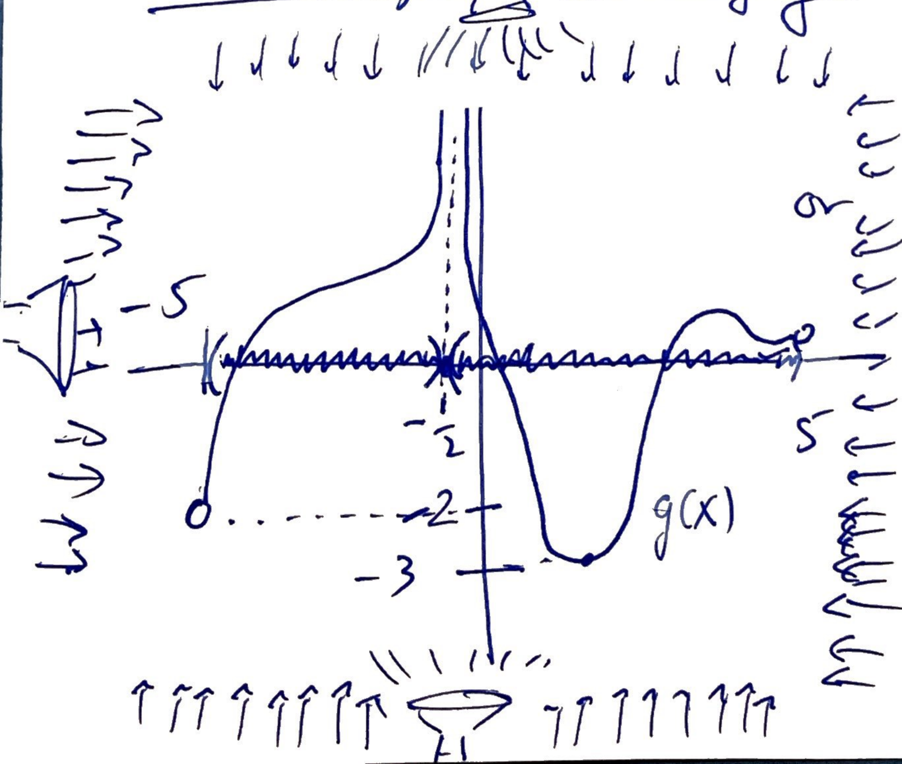
$$\boxed{D_f : \{x \mid -2 < x \leq 3\}}$$

such that

closed boundary

$$\boxed{R_f : \{y \mid -2.5 \leq y \leq 3\}}$$

• Flashlight analogy:



$$D_{g(x)} : \{x \mid x \in (-5, -\tfrac{1}{2}) \cup (-\tfrac{1}{2}, 5)\}$$

or $\{x \mid -5 < x < -\frac{1}{2} \text{ or } -\frac{1}{2} < x < 5\}$

or

$$\boxed{D_{g(x)} : \{x \mid x \in (-5, 5),\ x \neq -\tfrac{1}{2}\}}$$

$$\boxed{R_{g(x)} : \{y \mid -3 \leq y < \infty\}}$$

3

EX

State the domain and range of $f(x) = 3\sqrt{x-2}$

Domain: all valid inputs

$$\boxed{D_{f(x)} : \{x \mid x \geq 2\}}$$

Range: all possible outputs given a valid input

$$\boxed{R_{f(x)} : \{y \mid 0 \leq y\}}$$

EX

Find domain and Range

$$f(x) = \frac{(x^2-9x)/x^2}{(x^2-81)/x^2} = \frac{1-\cancel{\frac{9x}{x^2}}\to 0}{1-\cancel{81/x^2}\to 0} = 1 \quad \frac{17}{2}$$

- can't divide by 0 so $x \neq \pm 9$

$$\boxed{D_{f(x)} : \{x \mid x \neq \pm 9\}}$$

- Range: $\boxed{R_{f(x)} : \{y \mid y \neq 1\}}$ Future Knowlege

$$x^2 - 0x - 81 \overline{\big) x^2 - 9x + 0} \quad 1 + \frac{-9x+81}{x^2-81}$$

$$\frac{-(x^2-0x-81)}{-9x+81}$$

- Rational function

$$f(x) = \frac{x\cancel{(x-9)}}{(x+9)\cancel{(x-9)}} = \frac{x}{x+9} = 1 - \frac{9\cancel{(x-9)}}{(x+9)\cancel{(x-9)}}$$

↑ New function

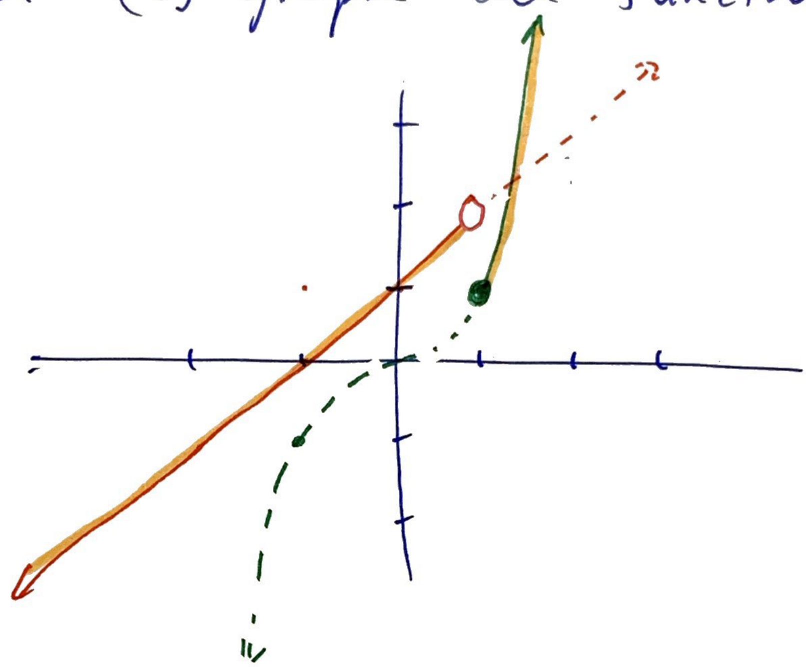
piecewise functions

we may want a function that behaves differently in different domain regions

Ex

$$f(x) = \begin{cases} x+1, & x < 1 \\ x^3, & x \geq 1 \end{cases}$$

- Graph (i) graph all functions



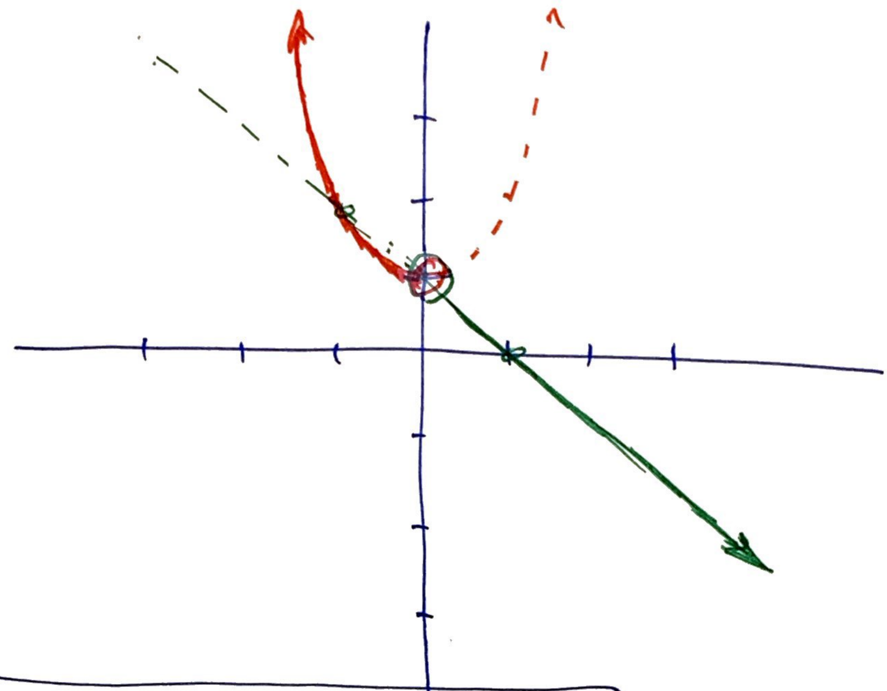
(ii) "white out" the parts of each function not in its region

- $D_f : \{x \mid x \in \mathbb{R}\}$

  $R_f : \{y \mid y \in \mathbb{R}\}$

EX Find the D_f & R_f for

$$f(x) = \begin{cases} \boxed{x^2+1} & x<0 \\ \boxed{1-x} & x>0 \end{cases}$$



$D_f : \{x \mid x \neq 0\}$

$R_f : \{y \mid y \neq 1\}$